A

COMPLETE EPITOME

OF

Practical Navigation,

AND

Nautical Astronomy,

CONTAINING

All Necessary Instructions for Keeping a Ship's Reckoning

AT SEA:

WITH THE MOST APPROVED METHODS OF ASCERTAINING THE

LATITUDE AND LONGITUDE,

AND EVERY REQUISITE TO FORM

THE COMPLETE NAVIGATOR;

THE WHOLE BEING RENDERED PERFECTLY EASY, AND ILLUSTRATED BY NUMEROUS EXAMPLES, DIAGRAMS AND CHARTS.

NEW EDITION.

Rearranged and Considerably Extended.

By J. W. SAUL.

LONDON:
IMRAY, LAURIE, NORIE AND WILSON
156 MINORIES.
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ORIGINAL PREFACE.

Having been for several years past engaged in the instruction of persons designed for, or belonging to the Sea, I have frequently had occasion to lament that most of the existing works on Practical Navigation, and particularly some that have been very generally circulated, are extremely erroneous, both in the instructive and tabular parts, and by no means calculated to answer all the purposes of the Mariner, Teacher, or Pupil.

With a view to remedy these defects, and to facilitate the acquirement of this most important art, and further stimulated by the flattering reception of my former labours, I have ventured to exert my best abilities in composing the present work; and, although I do not mean to arrogate to myself any superior professional merit, yet I humbly apprehend that my long experience and intimate connection with the subject have enabled me, in some measure, to form a competent judgment of what is most requisite to assist the industrious Mariner in acquiring a knowledge of the practical part of Navigation.

In order to accomplish my intended purpose as effectually as possible I have examined, with the greatest attention and caution, the various publications that have been written on Navigation, and placing them in a comparative point of view, have, I trust, been thence enabled to avoid the errors, and to improve the merits, of those who have preceded me in this branch of Science.

That nothing might be wanting to assist the student in his progress through the subject, I have commenced with a short treatise on Decimal Arithmetic, the nature of which he will find very necessary to be understood in going through the various computations that follow. In Geometry such definitions and problems only are introduced as appear most essential. Plane Trigonometry, both right and oblique-angled, being the foundation of the Sailings, is treated of at considerable length. Next follows Geography, containing a description of the form and magnitude of the Earth, with its various real and imaginary divisions, and an explanation of the nature of latitude and longitude: an account is then given of the Instruments used for measuring a Ship's way, with the manner of correcting their errors. This finishes the introductory part to Navigation.

We now proceed to the various Sailings in which the examples are resolved by construction, calculation, inspection and Gunter's Scales; then

follows a description of Charts, with the methods of using and constructing them. The art of Surveying Coasts and Harbours, being very essential to those who visit unknown parts, is treated in a manner which it is hoped will make its acquisition and practice perfectly easy.

We come next to the application of Astronomy to Navigation, and here I have thought proper to give a short but comprehensive view of the solar system, where the Earth is considered as a Planet; and have then described the various imaginary circles of the sphere. The nature of parallax, refraction, &c., are explained under this head. The Theory of the Winds and Tides, with the methods of finding the Time of High Water, follow next in order.

The most approved Methods of ascertaining the Latitude and Longitude at Sea by Celestial Observations, also the Variation of the Compass by Amplitudes and Azimuths, are explained by proper rules and examples: there is also given a particular description, with the uses of the various Astronomical Instruments employed in taking the observations. In this part of the work I have given Mr. Douwes' Rules for computing the Latitude by two Altitudes of the Sun, and four different methods of clearing the Distance, the last of which, invented by Captain Mendoza Rios, has the advantage of not requiring any distinction of cases. The method of finding the Longitude by a Time-keeper being now much practised, the necessary rules and examples are introduced for that purpose.

The learner is next led into the Rules for keeping a Journal at Sea, wherein are exhibited the Methods of correcting the Courses for Leeway, Variation, &c., with general Rules for working a Day's Work; and the whole is illustrated by separate days' works, and further by a Journal kept from England to Madeira.

The Explanation of Sea Terms and Technical Phrases, also the substance of an Examination of a Young Sea Officer in the working and of piloting a Ship, I have inserted because, although they may be considered by some as superfluous, and not immediately connected with the subject, they nevertheless appeared to be of too much importance to others to be omitted.

With respect to the Tables in this Work, I have only to observe here, that they are published under the title of "*Nautical Tables*," that they have been very generally adopted by the Officers in the Navy and in the Honourable East India Company's Service, and have received the approbation of Navigators in general.

J W. NORIE.

Note.—It may here be observed that Norie's *Complete Epitome of Practical Navigation* was originally dedicated *by permission* "to the Honourable the Court of Directors of the United Company of Merchants of England trading to the East Indies."

PREFACE TO THE NEW EDITION

THIS latest edition of this Epitome has been necessitated by a combination of circumstances, which need no description, but which are well known to students of Navigation.

The inclusion of so much "new work," together with the remodelling of a large portion of the old, afforded an opportunity of rearranging the book in a more systematic manner, having due regard to the order in which the various subjects fit into each other, and also to the order in which the young student of Navigation might, with advantage, pursue his studies.

It will be observed that those portions of the "Science of Navigation," which have always been regarded as *advanced* have been placed towards the end of the book, as it was felt that to place too many difficulties in the early part of the work often resulted in the complete neglect of the whole subject by the young student at sea, who is thrown entirely on his own resources when in pursuit of knowledge.

This work must be regarded as a "Standard Work on Navigation" rather than a book for cramming purposes, and the student who goes diligently through it will be amply rewarded by a thorough knowledge of the theory and practice of Navigation, which no system of cramming can achieve.

The following new sections have been inserted in this edition: (1) Spherical Trigonometry (Napier's Rules for the solution of Right-angled and Quadrantal Spherical Triangles); (2) Figure Drawing in Nautical Astronomy; (3) Marine Surveying; (4) Chart Construction; (5) Theory of the Station Pointer and its use; (6) Barometer, Thermometer, and Hydrometer; (7) Reduction to Soundings; (8) Meteorology; (9) Deviation of the Compass and Compass Adjustment; and lastly Proofs of certain Formulæ.

The following sections have been entirely remodelled and improved: (1) Logarithms; (2) Geometry; (3) Description and Use of Charts; (4) The Compass; (5) The Sextant; (6) Introduction to Nautical Astronomy; (7) Correction of Courses; (8) Day's Work; (9) Latitude by Reduction to the Meridian (Direct Method); (10) Computation of Altitudes; (11) Latitude by Double Altitudes (Direct Method); (12) Great Circle Sailing.

By the addition of the Natural Haversine Table to Norie's Nautical Tables, where it will be found adjacent to the Log. Haversine, it has been found possible to replace some very tedious and cumbersome calculations by equally sound but much shorter methods. This will be made obvious by an examination of the last method given in the Chronometer

Problem, and also in the Computation of Altitudes, and Latitude by Double Altitude problems, where its advantages are fully exemplified.

It is earnestly recommended that the young student of Navigation should study at an early stage the principles of Plane Trigonometry, and also the application of Napier's Rules for Circular Parts to the solution of right-angled spherical trigonometry.

The practical use of Napier's Rules for Circular Parts is fully demonstrated in the Ex-Meridian, Latitude by Double Altitudes of a Star, and Great Circle Sailing problems.

My thanks are due to Mr. W. R. Courtney for the very able manner in which he has assisted me in reading the proofs of this very extended Epitome.

J. W. SAUL.

London, July, 1917

CONTENTS

NAVIGATION.

INTRODUCTION.

NAVIGATION is the Art which instructs the mariner how to conduct a ship over the wide and trackless ocean, from one port to another, with the greatest safety, and in the shortest time possible.

Navigation may be divided into two branches: viz., *Seamanship*, comprehending the method of managing a vessel by disposing her sails, rudder, &c., so that she may move in any assigned course or direction the wind or weather will permit; and *Navigation Proper* (the part we intend to treat of in the present work), which comprehends those methods by which a mariner determines at any time the situation of his vessel, the course she is to be steered, and the distance she has to run, to gain her intended port: hence the requisites for a mariner, in order to understand this branch of the Nautical Art, are a competent knowledge of the figure of the earth, with the various imaginary circles drawn upon it, so as to be able to ascertain the distance and situation of places with respect to each other; the method of finding the ship's latitude and longitude, either by her course and distance run, or by astronomical observations; the use of various instruments, as the log, compass, log-glass, quadrant, sextant, chronometer, &c.; the different allowances necessary to be made in estimating a ship's way, as for leeway, the variation and deviation of the compass, and currents; the method of finding the time of high water at any place; the use of charts, with the method of constructing them: all of which particulars, depending upon mathematical and astronomical principles, we shall endeavour, in the following pages, to explain and illustrate in such a manner as to render every part as clear, concise, and methodical as possible.

TABLES USEFUL IN NAVIGATION AND NAUTICAL ASTRONOMY

MEASURE OF LENGTH

12 Inches	=	1 Foot.
3 Feet	=	1 Yard.
2 Yards, or 6 Feet	=	1 Fathom.
220 Yards	=	1 Furlong.
8 Furlongs, or 1760 Yards, or 5280 Feet	=	1 Statute Mile.
3 Miles	=	1 League.
6080 Feet	=	1 Nautical Mile.

SURVEYING AND CHART MEASURE

100 Fathoms, or 600 Feet	=	1 Cable.
10 Cables, or 6000 Feet	=	1 Mile.

MEASURE OF TIME

60 Seconds	=	1 Minute.
60 Minutes	=	1 Hour.
24 Hours	=	1 Day.
7 Days	=	1 Week.
28, 29, 30, or 31 Days	=	1 Calendar Month.
12 Calendar Months	=	1 Year.
365 Days	=	1 Common Year.
366 Days	=	1 Leap-Year.

1 Hour = 60 Minutes = 3600 Seconds.
24 Hours = 1440 Minutes = 86400 Seconds.

Number of Days in Each Month

January	31	May	31	September	30
February	28	June	30	October	31
March	31	July	31	November	30
April	30	August	31	December	31

Or, according to the old rhyme:—

Thirty days hath September,
April, June, and November;
February hath twenty-eight alone,
All the rest have thirty-one,
Except in leap-year, and then's the time
February's days are twenty-nine.

Leap-year, which gives February 29 days, may be found by dividing the date of the year by four; if there be no remainder, it is Leap-year, and if there be a remainder, that remainder is the number of years after Leap-year. But if the last two figures of the year are cyphers, then a Leap-year will occur only when the first two figures are exactly divisible by four, thus the year 2000 will be a Leap-year.

ANGULAR MEASURE

For Navigation and Nautical Astronomy

60″ (seconds)	=	1′ (minute).
60′ (minutes)	=	1° (degree).
90 degrees	=	1 quadrant.
180 degrees	=	1 semicircle.
360 degrees	=	1 circumference.

$1° = 60' = 3600''$.
$360° = 21600' = 1296000''$.

Degrees, etc., in Relation to Time

360° = 24 Hours.	15° = 1 Hour.
180° = 12 Hours.	1° = 4 Minutes.
90° = 6 Hours.	15′ = 1 Minute.
60° = 4 Hours.	15″ = 1 Second.

FRENCH AND ENGLISH MEASURES

Metre		Feet		Fath.
1	=	3·28	=	0·55
2	=	6·56	=	1·09
3	=	9·84	=	1·64
4	=	13·12	=	2·19
5	=	16·40	=	2·73
6	=	19·69	=	3·28
7	=	22·97	=	3·83
8	=	26·25	=	4·37
9	=	29·53	=	4·92
10	=	32·81	=	5·47
100	=	328·09	=	54·68

1 Milimetre = ·03937 inch.
1 Centimetre = ·39371 inch.

ARITHMETIC OF NAVIGATION

The Arithmetic of Navigation can be explained in a few pages, and these the beginner would do well to carefully read. It is, of course, supposed that he is well up in the four rules of *simple* arithmetic—addition, subtraction, multiplication, and division—which are as constantly required in Navigation as in daily business transactions.

Beginning with the arithmetic of the Circle and of Time, it is to be noted that the parts of both are divided *sexagesimally*, or, in other words, *sixty* of a less denomination make *one* of a greater. Two short TABLES furnish the basis of computation.

(A) DIVISIONS OF THE CIRCLE, OR ANGULAR MEASURE

60 seconds (″)	make	1 minute (′)
60 minutes	„	1 degree (°)

and these terms are respectively marked ° ′ ″, so that 5° 51′ 28″ is to be read 5 degrees, 51 minutes, 28 seconds.

(B) MEASUREMENT OF TIME

60 seconds (s.)	make	1 minute (m.)
60 minutes	„	1 hour (h.)

but in this instance the terms are respectively marked h. m. s., so that 6h. 31m. 24s. is to be read 6 hours, 31 minutes, 24 seconds.

It will here be perceived that though the lower denominations in both tables are known by similar names—seconds and minutes—yet have they different signs to distinguish them, and in speaking of them, the former are called seconds and minutes of arc, and the latter, seconds and minutes of time; nor are these signs (which represent values) interchangeable, for, as will (in due course) be shown, a second of time has fifteen times the value of a second of angular measure, and so also as regards the minutes.

This appears to be the proper place for drawing attention to the ARITHMETICAL SIGNS which are most frequently used in computation; they are as follows—

= *equal to*, is the sign of Equality; as 60 minutes = 1 hour; that is, 60 minutes are equal to 1 hour.

+ *plus* (more), is the sign of Addition; as $8 + 7 = 15$; that is, 8 added to 7 is equal to 15.

— *minus* (less) is the sign of Subtraction; as $9 - 3 = 6$; that is, 9 lessened by 3 is equal to 6.

× *multiplied by*, is the sign of Multiplication; as $9 \times 12 = 108$; that is, 9 multiplied by 12 is equal to 108.

÷ *divided by*, is the sign of Division; as $84 \div 12 = 7$, that is, 84 divided by 12 is equal to 7. *Divided by* is also expressed by placing one number over the other; as $\frac{84}{12} = 7$; that is, 84 divided by 12 is equal to 7. In all cases, the upper number is to be divided by the lower number.

B 2

ADDITION OF DEGREES, &c., AND TIME

Note.—It will be sufficient here to remark, once for all, that like denominations (as in every other computation of compound quantities) must stand directly under each other; thus, degrees must be placed under degrees, ′ under ′, and ″ under ″; and similarly hours under hours, minutes under minutes, and seconds under seconds.

RULE.—Take the sum of the column of seconds and write it down, if less than 60; if the sum exceeds 60, find how many minutes are contained in it, then write down the remaining seconds, and carry the minutes to the column of minutes. Next, take the sum of the column of minutes, and write it down if less than 60; if the sum exceeds 60, find how many degrees are contained in it, and write down the remaining minutes, carrying the degrees to the column of degrees. Finally, take the sum of the column of degrees.

Proceed in the same manner if the quantities are hours, minutes, and seconds.

					H.	M.	S.				
(1)	28°	17′	49″	(2)	3	46	15	(3)	108°	49′	
	49	30	40		3	48	0		3	52	
	79	4	57		3	49	20	Sum	112	41	
Sum	156	53	26		11	23	35				

In (1) the sum of the column of seconds is 146, but we write down 26 and carry 2′, because 146″ make 2′ 26″; the sum of the column of minutes, with 2 to carry is 53′, which we write down at once; the sum of the degrees is 156°.

The operation is the same in (2) and (3).

SUBTRACTION OF DEGREES, &c., AND TIME

RULE.—In the lower denominations, when the quantity to be subtracted is less than the other, the process is simple; if it be greater, we must borrow 1 of the next higher denomination, which, expressed in terms of the lower, is 60.

										H.	M.	S.
(1)	From	18°	49′	30″	(2)	16°	19′	30″	(3)	15	8	42
	Take	7	20	15		2	44	46		5	3	48
	Rem.	11	29	15		13	34	44		10	4	54

(1) is simple enough.

(2) requires explanation. In the column of seconds we cannot take 46 from 30, but by borrowing 1′ (*i.e.*, 60″) 30″ becomes 90″, and 46 from 90 leaves 44″, which write down. Then, having borrowed under the head of seconds, we say 45 from 19 in the column of minutes, but can get no result unless we borrow 1° (*i.e.*, 60′), and then 45′ from 79′ leaves 34′, which write down. Finally, having borrowed in the minute column, carry 1 to the 2°, and say 3° from 16° leaves 13°.

The principle is the same in the case of hours, minutes, and seconds.

In Navigation, it is very often required to take the upper line from the lower; this should be practised, as it does not look well to make a transposition of the quantities.

(1) Take	76°	0′	52″	(2)	127°	17′	41″	(3)	275°	11′	16″
From	90	0	0		180	0	0		360	0	0
Rem.	13	59	8		52	42	19		84	48	44

In (1) we say 52″ from 60″ leaves 8″; carry 1 to the minutes, then 1′ from 60′ leaves 59′; finally, carry 1 to the degrees, and 77° from 90° leaves 13°.

(2) and (3) are operations of a similar character, and it will be perceived that 60 is borrowed in each of the lower denominations.

Multiplication of Degrees, &c., and Time

An example will best explain the method; begin with the column of seconds.

(1) Mult.	48°	46′	13″	(2) Mult.	106°	4′	53″
By			2	By			4
Product	97	32	26		424	19	32

Take (1): twice 13 are 26, which write down in column of seconds, since 26 is less than 60. Twice 46 are 92, but 92′ = 1° 32′; therefore write down 32 in column of minutes, and carry 1°. Twice 48 are 96, and 1° to carry, make 97°.

Division of Degrees, &c., and Time

An example will best explain the method; begin with the degrees, then take the minutes, and lastly the seconds.

(1).—Divide 147° 15′ 40″ by 2.				(2).—Divide 152° 47′ 36″ by 4.			
2)	147°	15′	40″	4)	152°	47′	36″
	73	37	50		38	11	54

In (1): 2 into 147 gives 73 and 1 over (*i.e.*, 1° or 60′); then 60 and 15 make 75, and 2 into 75 gives 37′ with 1 over (*i.e.*, 1′ or 60″); finally, 60 and 40 make 100, and 2 into 100 gives 50″.

Reduction of Degrees, &c., and Time.

An important process in the arithmetic of Angular Measure and Time is *Reduction*, that is the converting or changing a quantity from one denomination to another without altering its absolute value. Take an example from money: we know that 100 shillings make £5; that is, the shillings here indicated have the same value as the pounds; to get the £5 we divide the 100 by 20 (since 20 shillings make £1); for the reverse process we multiply the 5 by 20, which gives us the 100 shillings, hence the following rule—

General Rule for Reduction.—Consider how many of the less denomination make one of the greater ; then multiply the higher denomination by this number, if the reduction is to be to a less name ; or divide the lower denomination by it, if the reduction be to a higher name.

In Angular Measure and in Time (by Tables, p. 3) 60 of the less denomination make one of the greater ; hence, as the case requires, we must multiply or divide by 60.

EXAMPLES

In 5h. 48m. 11s., how many seconds ?

H.	M.	S.
5	48	11
60		
348 minutes.		
60		
20891 seconds.		

Here, 5 multiplied by 60 gives 300, to which add 48, and the result is 348 minutes. Then 348 multiplied by 60 gives 20880, to which add 11, and the final result is 20891 seconds.

And similarly for any other quantity. Thus, 18′ 42″ reduced to ″ give 1122″.

In 505′ how many degrees ?

$$6'0)\ 50'5$$
$$8^\circ\ 25'$$

Here, for simplicity, by striking off the 0 from 60, we must also strike off the unit (last) place from 505 ; then say 6 into 50 goes 8 times and 2 over ; write down 8, and bring down the 5, placing the 2 before it ; the result will be 8° 25′.

And similarly for any other quantity. Thus, 763 seconds (of time) reduced to minutes give 12m. 43s.

(1) In 98° 0′ 56″ how many ′ (minutes) and ″ (seconds of arc) ?
Answer—5880′ ; and 352856″.

(2) In 3h. 59m. 14s. how many minutes and seconds ?
Answer—239min. ; and 14354 sec.

(3) In 43062 seconds how many hours ?
Answer—11h. 57m. 42s.

Enough has now been said on the arithmetic of the circle and of time, and we next proceed to explain the nature of Decimals, without a knowledge of which we should be unable to use either the Nautical Almanac or the usual Nautical Tables that aid us in finding a ship's place from day to day.

DECIMAL ARITHMETIC

Numbers such as 1, 52, 148, and so on to millions or more, consisting of any *whole number* of units, are called *Integers*, but when we speak of a number which is a portion of 1, or unity, as of an eighth ($\frac{1}{8}$), a quarter ($\frac{1}{4}$), a third ($\frac{1}{3}$), a half ($\frac{1}{2}$), two-thirds ($\frac{2}{3}$), or three-fourths ($\frac{3}{4}$) of anything, say of a mile, we mean a fractional part of the mile, and the arithmetic of these values is termed *Vulgar Fractions.* In computations connected with Navigation, the same values are expressed *decimally*—*i.e.*, as *Decimal Fractions*—because by their aid we get better results with fewer figures than if we used vulgar fractions.

Decimal fractions are distinguished by a dot placed *before* the figure, and are read as tenths, hundredths, thousandths, etc., only ; thus ·1 stands for $\frac{1}{10}$ (one-tenth), ·3 for $\frac{3}{10}$ (three-tenths), ·25 for $\frac{25}{100}$ (twenty-five-hundredths), ·125 for $\frac{125}{1000}$ (125-thousandths), and so on.

The relation of Decimal to Vulgar Fractions may be illustrated as follows : The vulgar fraction $\frac{1}{2}$, when written decimally, becomes ·5, *i.e.*, $\frac{5}{10}$ or 5 divided by 10, because 5 parts of anything that is divided into 10 parts is the same as one-half of the whole (unit) ; similarly, $\frac{1}{4}$ becomes a decimal in the form of ·25, *i.e.*, 25 divided by 100, because 25 parts of 100 parts is the same thing as one-fourth.

Cyphers after (or affixed to) decimal parts do not alter their value ; thus ·5, ·50, or ·500, each express an equal value—viz., $\frac{5}{10}$, $\frac{50}{100}$, or $\frac{500}{1000}$, *i.e.*, half a unit. But cyphers before (or prefixed to) decimal parts decrease the value tenfold for each cypher ; thus, while ·5 is $\frac{5}{10}$ or $\frac{1}{2}$, ·05 is only $\frac{5}{100}$ or $\frac{1}{20}$; and similarly ·005 becomes $\frac{5}{1000}$ or $\frac{1}{200}$.

This explanation is not to be taken as trivial, or out of the course of our main subject, because when we come to speak of the Nautical Almanac, we shall find most of the corrections extend to hundredths and thousandths.

Caution.—After what has been written here, avoid a very common error in expressing decimal parts ; for instance, never call ·75 seventy-five tenths, but 75-hundredths ; and never call it decimal seventy-five, but decimal seven five.

NOTE.—A Vulgar Fraction may always be turned into a Decimal Fraction, by dividing the figure above the line (or numerator) by the figure below the line (or denominator), affixing as many cyphers to the numerator as are required ; thus, $\frac{1}{8} = \frac{1 \cdot 000}{8} = \cdot 125$.

Having explained the nature of decimals, it may now be stated that in the arithmetic of Navigation the numbers are not entirely decimals, but in the majority of instances consist of whole numbers (or integers) *and* decimals, as when we write 28·8 miles for 28 and 8-tenths of a mile, the figures to the *left* of the decimal point are *integers*, and such as are to the *right* are *decimals.*

Addition and Subtraction of Decimals

RULE.—Addition and subtraction of decimals are performed exactly the same as in whole numbers, observing always to place the decimal points so that they may stand directly under one another ; and thus figures of the same denomination will range properly.

Examples in Addition

(1)	(2)	(3)	(4)
53·212	65·	720·1464	·5
79·464	246·3	39·	·75
2·304	19·24	7·246	·253
127·414	121·46	259·1703	·582
262·394	452·00	1025·5627	2·085

Examples in Subtraction

(5)	(6)	(7)	(8)
246·25	·75	176·014	174·
19·5	·5	29·008	2·561
226·75	·25	147·006	171·439

NOTE.—In example (8) as there are no figures in the upper line above 561 (in the lower one), 000 is considered to be expressed in the upper line, and then the subtraction is made in the usual way.

Multiplication of Decimals

RULE.—Multiply the given numbers together as if they were whole numbers, and point off as many decimals in the product, counting from the right hand, as there are decimals in the multiplicand and multiplier together.

When it happens that there are not so many figures in the product as there should be decimals, supply the defect by prefixing cyphers on the left hand (*see* Example 3).

Examples

	(1)	(2)	(3)
Multiplicand	3·275	·25	·2376
Multiplier	29·5	·42	·0062
	16375	50	4752
	29475	100	14256
	6550	·1050	·00147312
Product	96·6125		

Division of Decimals

Proceed as in *simple* division, but introducing cyphers into the dividend if required; and then, the division having been made, strike off in the quotient as many decimal places as the dividend has decimal places in excess of the divisor; if there are not so many, the defect must be supplied by *prefixing* cyphers.

When, after the division, there is a remainder, cyphers may be added to the dividend, and the operation continued as before until either there be no remainder, or a sufficient degree of exactness has been obtained in the quotient.

Examples

Divide 612 by 136

Divisor. Dividend. Quotient.

```
136) 612·0 (4·5
     544
      680
      680
```

Divide 276 by 345

Divisor. Dividend. Quotient.

```
345) 276·0 (·8
     276·0
```

Here 136 into 612 goes 4 times, and leaves remainder 68 ; to the latter is affixed 0 ; then 136 into 680 goes 5 times ; now the 0 in this case is a decimal, and gives the dividend one decimal place in excess of the divisor ; therefore the 5 in the quotient is a decimal.

Here the dividend 276 is not divisible by 345, except by affixing 0 ; after which the divisor goes 8 times into the dividend ; now the 0 being a decimal, the 8 in the quotient is also a decimal, according to the rule.

Divide 7234·5 by 6·5

Divisor. Dividend. Quotient.
6·5) 7234·5 (1113
65
73
65
84
65
195
195

Divide ·45625 by 12·5

Divisor. Dividend. Quotient.
12·5) ·45625 (·0365
375
812
750
625
625

Reduction of Decimals

To reduce a Vulgar Fraction to a Decimal of the same value

RULE.—Add cyphers at pleasure to the numerator, and divide by the denominator ; the quotient will be the decimal fraction required.

Example

Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{64}$, $2\frac{9}{16}$ to a decimal fraction.

$$\tfrac{1}{2} = {\cdot}5, \quad \tfrac{3}{4} = {\cdot}75, \quad \tfrac{7}{64} = {\cdot}109375, \quad \tfrac{41}{16} = 2{\cdot}5625$$

Express the following decimals as vulgar fractions.

$${\cdot}7, \ {\cdot}75, \ {\cdot}0875, \ {\cdot}0064$$

$$\tfrac{7}{10}, \ \tfrac{75}{100} = \tfrac{3}{4}, \ \tfrac{0875}{10000} = \tfrac{175}{2000}, \ \tfrac{{\cdot}0064}{10000} = \tfrac{16}{2500} = \tfrac{4}{625}$$

Reduce $\frac{4}{5}$ of a mile to a decimal fraction.

5)4·0
·8

which is a decimal of the same value with the proposed vulgar fraction $\frac{4}{5}$.

Reduce $\frac{23}{60}$ of a degree to a decimal fraction.

60)23·000(·383
180
500
480
200
180 *etc.*

Every quantity may be considered as a fraction of a larger quantity of the same kind ; thus a minute is the $\frac{1}{60}$ of a degree ; an inch, the $\frac{1}{12}$ of a foot, etc. ; and therefore may be reduced to a decimal fraction, as in the following examples :—

Reduce 10 hours 35 minutes to the decimal of a day.

60)35·0

24)10·5833(·44097 Answer.

96

98
96

233
216

170
168

Reduce 24 minutes 20 seconds to the decimal of an hour.

60)20·0

60)24·333

·40555 Answer.

Reduce 1 foot 6 inches to the decimal of a yard.

ft. in.	ft.
1 6	3 = 1 yard
12	12
18 Numerator.	36 Denominator.

36) 18·0 (·5 Answer.
180

Thus, $\frac{18}{36}$ of an inch = ·5 or $\frac{1}{2}$ an inch.

Reduce 21′ (minutes) 54″ (seconds) to the decimal of a degree.

21′ 54″ = 1314″ Numerator

1 deg. or 60′ = 3600″ Denominator

3600) 1314·000 (·365 Answer

10800

23400
21600

18000
18000

To find the Value of a Decimal Fraction

RULE.—Multiply the decimal by the number of parts of the next inferior denomination contained in the integer, pointing off in the product as many places for decimals, to the right hand, as the given decimal consists of, and those to the left hand will be an integral number: then multiply the remaining decimals by the number of parts contained in the next inferior denomination, and point off the decimals as before. Proceed thus till it be brought to the lowest denomination.

What is the value of ·259 of a league?

	·259
	3
Miles	·777
	8
Furlongs	6·216
	220
Yards	47·520

Answer, 6 furlongs 47·52 yards.

What is the value of ·42 of a degree?

	·42
	60
Minutes	25′·20
	·60
Seconds	12″·00

Answer, 25′ 12″

When tenths of a degree or minute are to be reduced into minutes or seconds, it may be expeditiously done by multiplying the tenths by 6, and the product will give the minutes or seconds required: for example, ·5 of a degree multiplied by 6 gives 30 minutes, and ·9 of a minute, 54 seconds.

On the contrary, to reduce minutes and seconds to tenths of a degree or minute, divide them by 6.

The form this rule most commonly takes for the purposes of navigation is reduction of seconds to the decimal of a minute, or of minutes to the decimal of an hour, generally the latter, as when we have to correct the elements taken from the Nautical Almanac for any other time than Greenwich noon.

RULE for reducing a lower denomination to the decimal of a higher one. By Table A or B, p. 3, the divisor is 60; therefore write down the given number of minutes (or seconds, as the case may be) and divide them by 60.

Examples

Reduce 42 minutes to the decimal of an hour.

```
60) 42·0 (·7
    420
```

Therefore, 42 min. is seven-tenths of an hour; and if, in this case, the given time had been 10h. 42m., we should express it, decimally, as 10·7h., *i.e.*, 10 hours and 7-tenths of an hour.

Reduce 33 minutes to the decimal of an hour.

```
60) 33·0 (·55
    300
    ---
     300
     300
```

Here we see 33 min. is 55-hundredths of an hour, and if the given time had been 8h. 33m. we should write 8·55h.

We can adopt a *shorter method* of getting the same result, as follows: The unit place in the 60 being 0, we can reject it if we make the unit place in the minutes a decimal; and the divisor is 6. Thus, using the examples above—

```
6) 4·2        6) 3·3
   ·7            ·55
```

And by a similar process we can make a short table.

(*c*) *Minutes expressed as the Decimal of an Hour*

min.	hour.	min.	hour.	min.	hour.
3 =	·05	21 =	·35	39 =	·65
6 =	·1	24 =	·4	42 =	·7
9 =	·15	27 =	·45	45 =	·75
12 =	·2	30 =	·5	48 =	·8
15 =	·25	33 =	·55	51 =	·85
18 =	·3	36 =	·6	54 =	·9
				57 =	·95

But the table may be adapted to various denominations. Thus, using the column of minutes as seconds, the hour column becomes decimal of a minute; or using the column of minutes as minutes (′) of angular measure, the hour column becomes decimal of a degree (°); for instance, $18' = 0^\circ{\cdot}3$, *i.e.*, $18' = 3$-tenths of a degree.

In practice it is sufficiently accurate to use but one decimal place; thus we should say 14m. = ·2 of an hour, since 14 is nearer to 12 than to 18; but 16m. = ·3 of an hour, because 16 is nearer to 18 than to 12; and so for other quantities; always avoid an excess of figures, for otherwise you may "strain at a gnat and swallow a camel."

Proportion or Rule of Three

Problems which have the idea of proportion in them are now usually solved by what is called the *Unitary Method.*

If the given numbers consist of several denominations, they are to be reduced to decimals by the preceding Rules.

Examples

If a ship sail 49·5 miles in 8 hours, how many miles will she run in 24 hours, supposing her to go at the same rate?

In 8 hours the ship sails 49·5 miles.

„ 1 hour „ „ $\frac{49{\cdot}5}{8}$ „

„ 24 hours „ „ $\frac{49{\cdot}5}{8} \times 24$ „

$$\begin{array}{r} 49{\cdot}5 \\ 24 \\ \hline 1980 \\ 990 \\ \hline 8)1188{\cdot}0 \\ \hline 148{\cdot}5 \end{array}$$

Answer 148·5 Miles.

Suppose a watch or chronometer gain 14 seconds in 5 days 6 hours, how much will it gain in 17 days 15 hours?

6 hours = ·25 of a day.
15 hours = ·625 of a day.

In 5·25 days the watch gains 14 sec.

„ 1 day the watch gains $\frac{14}{5{\cdot}25}$ sec.

„ 17·625 days the watch gains $\frac{14}{5{\cdot}25} \times 17{\cdot}625$ sec.

$$\begin{array}{r} 14 \\ 17{\cdot}625 \\ \hline 70500 \\ 17625 \\ \hline 5{\cdot}25)246{\cdot}750(47 \\ 2100 \\ \hline 3675 \\ 3675 \\ \hline \cdots\cdot \end{array}$$

47 Sec. Ans.

LOGARITHMS

Logarithms are a series of numbers invented, and first published in 1614, by Lord Napier, Baron of Merchiston in Scotland, for the purpose of facilitating troublesome calculations in plane and spherical trigonometry. These numbers are so contrived, and adapted to other numbers, that the sums and differences of the former shall correspond to, and show, the products and quotients of the latter.

Logarithms may be defined to be the numerical exponents of ratios, or a series of numbers in arithmetical progression, answering to another series of numbers in geometrical progression; as,

Thus—

0.	1.	2.	3.	4.	5.	6.	7.	8. ind. or log.
1.	2.	4.	8.	16.	32.	64.	128.	256. geo. prog.

Or—

0.	1.	2.	3.	4.	5.	6.	7.	8. ind. or log.
1.	3.	9.	27.	81.	243.	729.	2187.	6561. geo. pro.

Or—

0.	1.	2.	3.	4.	5.	6	7.	8.	ind. or log.
1.	10.	100.	1000.	10000.	100000.	1000000.	10000000.	100000000	geo. pro.

Whence it is evident that the same indices serve equally for any geometrical series; and, consequently, there may be an endless variety of systems of logarithms to the same common number, by only changing the second term 2, 3, or 10, etc., of the geometrical series of whole numbers.

In these series it is obvious that if any two indices be added together, their sum will be the index of that number which is equal to the product of the two terms, in the geometrical progression to which those indices belong: thus, the indices 2 and 6 being added together make 8; and the corresponding terms 4 and 64 to those indices (in the first series), being multiplied together, produce 256, which is the number corresponding to the index 8.

It is also obvious that if any one index be subtracted from another the difference will be the index of that number which is equal to the quotient of the two corresponding terms: thus, the index 8 minus the index 3 = 5; and the terms corresponding to these indices are 256 and 8, the quotient of which, viz., 32, is the number corresponding to the index 5, in the first series.

And, if the logarithm of any number be multiplied by the index of its power, the product will be equal to the logarithm of that power; thus, the index, or logarithm of 16, in the first series, is 4; now, if this be multiplied

by 2, the product will be 8, which is the logarithm of 256, or the square of 16.

Again: if the logarithm of any number be divided by the index of its root, the quotient will be equal to the logarithm of that root: thus, the index or logarithm of 256 is 8; now, 8 divided by 2 gives 4, which is the logarithm of 16, or the square root of 256, according to the first series.

The logarithms most convenient for practice are such as are adapted to a geometrical series increasing in a tenfold ratio, as in the last of the foregoing series; being those which are generally found in most mathematical works, and which are usually called *common logarithms*, in order to distinguish them from other species of logarithms.

In this system of logarithms, the index or logarithm of 1 is 0; that of 10 is 1; that of 100 is 2; that of 1000 is 3; that of 10000 is 4; etc., etc.; whence it is manifest that the logarithms of the intermediate numbers between 1 and 10 must be 0, and some fractional parts; that of a number between 10 and 100 must be 1, and some fractional parts; and so on for any other number; those fractional parts may be computed by the following

Rule.—To the geometrical series 1, 10, 100, 1000, 10000 etc., apply the arithmetical series 0, 1, 2, 3, 4, etc., as logarithms. Find a geometrical mean between 1 and 10, or between 10 and 100, or any other two adjacent terms of the series between which the proposed number lies. Between the mean thus found and the nearest extreme, find another geometrical mean in the same manner, and so on till you arrive at the number whose logarithm is sought. Find as many arithmetical means, according to the order in which the geometrical ones were found, and they will be the logarithms of the said geometrical means, the last of which will be the logarithm of the proposed number.

EXAMPLE.—To compute the Log. of 2 to Eight Places of Decimals—

Here the proposed number lies between 1 and 10.

FIRST.—The log. of 1 is 0, and the log. of 10 is 1; therefore $0 + 1 \div 2 = \cdot 5$ is the arithmetical mean, and $\sqrt{1 \times 10} = 3{\cdot}1622777$ is the geometrical mean: hence the log. of 3·1622777 is ·5.

SECOND.—The log. of 1 is 0, and the log. of 3·1622777 is ·5; therefore $0 + \cdot 5 \div 2 = \cdot 25$ is the arithmetical mean, and $\sqrt{1 \times 3{\cdot}1622777} = 1{\cdot}7782794$ the geometrical mean: hence the log. of 1·7782794 is ·25.

THIRD.—The log. of 1·7782794 is ·25, and the log. of 3·1622777 is ·5; therefore $\cdot 25 + \cdot 5 \div 2 = \cdot 375$ is the arithmetical mean, and $\sqrt{1{\cdot}7782794 \times 3{\cdot}1622777} = 2{\cdot}3713741$ the geometrical mean: hence the log. of 2·3713741 is ·375.

FOURTH.—The log. of 1·7782794 is ·25, and the log. of 2·3713741 is ·375; therefore $\cdot 25 + \cdot 375 \div 2 = \cdot 3125$ is the arithmetical mean, and $\sqrt{1{\cdot}7782794 \times 2{\cdot}3713741} = 2{\cdot}0535252$ the geometrical mean: hence the log. of 2·0535252 is ·3125.

FIFTH.—The log. of 1·7782794 is ·25, and the log. of 2·0535252 is ·3125; therefore $\cdot 25 + \cdot 3125 \div 2 = \cdot 28125$ is the arithmetical mean, and

$\sqrt{1 \cdot 7782794 \times 2 \cdot 0535252} = 1 \cdot 9109530$ the geometrical mean: hence the log. of 1·9109530 is ·28125.

SIXTH.—The log. of 1·9109530 is ·28125, and the log. of 2·0535252 is ·3125; therefore ·28125 + ·3125 ÷ 2 = ·296875 is the arithmetical mean, and $\sqrt{1 \cdot 9109530 \times 2 \cdot 0535252} = 1 \cdot 9809568$ the geometrical mean: hence the log. of 1·9809568 is ·296875.

SEVENTH.—The log. of 1·9809568 is ·296875, and the log. of 2·0535252 is ·3125; therefore ·296875 + ·3125 ÷ 2 = ·3046875 is the arithmetical mean, and $\sqrt{1 \cdot 9809568 \times 2 \cdot 0535252} = 2 \cdot 0169146$ the geometrical mean: hence the log. of 2·0169146 is ·3046875.

EIGHTH.—The log. of 2·0169146 is ·3046875, and the log. of 1·9809568 is ·296875; therefore ·3046875 + ·296875 ÷ 2 = ·30078125 is the arithmetical mean, and $\sqrt{2 \cdot 0169146 \times 1 \cdot 9809568} = 1 \cdot 9988548$ the geometrical mean: hence the log. of 1·9988548 is ·30078125.

Proceeding in this manner, it will be found, after 25 extractions, that the log. of 1·9999999 is ·30103000; and since 1·9999999 may be considered as being essentially equal to 2 in all the practical purposes to which it can be applied, therefore the log. of 2 is ·30103000.

If the log. of 3 be determined, in the same manner, it will be found that the twenty-fifth arithmetical mean will be ·47712125, and the geometrical mean 2·9999999; and since this may be considered as being in every respect equal to 3, therefore the log. of 3 is ·47712125.

Now, from the logs. of 2 and 3, thus found, and the log. of 10, which is given = 1, a great many other logarithms may be readily raised; because the sum of the logs. of any two numbers gives the log. of their product; and the difference of their logs. the log. of the quotient; the log. of any number, being multiplied by 2, will give the log. of the square of that number; or, multiplied by 3, will give the log. of its cube; as in the following examples—

EXAMPLES

(1) To find the Log. of 4—

To the log. of 2 =	·30103000
Add the log. of 2 =	·30103000
Sum is the log. of 4 =	·60206000

(2) To find the log. of 5—

From the log. of 10 =	1·00000000
Take the log of 2 =	·30103000
Rem. is the log. of 5 =	·69897000

(3) To find the log. of 6—

To the log of 3 =	·47712125
Add the log. of 2 =	·30103000
Sum is the log. of 6 =	·77815125

(4) To find the log. of 8—

To the log. of 4 =	·60206000
Add the log. of 2 =	·30103000
Sum is the log. of 8 =	·90309000

EXAMPLES

(5) To find the log. of 9—

To the log. of 3 =	·47712125
Add the log. of 3 =	·47712125
Sum is the log. of 9 =	·95424250

(6) To find the log. of 15—

To the log. of 5 =	·69897000
Add the log. of 3 =	·47712125
Sum is the log. of 15 =	1·17609125

(7) To find the log. of 81 = the square of 9—

Log. of 9 =	·95424250
Multiply by	2
Product is the log. of 81 =	1·90848500

(8) To find the log. of 729 = the cube of 9—

Log. of 9 =	·95424250
Multiply by	3
Product is the log. of 729 =	2·86272750

Since the odd numbers 7, 11, 13, 17, 19, 23, 29, etc., cannot be exactly deduced from the multiplication or division of any two numbers, the logs. of those must be computed in accordance with the rule by which the logs. of 2 and 3 were obtained; after which the labour attending the construction of a table of logarithms will be greatly diminished, because the principal part of the numbers may then be very readily found by addition, subtraction, and composition.

Having shown the construction of the Tables of Logarithms, we shall now proceed to the manner of using them.

PROPERTIES OF LOGARITHMS

It being understood that when we write or say 4, or 56, or 749, or 8476, and so forth to any extent, we mean that we are giving expression to *integers* (or *whole numbers*) which may consist of any number of figures (or digits) ranging from one to millions or more.

On the other hand, if we say or write five-tenths and express it by a figure, thus, ·5; or again, if we say five-hundredths and write it as ·05; and yet further, if we say thirty-seven hundredths and write it as ·37, we are giving expression to a *decimal fraction.*

And again, if we write 37·04 or 349·67, we have numbers that are partly integers and partly decimals; the figures to the left of the dot (or decimal point) being the integers, and those to the right of the dot being decimals. All these are natural numbers, or such as appertain to common arithmetic.

The table in Norie's "Epitome" to which the following remarks apply is the Table of Logs., and to it we direct special attention. In the first part of it you will see a column marked "No." and by its side another column marked "Log." The meaning of "No." is "number," and "Log." is the abbreviation for logarithm. You will also see that the numbers run from 1 to 100, and that there is a logarithm corresponding to each number, and standing by its side—

Thus, No.	8 has for its Log.		0·903090
No.	49 has	,,	1·690196, and
No.	100 has	,,	2·000000

It is important to note this, for you will see that each logarithm consists of two parts, viz., a figure to the left of the decimal point, and six figures to the right of the decimal point. The figure to the left is called the *index*, and the figures to the right are the *mantissa*, or, in a plain word, *decimals.*

If you now look in Table of Logs to the numbers beyond 100, you will see that the logarithmic columns by the side of the numbers have no index, but only the decimal part, and for this reason,—that though no logarithm is complete without an index *and* a decimal part, the index is readily supplied according to a fixed rule, and one very easily to be remembered.

Rule for the Index.—*The index of a logarithm is always less by one than the number of figures* (or digits) *in the integer or whole number.* On this basis you will see that the number 8 has index 0; number 49 has index 1; and number 100 has index 2. In a similar way number 300 or 465 would have index 2; number 4876 would have index 3; and number

75687 would have index 4; also number 3400757 would have index 6. Now, using 3400756 as a special illustration of the rule, we see that it consists of seven figures or digits; one less than seven is six, hence 6 is the logarithmic index of the number. Similarly 49 consists of two figures; one less than two is one; hence 49 has 1 for its logarithmic index.

If the number is partly an integer and partly a decimal, as 84·674, then the number of figures in the integer (or to the left hand of the dot or decimal point) only is counted to give the index; and as 84 consists of two figures, we have 1 for the index: similarly 846·74 gives index 2; and 8·4674 gives index 0.

If the number is entirely decimal, the *index* of the logarithm is properly *negative;* thus the index of the logarithm of ·9, or of ·94, or of ·949, is − 1 (*minus* one); the index of ·09, or ·094, or of ·0949, is − 2; also of ·009, or ·0094, the index is − 3. But to avoid confusion in the addition and subtraction of indices of different characters it is customary to use the arithmetical complement (that is, the number subtracted from 10) of the negative indices, and consider these complements as positive; hence for − 1 we say 9; for − 2 we say 8; for − 3 we say 7, and so on; since 9 is the arithmetical complement of 10 *lessened* by 1; and 8 is the arithmetical complement of 10 *lessened* by 2, etc.—

Thus we have for		8764	the index	3
,,	,,	876·4	,,	2
,,	,,	87·64	,,	1
,,	,,	8·764	,,	0
,,	,,	·8764	,,	9 or − 1
,,	,,	·08764	,,	8 or − 2
,,	,,	·008764	,,	7 or − 3

When you read further on you will see that the decimal part of the logarithm of 8764 is ·942702, and if we complete the logarithm by prefixing its index, we get—

For number 8764	the completed log is	3·942702
876·4	,,	2·942702
87·64	,,	1·942702
8·764	,,	0·942702
·8764	,,	9·942702
·08764	,,	8·942702
·008764	,,	7·942702

From these numbers with their accompanying logarithms we also learn another fact, viz., that the same significant figures (as 8764), whether they are whole numbers, or partly whole and partly decimal, or entirely decimal, give the same logarithm in so far as its decimal part is concerned: the only change is in the index of the logarithm.

It, therefore, follows that the index of a logarithm must be known before any definite value can be assigned to the corresponding natural number. For example, the logarithm of 8764 is 942702, and if the index of the logarithm be 3 the natural number will be 8764, and if the index be 1 the natural number will be 87·64; but if the index be 6 the natural number will be 8764000. It will thus be seen that the number of figures in the whole number is one more than the number indicated by the index.

Example.—Required the logarithm of 25047. First, look in the left-hand column for 250; opposite to this, in the column with 4 at the top, is the decimal part of the logarithm, which is ·398634; in the right-hand column is the difference 173; this multiplied by 7, the fifth figure, gives 1211; from which cut off the last figure, and the remainder 121 added to ·398634 will give, with the proper index prefixed, 4·398755, the logarithm required.

In the same manner the logarithm of 598765 will be found to be 5·777256; obtained as follows—

Diff. ..	73	The Log. of 5987 in the Table of Logs. is..	777209
Mult. by	65	Increase for 65, the excess figures =	+ 47
	365	Therefore the Log. of 598765 is	5·777256
	438		
	47·45		

Here, in getting the correction for 65, two figures are cut off to the right because we have multiplied the difference by two figures; and having added 47 (the figures to the left) to the logarithm of 5987 we get the decimal part of the required logarithm; and finish by prefixing the proper index, which in this case is 5 because 598765 is a whole number, and consists of six figures. But the logarithm of 598·765 would be 2·777256.

N.B.—When correcting logarithms for the number of figures in excess of four, always cut off as many figures on the right-hand side as you multiply by.

For example, find the logarithm of 1746007—

Diff. ..	249	Log. 1746 =	242044
	·007	Increase for 007 =	+ 2
	1·743	∴ the Log. 174007 =	6·242046

Find the logarithm of 374909—

Diff. ..	116	The Log. of 3749 is	573915
Mult. by	09	Increase for 09	+ 10
	10·44	Log. of 374909	5·573925

The index 5 being supplied on the basis that there are six figures in the whole number, and the proper decimal part of the logarithm being found through the column of differences, as before. Similarly, the logarithm of 374·909 would be 2·573925.

To find the number corresponding to a given Logarithm

If the *Logarithm is found exactly*, the corresponding number is taken out at once; thus, seek for the logarithm in the Log. Tables and take out its numerical value in column "No."

For Log. 0·000000	the number is	....	1
0·845098	"	....	7
1·518514	"	....	33
2·000000	"	....	100
2·210397	"	....	159
3·203848	"	....	1599
4·203848	"	....	15990
5·203848	"	....	159900

Examples for Practice

Find the Logarithms for the following Numbers—

(1)	7	(4)	8470	(7)	67492	(10)	·897
(2)	59	(5)	8479	(8)	65208	(11)	·0764
(3)	785	(6)	67490	(9)	468219	(12)	·0059

Answers—

(1)	0·845098	(4)	3·927883	(7)	4·829252	(10)	9·952792
(2)	1·770852	(5)	3·928345	(8)	4·814301	(11)	8·883093
(3)	2·894870	(6)	4·829239	(9)	5·670449	(12)	7·770852

Find the Numbers corresponding to the following Logarithms—

(1)	0·000000	(4)	2·037426	(7)	3·000000	(10)	4·809654
(2)	0·903090	(5)	2·201397	(8)	3·543323	(11)	3·649101
(3)	1·959041	(6)	3·380030	(9)	4·911232	(12)	9·650016

Answers—

(1)	1	(4)	109	(7)	1000	(10)	64514
(2)	8	(5)	159	(8)	3494	(11)	4457·598
(3)	91	(6)	2399	(9)	81514	(12)	·4467

To find the Arithmetical Complement of a Logarithm

The arithmetical complement of a logarithm is the number it wants of 10·000000; and the easiest way to find it is, beginning at the left hand, to subtract every figure from 9, except the last significant figure, which is to be taken from 10. Thus the arithmetical complement of 4·478309 is 5·521691: it is frequently used in the Rule of Proportion, and in trigonometrical calculations, to change subtraction into addition.

Accuracy of Logarithms

As regards the accuracy of logarithms to six places, they cannot be taken out correctly for numbers in excess of 435000, because the difference then ceases to change at the rate of 100 for 1 in the fourth figure.

MULTIPLICATION BY LOGARITHMS

The logarithm of the product of any two numbers is equal to the sum of their logarithms.

Let n, n^1, be any two numbers, l, l^1, their logarithms.

Then $\log._{10} (n \times n^1) = l + l^1$

RULE.—Add together the logarithms (Table of Logs.) of the two numbers to be multiplied and their sum will be a logarithm, the natural number corresponding to which will be the product required: if either the multiplicand or multiplier, or both of them, should consist wholly of decimals, and the index of the sum exceed 10, reject the 10, and the remainder will be the index of the logarithm answering to the product.

EXAMPLES

Multiply	25	Log.	1·397940	Multiply	23·2	Log.	1·365488
by	3	Log.	0·477121	by	·6	Log.	9·778151
Product	75	Log.	1·875061	Product	13·92	Log.	1·143639
Multiply	371	Log.	2·569374	Multiply	·246	Log.	9·390935
by	25	Log.	1·397940	by	700	Log.	2·845098
Product	9275	Log.	3·967314	Product	172·2	Log.	2·236033

Diff. 259	Multiply	1675·6	Log.	3·224170
6	by	35	Log.	1·544068
Corr. for 6 = 155·4	Product	58646	Log.	4·768238
				768194
1675 Log. ·224015				74)440(6
Corr. for 6 + 155				444
1675·6 Log. 3·224170				

Examples for Practice

(1)	Multiply	87 by 96	(4)	Multiply	385·5 by 28·4
(2)	,,	649 by 84	(5)	,,	19·48 by 1·65
(3)	,,	3876 by 50	(6)	,,	48708 by ·55

Answers—

(1)	Log. 3·921790	gives	8352	(4)	Log. 4·039342	gives	10948·2
(2)	,, 4·736524	,,	54516	(5)	,, 1·507073	,,	32·142
(3)	,, 5·287354	,,	193800	(6)	,, 4·427963	,,	26789·4

Note.—In all these problems, if the resulting logarithm differs from a logarithm in the Tables by no more than 1, the number corresponding to the tabular logarithm may be generally taken as the correct answer; thus logarithm 3·763876 or 3·763878 gives natural No. 5806.

DIVISION BY LOGARITHMS

The logarithm of the quotient of any two numbers is equal to the difference of their logarithms.

Let n, n^1, be any two numbers, l, l^1, their logarithms.

Then $\log._{10}\frac{n}{n^1} = l - l^1$.

RULE.—From the logarithm of the dividend subtract the logarithm of the divisor, and the remainder will be a logarithm, whose corresponding number will be the quotient required. When the index of the divisor exceeds that of the dividend, borrow 10, and the remainder will be the index of the quotient.

EXAMPLES

Divide	75	Log.	1·875061
by	3	Log.	0·477121
Quotient	25	Log.	1·397940
Divide	927500	Log.	5·967314
by	250	Log.	2·397940
Quotient	3710	Log.	3·569374

Divide	139·2	Log.	2·143639
by	·6	Log.	9·778151
Quotient	232	Log.	2·365488
Divide	·01722	Log.	8·236033
by	·07	Log.	8·845098
Quotient	·246	Log.	9·390935

Divide	247296	Log.	5·393217
by	·06	Log.	8·778151
Quotient	4121600	Log.	6·615066
			615003

105)63000(600
630
00

Diff. 176
96
1056
1584
168·96

2472 Log. ·393048
Corr. for 96 + 169
247296 Log. 5·393217

Examples for Practice

(1)	Divide	1728 by 12
(2)	,,	10065 by 55
(3)	,,	72728 by 4
(4)	Divide	1000 by ·99
(5)	,,	75026 by ·25
(6)	,,	8007 by ·08007

Answers—

(1)	Log.	2·158363	gives	144
(2)	,,	2·262451	,,	183
(3)	,,	4·259642	,,	18182
(4)	Log.	3·004365	gives	1010·1
(5)	,,	5·477272	,,	300104
(6)	,,	5·000000	,,	100000

INVOLUTION

Involution is the raising of powers from a given root. When a number is multiplied by itself, the product is called its *second* power, or *square*; when this product is multiplied by the given number, the last product is called its *third* power, or *cube*; and when the multiplication is again repeated, the fourth power, and so on. The first power, or number thus

raised, is called the *root*, and the number of the power to which the given number is raised, the *index* of that power: hence, to raise or involve a number to a given power, multiply its logarithm by the index of the power to which it is to be raised, and the product will be the logarithm of the power sought.

When the given number is a decimal fraction, reject the tens resulting from the multiplication of the index of the logarithm by the power.

Involution is expressed in the following manner: 10^2 means the square of 10, 10^3 means the cube of 10, 10^4 means the fourth power of 10, and so on.

The logarithm of the power of any number is equal to the logarithm of the number multiplied by the index of the power.

Formula.—

$$\text{Log. } M^r = \text{Log. } M \times r.$$

where M = any number, and r any power.

EXAMPLES

Required the square, or 2nd power of 15.

15	Log. 1·176091
Index	2
Reqd. Power 225	Log. 2·352182

Required the cube, or 3rd power of 2·5.

2·5	Log. 0·397940
Index	3
Reqd. Power 15·625	Log. 1·193820

Required the square of ·174.

·174	Log. 9·240549
Index	2
Reqd. Power ·030276	Log. 8·481098

Required the 5th power of ·2.

·2	Log. 9·301030
Index	5
Reqd. Power ·00032	Log. 6·505150

Examples for Practice

(1) Required the cube of 3·725.
(2) Required the square of ·0975.
(3) Find the value of $(12{\cdot}75)^2$
(4) Find the value of $({\cdot}375)^3$

Answers—

(1) 51·687
(2) ·0095063
(3) 162·56
(4) ·052734

EVOLUTION

Evolution is the extracting of the root of a given number, or finding a number which, when raised to the given power, will produce the given number: it is consequently the reverse of involution, and is performed by dividing the logarithm of the number by the index of the power, and the quotient will be the logarithm of the root required.

When the given number is a decimal fraction, prefix to the index of its logarithm a figure less by one than the index of the root, and divide the whole by the index of the root. See the two examples of decimals below.

Evolution is expressed in the following manner. $\sqrt{10}$ means the square root of 10, $\sqrt[3]{10}$ means the cube root of 10, and so on. It is also expressed in fractional indices, thus $10^{\frac{1}{2}}$ means the square root of 10, $10^{\frac{1}{3}}$ means the cube root of 10, and so on.

The logarithm of the root of any number is equal to the logarithm of the number divided by the index of the root.

Formula.—

$$\text{Log. } \sqrt[m]{M} = \text{Log. } M \div m$$

where M = any number, and $\sqrt[m]{\ }$ the required root.

EXAMPLES

Required the square root of 225.

225	Log. 2)	2·352182
Reqd. Root 15	Log.	1·176091

Required the cube root of 15·625.

15·625	Log. 3)	1·193820
Reqd. Root 2·5.	Log.	0·397940

Required the square, or 2nd root of ·030276.

·030276	Log. 2)	18·481098
Reqd. Root ·174	Log.	9·240549

Required the 5th root of ·00032.

·00032	Log. 5)	46·505150
Reqd. Root ·2	Log.	9·301030

Examples for Practice

(1) Required the square root of 20·736.

(2) Required the cube root of ·003375.

(3) Find the value of $\sqrt[3]{1728}$.

(4) Find the value of $\cdot6561^{\frac{1}{4}}$.

Answers—

(1) 4·5537
(2) ·15
(3) 12
(4) ·9

PROPORTION

Rule.—Add together the logarithms of the quantities multiplied together, and subtract the logarithm of the divisor.

Or, add together the logarithm of the quantities multiplied and the arithmetical complement of the divisor.

EXAMPLES

If a ship sail 49·5 miles in 8 hours, how many miles will she run in 24 hours, supposing her to go at the same rate?

24	Log. 1·380211
49·5	Log. 1·694605
	Sum 3·074816
8	Log. 0·903090
148·5 miles	Log. 2·171726

Or thus,

24	Log. 1·380211
49·5	Log. 1·694605
8 Arith. Co.	Log. 9·096910
148·5 miles	Log. 2·171726

Suppose a watch or chronometer gain 14 seconds in 5 days 6 hours, how much will it gain in 17 days 15 hours?

17·625	Log. 1·246129
14	Log. 1·146128
	Sum 2·392257
5·25	Log. 0·720159
47 seconds	Log. 1·672098

Or thus,

17·625	Log. 1·246129
14	Log. 1·146128
5·25 Arith. Co.	Log. 9·279841
47 seconds	Log. 1·672098

GEOMETRY

GEOMETRY * is the science which treats of extension, or form ; that is extent of distance, extent of surface, and extent of capacity, or solid content.

DEFINITIONS

I

A SOLID is a magnitude which has length, breadth, and thickness.

II

A SURFACE, or SUPERFICIES, is the boundary of a solid, and has length and breadth only.

III

A LINE is the boundary of a surface, and has length only.

IV

A POINT is that which has no dimensions of any kind—neither length, nor breadth, nor thickness ; it has position, but not magnitude, being the extremity of a line.

V

A STRAIGHT LINE, or RIGHT LINE, is that which lies evenly between its extreme points, without changing its direction, and is the nearest distance between the two points that terminate it, as A B.

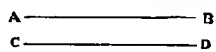
NOTE.—When the word " line " is used, it is understood to be a *straight* line, and is generally expressed by two letters at its extremes.

VI

A CURVED LINE is that which is continually changing its direction, as C D.

VII

PARALLEL LINES are such as extend in the same direction, being in every part at the same distance from each other, and which, if infinitely produced, would never meet ; as the lines A B and C D.

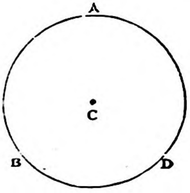


VIII

A PLANE SURFACE, or PLANE, is that with which a straight line will wholly coincide when drawn between any two points on it.

* Geometry (derived from two Greek words signifying *land* and *to measure*) originally meant nothing more than the measurement of land. The Egyptians are credited with its invention, as they had recourse to it in order to ascertain the effaced boundaries of their land after the annual inundations of the Nile ; but the Science, in its present extended sense, constitutes the foundation of Mathematics.

IX

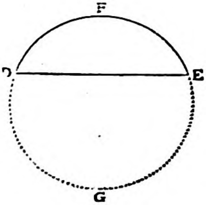


A CIRCLE is a plane figure bounded by a curved line, called the CIRCUMFERENCE, as A B D, which is in every part equally distant from a point within it, called the CENTRE, as C; it is formed by the revolution of a line about one of its extremities, which remains fixed.

The circumference is often called the circle; but properly the circle is the space contained within the circumference.

The circumference of a circle is divided into 360 equal parts, called DEGREES, which are subdivided into minutes and seconds (*see* p. 3). It is also divided into 32 equal parts of 11° 15′ each, which are called the POINTS OF THE COMPASS.

X

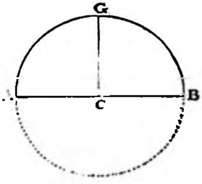


An ARC of a circle is any part of the circumference, as D F E.

A CHORD is a line joining the ends of an Arc, as D E; thus the line D E is the chord of the arc D F E.

A SEGMENT is a portion of a circle cut off by a chord; and in this case divides the circle into two unequal parts, as D F E and D G E.

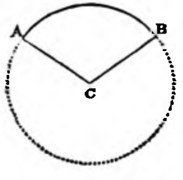
XI



A DIAMETER is a straight line drawn through the centre of a circle, and terminated at both ends by the circumference, as A C B; it divides the circle into two equal parts, called SEMICIRCLES, as A G B and A F B; a semicircle contains 180°.

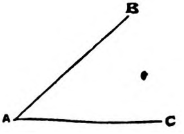
A QUADRANT is half a semicircle, or the fourth part of the whole circle, as A C G or G C B; a quadrant contains 90°.

A RADIUS, or SEMIDIAMETER, is a straight line drawn from the centre to any part of the circumference, and is the extent taken in the compasses to describe a circle, as C A, C G, or C B.



XII

A SECTOR of a circle is any part of a circle comprehended between two radii and their included arc, as A C B.



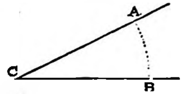
XIII

An ANGLE is the inclination or opening of two straight lines meeting in a point: the point where they meet is called the ANGULAR POINT, as A; and the lines that include the angle are the SIDES or LEGS, as A B, or A C.

An angle is sometimes expressed by three letters, the middle one always denoting the angular point, and the other two the legs that include it; but generally by the letter at the angular point only; as the angle B A C, or the angle A.

XIV

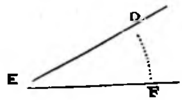
An angle is measured by an arc of a circle contained between its legs, making the angular point the centre of the circle; thus the arc A B is the measure of the angle A C B.



Increase or decrease in the length of the legs does not alter the angle, as the *length* of the lines is in no way connected with their *direction* towards each other.

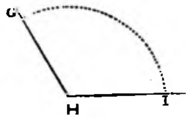
XV

Angles are said to be equal to each other when the arcs that measure them are equal, thus the angle D E F and the angle A C B (in Def. XIV.) are equal, since the arcs D F and A B are equal.



XVI

One angle is greater or less than another angle, according as the arc between its legs is greater or less; thus the angle G H I is greater than the angles A C B or D E F (in Def. XIV. and XV.).



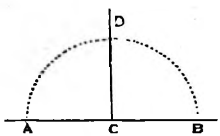
XVII

As all circles are divided into 360 equal parts, called degrees, etc., a certain number of these divisions will be contained between the two legs of the angle; therefore an angle is said to measure as many degrees, minutes, etc., as are contained in the arc between the legs.

The arc which measures an angle may be described with any radius; for, since the whole circumference of every circle is supposed to be divided into the same number of parts, it hence follows that the divisions will be greater or less in the same proportion as the whole circumference.

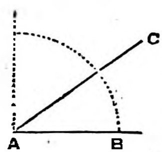
XVIII

A RIGHT ANGLE.—When one line falls upon another, so as to make the angles on each side of it equal, it is called a PERPENDICULAR; and the angles formed by these lines, as the angles A C D, D C B, are called RIGHT ANGLES. Now as the semicircle A D B contains 180 degrees (the half of 360), all right angles will contain an arc of 90 degrees, equal to the fourth part of the whole circle.



XIX

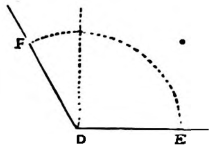
An ACUTE ANGLE is that which contains less than a right angle, or 90 degrees, as the angle C A B.



XX

An Obtuse Angle is that which contains more than a right angle, or 90 degrees, as the angle F D E.

Acute and obtuse angles are called Oblique Angles.

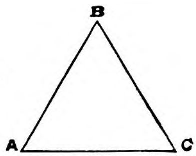


XXI

A Plane Triangle is a figure bounded by three right lines, and contains three angles, of which there are several kinds, both with respect to their sides and angles.

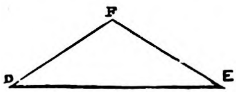
XXII

An Equilateral Triangle is that which has its three sides equal to one another, as A B C.



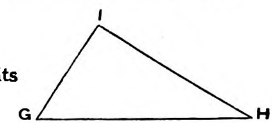
XXIII

An Isosceles Triangle is that which has only two of its sides equal, as D E F.



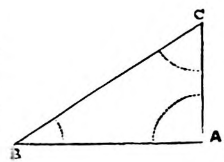
XXIV

A Scalene Triangle is that which has all its sides unequal, as G H I.



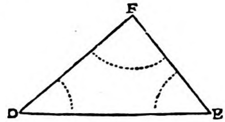
XXV

A Right Angled Triangle is that which has one of its angles a right angle, or containing 90 degrees, as the angle A : the side opposite the right angle is called the Hypothenuse, as B C ; and of the other two sides or Legs, that which stands upright is the Perpendicular, as A C ; and the other is the Base, as B A.



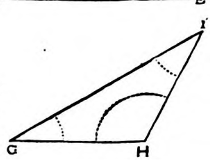
XXVI

An Acute Angled Triangle is that which has all its angles acute, as D E F.



XXVII

An Obtuse Angled Triangle is that which has one of its angles obtuse, as the angle H in the triangle G H I.

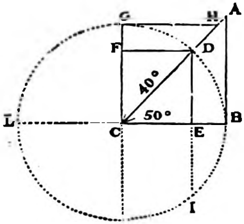


XXVIII

All triangles that are not right angled, whether they are acute or obtuse, are, in general terms, called OBLIQUE ANGLED TRIANGLES, without any other distinction.

Also, the three sides of a triangle are frequently distinguished by giving to one of them the name of *base*, in which case the other two are called *the two sides*, and the angular point opposite to the base is called the *vertex* (or summit). In an *isosceles* triangle the *vertex* is the angular point between the two equal sides, and the *base* the side opposite to it.

XXIX



The SINE of an arc is a line drawn from one extremity of the arc, perpendicular to a diameter drawn to the other extremity, and is equal to half the chord of double the arc: thus D E is the sine of the arc D B, and is equal to half the chord D I, which is the chord of the arc D B I—the double of the arc D B.

The VERSED SINE of an arc is that part of a diameter contained between the sine and the arc; thus E B is the versed sine of the arc D B; and E L is sometimes called the SUVERSED SINE.

The TANGENT of an arc is a line drawn perpendicular to the end of a diameter passing through one extremity of the arc, and continued till it meets a line drawn from the centre through the other end of the arc; thus A B is the tangent of the arc D B.

The SECANT of an arc is a line drawn from the centre of the circle through one end of the arc, till it meets the tangent drawn from the other end; thus C A is the secant of the arc D B.

The COMPLEMENT of an arc is what it wants of a right angle, or 90 degrees; thus G D is the complement of D B, or D B of D G.

The SUPPLEMENT of an arc is what it wants of two right angles, or 180 degrees; thus L D is the supplement of D B, or D B of L D.

The CO-SINE, CO-TANGENT, CO-SECANT, and CO-VERSED SINE of an arc, are the sine, tangent, secant, and versed sine of the complement of that arc; Co. being a contraction of the word *complement*: thus D F is the cosine, G H the cotangent, C H the cosecant, and G F the coversed sine of the arc D B; being the sine, tangent, etc., of the arc D G, the complement of the arc D B.

The sine, tangent, and secant of an arc, as of D B, is likewise the sine, tangent, and secant of the supplement of that arc, as of L D.

An angle being measured by an arc of a circle (*see* Def. XIV.), the sine, tangent, etc., of an arc is the sine, tangent, etc., of the angle which is measured by the arc, or of the degrees and minutes, etc., that the arc contains; hence, supposing the arc D B, which measures the angle D C B, to contain 50 degrees, the lines D E, A B, A C, and E B, will be respectively the sine, tangent, secant, and versed sine of the angle A C B, or of 50 degrees; and consequently the cosine, cotangent, cosecant, and coversed sine, of the angle G C D, or of 40 degrees, which is the complement of 50 degrees.

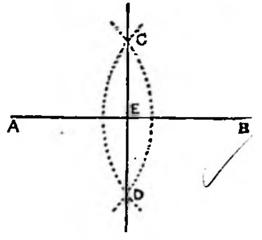
GEOMETRY

PROBLEM I

To divide a given Line A B *into two equal Parts*

Take any extent in the compasses greater than half the line A B, and with one foot in B describe an arc; with the same radius, and one foot in A, describe an arc cutting the former in C and D; through C and D draw a line; and this line will divide the given line A B into two equal parts at the point E.

In this manner any arc of a circle may be divided into two equal parts

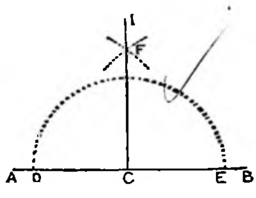


PROBLEM II

From a given Point C, *in a given Line* A B, *to raise a Perpendicular*

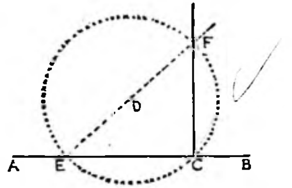
CASE 1st. When the given point C is near the middle of the line A B.

With one foot of the compasses in C, at any distance, draw an arc cutting the line A B in D and E; from the points D and E, with any distance greater than C E or C D, describe two arcs cutting each other in F; through the points F and C draw the line I C, and it will be perpendicular to the given line A B.

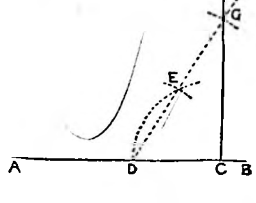


CASE 2nd. When the given point C is at, or near the end of the line A B.

Take any point out of the line, as D, and with the distance D C describe a circle, cutting the line A B in E and C; through the centre D and the point E draw the line E F, cutting the circle in F; then a line drawn through F and C will be the perpendicular required.



Or thus: Describe the arc D E at any distance from C; and with one foot of the compasses in D, with the same extent, describe an arc cutting the arc D E in E; from this point, keeping the same extent in the compasses, draw the arc G; through D and E draw the line D G, cutting the arc in G; then draw a line through G and C, and it will be the perpendicular required.

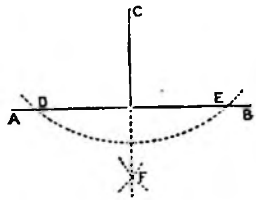


PROBLEM III

From a given Point C *to let fall a Perpendicular on a given Line* A B

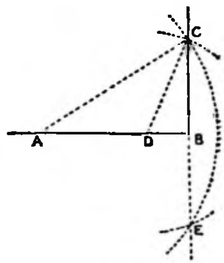
CASE 1st. When the point C is nearly opposite the middle of the line A B.

With one foot of the compasses in C, describe an arc cutting the line A B in D and E; from these points, at any distance, describe two arcs cutting each other in F; through the points C and F draw a line, and it will be perpendicular to the given line A B.



CASE 2nd. When the given point C is nearly opposite to the end of the line A B.

Place one foot of the compasses in any part of the given line, as at A, and with the distance A C describe the arc C E; then from any other part of the given line nearly under the point C, as at D, with the distance D C describe a small arc cutting the arc C E in E; then a line drawn through the points C and E will be perpendicular to the line A B.

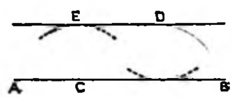


PROBLEM IV

To draw a Line parallel to a given Line A B

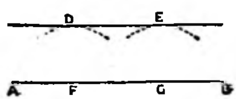
CASE 1st. When the parallel line is to pass through a given point D.

Take the nearest distance between the given point D and the line A B; with that distance set one foot of the compasses on any part of the line A B, as at C, and describe the arc E; from the point D draw a line so as just to touch the arc E without cutting it; then that line will be parallel to the given line A B, and pass through the given point D.



CASE 2nd. When the parallel line is to be at a given distance from the line A B.

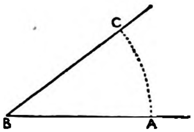
With the given distance in the compasses, describe two arcs D and E, from any two points, as F and G in the given line; then a line D E drawn just touching the two arcs without cutting them, will be parallel to the given line A B.

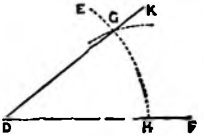


PROBLEM V

At a given Point in a Line to make an Angle equal to a given Angle

The given angle C B A; and D is the point in the line D F. With one foot of the compasses in B describe the arc A C: with the same extent in the compasses, place one foot in D and describe the arc H E; then take





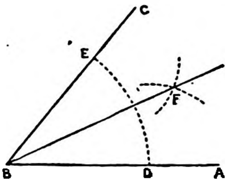
the distance A C and apply it to the arc H E from H to G, and through the points D and G draw the line D K; the angle G D H will then be equal to the angle C B A, as required.

PROBLEM VI

To divide a given Angle A B C *into two Equal Parts*

From the angular point B, with any extent in the compasses, describe the arc D E; from D and E, with the same or any other extent, describe two arcs cutting each other in F; through the points B and F draw a line, and it will divide the angle into two equal parts.

In the same manner any given arc of a circle is bisected, when the centre of the circle is given.

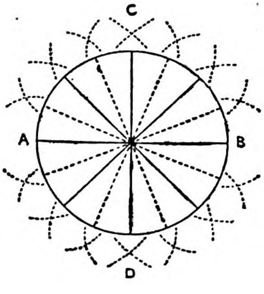


PROBLEM VII

To divide a Circle A B C D *into two, four, eight, sixteen, thirty-two, etc., Equal Parts*

Draw a diameter A B, and it will divide the circle into two equal parts; from the points A and B describe two arcs at C and D; a line drawn through these will divide the circle into four equal parts; then bisect the arcs A C, C B, etc., by Problem VI., and the circle will be divided into eight equal parts, and so on by continual bisections.

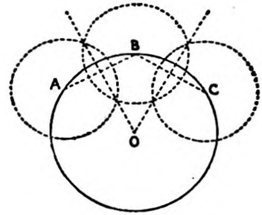
This problem is useful in constructing the Mariner's Compass.



PROBLEM VIII

To describe the Circumference of a Circle through any three given Points A B C, *not situated in a straight Line*

Draw lines joining A B and B C, and bisect them by lines meeting in O, as directed in Problem I.; then from O, at the distance of any one of the points, as O A, describe a circle, and it will pass through the other points B and C, as required.

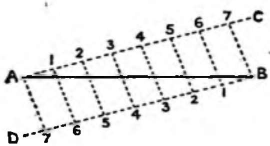


In this manner the centre of a circle may be found; for, taking any three points in the circumference, and proceeding as directed above, the lines meeting at O will give the centre required.

PROBLEM IX

To divide a given Line A B *into any proposed Number of Equal Parts*

Let it be required to divide the line A B into *seven* equal parts; from one end A, of the given line A B, draw a line A C, making any angle with A B, and from the other end B draw a line B D parallel to A C; on each of the lines A C, B D, beginning at A and B, set off as many equal parts as A B is to be divided into, viz. seven; then lines drawn from A to 7, 1 to 6, 2 to 5, 3 to 4, etc., will divide the given line into seven equal parts.



PROBLEM X

To construct Scales of Equal Parts

The simplest scale of equal parts is made by drawing a straight line, and dividing it with a pair of compasses into as many primary divisions as convenient, which, if the line be of a definite length, may be done by Problem IX.; subdivide one of these divisions decimally, *i.e.* into 10 equal parts; then each of the former may represent 10 units, as leagues, miles, etc., and in that case the latter will represent one of these units: or if the larger divisions be supposed to be 100, then the subdivisions will be tens, and so on.

There are frequently several of these scales drawn parallel to each other, of different lengths, on a flat rule (as Fig. 1, Plate I.); they are divided into as many equal parts as the length of the rule will admit; the numbers placed on the left hand showing how many parts in an inch each scale is divided into.

But the most correct scale of equal parts is the DIAGONAL SCALE (Fig. 2, Plate I.), the larger divisions of which are commonly an inch or half an inch, and sometimes a quarter of an inch, subdivided into 100 equal parts. To construct this scale, draw 11 equidistant parallel lines; divide the upper of these lines A E into such a number of equal parts as the scale is intended to contain; from each of these divisions draw perpendicular lines through

the 11 parallels to the line C F; subdivide the first of these divisions A B and C D into 10 equal parts, and from the point C to the first division in the line A B, draw a diagonal line; and then lines parallel to this through each succeeding subdivision. Proceed similarly with the subdivision of the part of the scale on the left hand.

Then, if the larger divisions be reckoned as units, the first subdivisions will be tenths, and the second (marked by the diagonals upon the parallels) hundredths; but if we take each of the larger divisions to represent 10, then the first subdivisions will be units, and the second tenths; or if the larger divisions be hundreds, then will the first subdivisions be tens, and the second units; so that the value of the subdivisions depends on that of the larger divisions.

The numbers 376, 37·6, 3·76 may therefore all be expressed by the same extent of the compasses: thus, setting one foot in the line marked 3 of the larger division, on the sixth parallel, and extending the other along the same parallel to the seventh diagonal, that distance will be the extent required; for if the three larger divisions be taken for 300, seven of the first subdivisions will be 70, which, upon the sixth parallel, taking in six of the second subdivisions for units, make the whole number 376: or if the three larger divisions be taken for 30, seven of the first subdivisions will be seven units, and the six subdivisions, upon the sixth parallel, will be six-tenths of a unit: lastly, if the three larger divisions be esteemed as only three, then will the first subdivisions be seven-tenths, and the six second subdivisions be the six-hundredth part of a unit.

PROBLEM XI

To construct Lines of Chords, Rhumbs, Tangents, Sines, etc.

Describe a semicircle A D B with any convenient radius (Fig. 3, Plate I.), and from the centre C erect the perpendicular C D, continued at pleasure to F; through B draw B E parallel to C F; and draw the lines A D and D B. Divide the quadrant D B into nine equal parts, and with one foot of the compasses in B and the distances B 10, B 20, etc., transfer them to the line B D, which will be a LINE OF CHORDS.

Divide the quadrant A D into eight equal parts, and with one foot of the compasses in A, and the distance A 1, and A 2, etc., transfer them to the line A D, and it will be a LINE OF RHUMBS, containing eight points of the compass.

From the points 10, 20, 30, etc., in the arc B D, draw lines parallel to D C, which will divide the radius C B into a LINE OF SINES, reckoning from C to B, or of VERSED SINES, if it be numbered from B to C; which may be continued to 180, if the same divisions be transferred to the line C A, the other half of the diameter.

From the centre C draw lines through the several divisions of the quadrant D B until they cut the line B E, which will become a LINE OF TANGENTS.*

Transfer the distances between the centre C and the divisions on the line of tangents to the line D F, and these will give the divisions of the LINE OF SECANTS, which must be numbered from D towards F.

From A draw lines through the several divisions of the arc B D, and

* From the construction of the lines of chords, sines, and tangents, it is obvious that the chord of 60°, the sine of 90°, and the tangent of 45° are all equal to the radius of the circle.

CONSTRUCTION OF THE LINES

ON THE

PLANE SCALE &c.

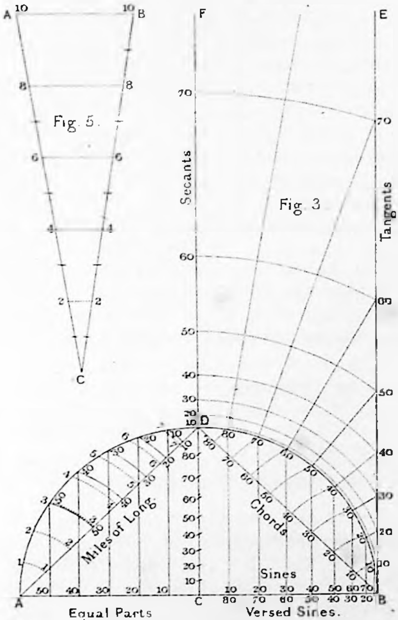


Fig. 5. Fig. 3.

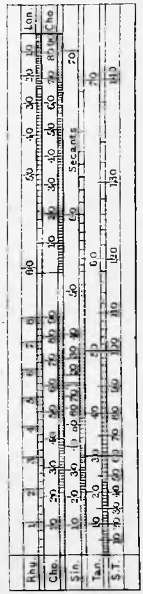


Fig. 4.

Fig. 1. COMMON SCALE OF EQUAL PARTS.

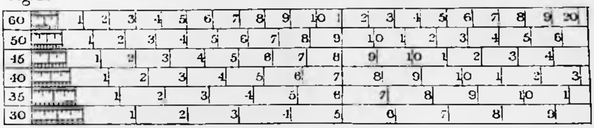
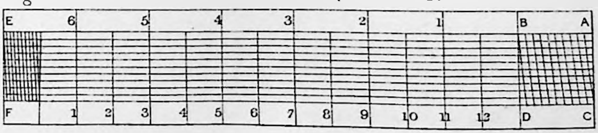


Fig. 2. DIAGONAL SCALE OF EQUAL PARTS.



J.W. Appleton del. Imray, Laurie, Norie & Wilson. H.& Homewood sc.

they will divide the radius C D into a LINE OF SEMI-TANGENTS, which are to be marked with the corresponding figures of the arc D B.

Divide the radius A C into six equal parts; through each of these draw lines parallel to C D, intersecting the arc A D; then, with one foot of the compasses in A, and the distances of the arc A 50, A 40, etc., transfer these to the line A D, and it will give the divisions of the LINE OF LONGITUDE.

If this line be laid upon the scale close to the line of chords, so that 60 on the line of longitude be opposite 0 on the chords, and any degree of latitude be counted on the chords, there will stand opposite to it, on the line of longitude, the miles contained in one degree of longitude in that latitude, the measure of a degree at the equator being 60 miles.

In the figure the divisions are given only to every tenth degree, and each point of the compass, which is sufficient to explain the *method of construction;* but in Fig. 4 these lines are graduated to degrees, and the rhumbs to quarters, and placed parallel, as exhibited on one side of a flat rule, which, with the scale of equal parts on the other side, constitutes the instrument called a PLANE SCALE.

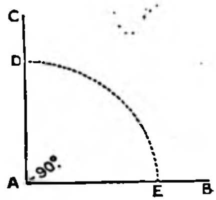
Besides the lines already mentioned, there are frequently on the Plane Scale a few other lines, but these are only so many scales of equal parts, each having equal divisions of different lengths, for the more readily laying down lines and figures of different lengths and magnitudes.

PROBLEM XII

To make an Angle that shall contain any proposed Number of Degrees

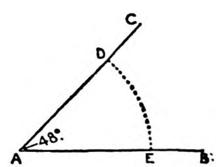
NOTE.—Angles are measured or laid off by means of the Scale of Chords (*see* Fig. 4). But a brass semicircle, or transparent *horn* semicircle, is a very useful instrument for all chart purposes.

CASE 1st. When the given angle is right, that is, contains 90 degrees.



Draw the line A B, and from the scale take the extent of the chord of 60 degrees in the compasses; then set one foot of the compasses in A, and with the other describe the arc E D, and set off thereon, from E to D, the distance of the chord of 90°; through A and D draw the line A C, then will the angle B A C be a right angle. By this method a perpendicular may easily be raised on a given line, since the angle formed by one line that is perpendicular to another is always a right angle.

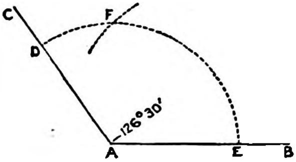
CASE 2nd. When the angle is to be acute; suppose one that shall contain 48 degrees.



Draw the line A B, and with one foot of the compasses in A (the chord of 60 degrees being taken as before), draw the arc E D, on which set off 48 degrees from E to D; through A and D draw the line A C; then will the angle B A C be made, containing 48 degrees, as was required.

CASE 3rd. When the angle is to be obtuse; suppose one that shall contain 126° 30′.

Draw the line A B, and from the point A, with the chord of 60° as before, draw the arc D E, and, as the divisions on the scale extend no further than 90°, first set off 90° from E to F; then set off the remainder, or excess above 90°, that is 36° 30′, from F to D; through A and D draw the line A C, and the angle B A C will contain 126° 30′.



PROBLEM XIII

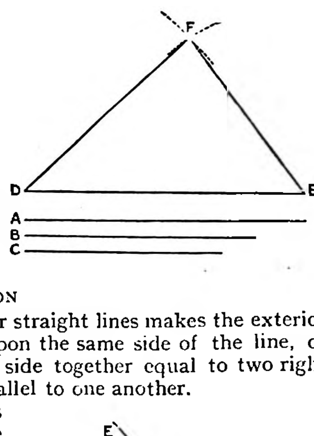
To measure a given Angle B A C

With one foot of the compasses in the angular point, and with the chord of 60 degrees, describe the arc D E (*see* the figures in Problem II.) cutting the legs in D and E; then the distance D E applied to the line of chords, from the beginning, will show the measure of the angle B A C, if it contain less than 90 degrees; but when the arc exceeds that quantity, take 90 degrees from the line of chords, and set it off from E to F; then measure the excess F D, and their sum will give the measure of the angle required.

PROBLEM XIV

To describe a triangle of which the three sides shall be respectively equal to three given straight lines, two of these lines being greater than a third.

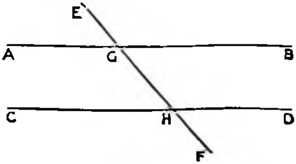
Let A B C be the three given lines. Take a straight line equal in length to A and call it D E. About D as a centre with a radius equal to B describe an arc of a circle, and about E as a centre with a radius equal to C describe another arc intersecting the former in the point F. Join D F and E F, and F D E is the triangle required.



PROPOSITION

If a straight line falling upon two other straight lines makes the exterior angle equal to the interior and opposite upon the same side of the line, or makes the interior angles upon the same side together equal to two right angles, the two straight lines shall be parallel to one another.

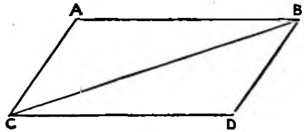
Let the straight line E F, which falls upon the two straight lines A B, C D, make the exterior angle E G B equal to the interior and opposite angle G H D upon the same side; or make the two interior angles B G H, G H D, on the same side, together equal to two right angles; A B is parallel to C D.



Because the angle E G B is equal to the angle G H D, and the angle E G B is equal to the angle A G H, the angle A G H is equal to the angle G H D, and they are alternate angles; therefore A B is parallel to C D. (Euclid 1-27).

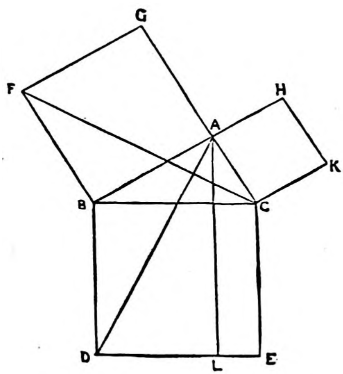
Parallelogram of Forces

If two adjacent sides represent in direction and amount two given forces acting at the point of meeting, a diagonal from this point will represent in direction and amount the equivalent force, or resultant of the two given forces.



The opposite sides of a parallelogram, as A B, C D, are equal, also the opposite angles are equal; and the diameter or diagonal C B divides it into two equal parts.

Since A B and C D are parallel, and C B meets them, the angle A B C is equal to the alternate angle B C D (1,29). And because A C is parallel to B D and B C meets them, therefore the angle A C B is equal to the alternate angle C B D (1,29). Hence in the two triangles A C B, C B D, because the angles A B C, B C A in the one are equal to the two angles B C D, C B D in the other, each to each; and one side B C which is adjacent to their equal angles, common to the two triangles; therefore their other sides are equal, each to each, and the third angle of the one equal to the third angle of the other (1,26), namely, the side A B to the side C D, and A C to B D, and the angle B A C to the angle B D C, and because the angle A B C is equal to the angle B C D, and the angle C B D to the angle A C B, therefore the whole angle A B D is equal to the whole angle A C D; and the angle B A C has been shown to be equal to B D C; therefore the opposite sides and angles of a parallelogram are equal to one another. Also the diameter B C bisects it. For since A B is equal to C D, and B C common, the two sides, A B, B C, are equal to the two D C, C B, each to each, and the angle A B C has been proved to be equal to the angle B C D; therefore the triangle A B C is equal to the triangle B C D (1,4); and the diameter B C divides the parallelogram A C D B into two equal parts.—Q. E. D.



In a right-angled triangle the square described on the hypotenuse is equal to the sum of the squares described on the other two sides.

Let A B C be a right-angled triangle, having the angle B A C a right angle; then shall the square described on the hypotenuse B C be equal to the sum of the squares described on B A, A C.

On B C describe the square B D E C; and on B A, A C describe the squares B A G F, A C K H. Through A draw A L parallel to B D or C E; and join A D, F C. Then because each of the angles B A C, B A G is a right angle, C A and A G are in the same straight line. For the same reason, B A and A H are in the same straight line. Now the angle C B D is equal to the angle F B A,

for each of them is a right angle. Add to each of these angles the angle A B C, therefore the whole angle D B A is equal to the whole angle F B C; and because the two sides A B, B D, are equal to the two sides F B, B C, each to each, and the included angle A B D is equal to the included angle F B C, therefore the base A D is equal to the base F C, and the triangle F B C to the triangle A B D.

Now the parallelogram B L is double of the triangle A B D (I. 41) because they are upon the same base and between the same parallels B D and A L; also the square G B is double of the triangle F B C, because these also are on the same base F B, and between the same parallels F B, G C.

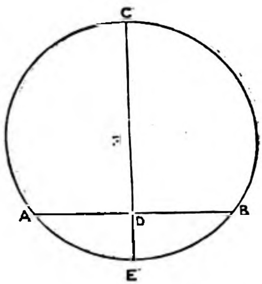
But the doubles of equals are equal to one another; therefore the parallelogram B L is equal to the square B G. Similarly, by joining A E, B K, it can be proved that the parallelogram C L is equal to the square H C. Therefore the whole square B D E C is equal to the two squares G B, H C.—Q. E. D.

Proposition

To find the Centre of a given Circle

Let A B C be the given circle; it is required to find its centre

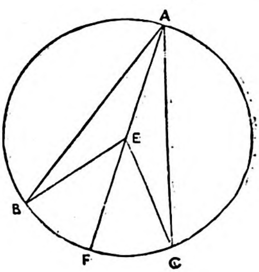
Draw within it any straight line A B and bisect A B at D; from the point D draw D C at right angles to A B, produce C D to meet the circumference at E and bisect C E at F. The point F shall be the centre of the circle A B C. It is manifest that if, in a circle, a straight line bisect another at right angles, the centre of the circle is in the straight line which bisects the other.



Proposition

The angle at the centre of a circle is double the angle at the circumference on the same base, that is, on the same arc.

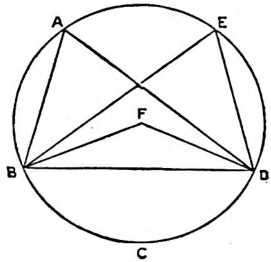
Let A B C be a circle and B E C an angle at the centre, B A C an angle at the circumference which have the same arc B C for their base; the angle B E C shall be double of the angle B A C. Let the centre of the circle be within the triangle B A C; join A E and produce it to F; then because E A is equal to E B, the angle E B A is equal to the angle E A B; the angle E B A plus the angle E A B equals the supplement of the angle A E B, but the angle B E F is also the supplement of A E B, therefore the angle B E F is double of the angle E A B. It can be shown that the angle F E C is double of the angle E A C; therefore the whole angle B E C is double of the whole angle B A C.



PROPOSITION

The angles on the same segment of a circle are equal to one another.

Let A B C D be a circle, and B A D, B E D angles in the same segment, B A E D; the angles B A D, B E D shall be equal to one another. Take F the centre of the circle A B C D.



Let the segment B A E D be greater than a semicircle. Join B F, D F.

Then because the angle B F D is at the centre and the angle B A D is at the circumference, and that they have the same arc for their base, namely B C D; therefore the angle B F D is double the angle B A D (M, 20). For the same reason the angle B F D is double the angle B E D.

Therefore the angle B A D is equal to the angle B E D.

Wherefore the angles in the same segment of a circle are equal to one another, whether the segment be greater or less than a semicircle.

The construction of triangles will be explained in Trigonometry.

PLANE TRIGONOMETRY

Plane Trigonometry is that branch of Mathematics which teaches us how to *compute* the sides and angles of plane triangles; it is divided into *right-angled* and *oblique-angled* Trigonometry, according as it is applied to the mensuration of right-angled triangles or oblique-angled triangles. It is used by the navigator to solve all the problems in the sailings except great circle sailing, which is solved by right-angled and oblique-angled spherical trigonometry.

RIGHT-ANGLED TRIGONOMETRY

1. Every triangle consists of six parts; namely, three sides and three angles.

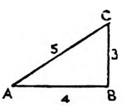
2. The sum of three angles of every plane triangle is equal to two right angles, or 180°; hence, if one of the angles be known, the sum of the other two may be found by subtracting the given angle from 180°: also, if two of the angles be known, their sum subtracted from 180° will give the third angle. Again, in a right-angled triangle (since the right angle contains 90°), the sum of the two acute angles is equal to 90°: therefore, if one of the acute angles be given, the other will be found by subtracting the given angle from 90°.

3. Any two sides of a triangle added together will be greater than the third side.

4. The greatest side of a triangle is opposite the greatest angle, and the least side opposite the least angle; also, in the same triangle, equal sides are opposite to equal angles.

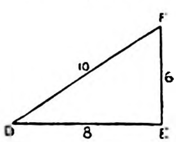
5. In any right-angled triangle, the side which is opposite to the right angle is called the *hypotenuse*; and of the other two sides, one is frequently termed the base, and the other, the perpendicular.

In every right-angled triangle, the square of the hypotenuse (or side opposite to the right angle) is equal to the sum of the squares of the sides which contain that angle.



Hence also in a right-angled triangle, the square of either of the two sides is equal to the difference of the squares of the hypotenuse and the other side.

6. Two right-angled triangles are *equal* to one another in all respects, when they have



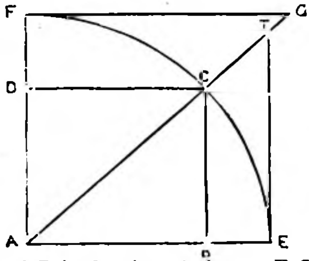
1. The hypotenuse, and a side of the one equal to the hypotenuse and a side of the other, each to each;
2. The hypotenuse and an acute angle equal;
3. Two sides equal;
4. A side and the adjoining acute angle equal;
5. A side and the opposite acute angle equal;

7. Two triangles are said to be *similar* when all the angles of the one are respectively equal to all the angles of the other; as, for instance, the triangle A B C is similar to the triangle D E F, because the angles A, B, and C are respectively equal to the angles D, E, and F.

The sides of similar triangles, opposite to equal angles, are proportional; thus in the triangles A B C and D E F, as A B is to D E, so is A C to D F, and so is B C to E F. Or as 4 : 8 : : 5 : 10 : : 3 : 6. But this is better illustrated on the basis of the ratio of the sides, as will be seen presently.

8. The old method of computing the sides and angles of a triangle was on the basis of proportion, and the names of the sides (as lines) bore reference to the *arc* of a circle and the angle that it subtended or measured. It is not wholly inappropriate to briefly note this method in connection with the annexed figure, bearing in mind that these definitions appertain to a quadrant, and an arc less than a quadrant.

Two arcs, the sum of which is a quadrant, or quarter of a circle, are called *complements* of each other, thus F C is the complement of E C, and E C the complement of F C.



In the quadrant E C F, let E C be an arc of a circle, and E A C the corresponding angle at the centre of the circle; then F C is the complemental arc, and F A C the complemental angle.

SINE.—The *sine of an arc* is a straight line drawn from one extremity of the arc perpendicular to the radius which passes through the other extremity of the arc; here C B is the sine of the arc E C, or of the angle E A C, to radius A C.

TANGENT.—The *tangent of an arc* is a straight line which touches the measuring arc at its commencement, and terminates in the radius *produced* through and beyond the other extremity of the arc: here E T is the tangent of the arc E C, or of the angle E A C, to radius A E, which is equal to A C.

SECANT.—The *secant of an arc* is a straight line extending from the centre of the circle, through one end of the arc, until it meets the further extremity of the tangent drawn from the commencement of the arc; here A T is the secant of the arc E C, or of the angle E A C, to radius A E or A C.

Since E is taken to be the origin or commencement of the arc which measures the angle E A C, so F may be taken to be the commencement of the arc F C which subtends the angle F A C; to the latter arc or angle, C D is the sine, F G is the tangent, and A G is the secant.

The complement of an arc being what the arc wants of a quadrant or 90°; in the figure, the arc C F is the complement of E C; and C A F, the angle which C F subtends, is the complement of the angle E A C. Now the *sine, tangent,* and *secant of the complement* of an arc are called the COSINE, COTANGENT, and COSECANT of the original arc.

Thus, generally, with respect to the arc E C or the angle E A C—

	C B is called the sine			*abbreviated*		sin.
	E T	,,	,, tangent,	,,	,,	tan.
	A T	,,	,, secant,	,,	,,	sec.
Also,	A B	,,	,, cosine,	,,	,,	cos.
	F G	,,	,, cotangent,	,,	,,	cot.
	A G	,,	,, cosecant,	,,	,,	cosec.
And,	E B	,,	,, versed sine,	,,	,,	ver. sin., or vers.

To the arc F C, it follows that C D will be its sine; F G its tangent; A G its secant; D A its cosine; E T its cotangent; and A T its cosecant.

Since F C and E C are complements of each other; also, since C B = D A, and C D = B A, therefore—

C B, the sine of E C = D A, the cosine of F C
B A, the cosine of E C = C D, the sine of F C
E T, the tangent of E C, is the cotangent of F C
F G, the cotangent of E C, is the tangent of F C

Thus:—

The *sine* of an angle is equal to the *cosine* of its *complement*; the *cosine* to the *sine* of its *complement*; the *tangent* to the *cotangent* of its *complement*.

9. The lines to which names (as sines, tangents, etc.) have been given in the preceding sections, are sufficiently correct as *geometrical* lines, appertaining to the arc E C, or angle E A C, which corresponds to the radius A E; thus, generally, lines so defined refer to circular arcs and the angles they subtend (or measure). To a circle less, or greater, than that in the fig., there would be no change in the angle by the diminution, or extension, of radius A C and A E; but the sines, tangents, &c., would differ considerably in length, and in fact before the value of the lines could be found we must know the length of the radius proper to the lines. There is a way out of this difficulty.

Trigonometry deals with the angles and sides of triangles, irrespective of arcs that measure angles; and since an invariable angle must have an invariable sine, cosine, tangent, etc., the difficulty is overcome by always considering radius as *unity*, or the abstract number 1, and treating the lines on the basis of ratios. To this method of treatment belong the trigonometrical functions, as distinguished from the geometrical definitions and their consequences.

RATIO.—It is well that you should have a clear notion of the *meaning of* the term *ratio*, which is commonly used in Trigonometry. Instead of saying that A is to B in the proportion of 3 to 5, we say "in the ratio of 3 to 5," and it is usually expressed as a fraction.

The ratio of one magnitude to another is independent of the *kind* of magnitudes compared: thus, one may contain the other, or the fifth, or twentieth, or hundredth part of the other the same number of times, whether they be lines, or surfaces, or solids, or again, weights or parts of duration.

It is required only for the comparison that the magnitudes be of the same kind, containing the same magnitude, each of them a certain number of times, or a certain number of times nearly. Upon these numbers, and upon these only, the ratio depends.

In brief then, ratio may be defined as the relative values of two quantities of the same kind, or the number of times that one contains the other.

TRIGONOMETRICAL TABLES.—There are two kinds of Trigonometrical tables that may be used in the computation of the sides and angles of a triangle. One kind contains the sines, cosines, tangents, etc., of every degree and minute of the quadrant, calculated as decimal fractions, to radius *unity*, or the abstract number 1. The sines, tangents, &c., of these tables are called NATURAL SINES,—and thus we speak of *natural* sines, *natural* cosines, *natural* tangents, &c., but they are not generally used in naviga-

tion, and some works do not even contain them. In the use of these tables, the product of two quantities is got only by multiplication, and the quotient by division, which is sometimes tedious. Tables constructed to radius 1 are called *natural* to distinguish them from another description of trigonometrical tables which are called *logarithmic.*

The *logarithmic* sines, cosines, tangents, &c., are constructed on the base of the natural sines, cosines, tangents, &c. As in the latter case, *all* the sines and cosines, all the tangents from 0° to 45°, and all the cotangents from 45° to 90°, are less than *unity* (or 1), the logarithms of these quantities have negative characteristics or indices. But, in order to avoid the necessity of entering negative numbers, the logarithmic tables are constructed by adding 10 to every index, and so registered. By this contrivance, addition does the work of multiplication, and division that of subtraction, and thus, in trigonometrical calculations, by the aid of logarithms of numbers on the same construction, much time and labour are saved. The tables here referred to are Log. Sines, etc., and Logarithms; when using them it is better to distinguish the logarithms of numbers as *log.;* those of sines, cosines, and tangents, etc., as L sin., L cos., L tan., etc.

TRIGONOMETRICAL FUNCTIONS OF AN ANGLE

In Trigonometry we deal with sides and angles of a triangle, and the trigonometrical functions of an angle are *independent of the magnitude of the radius.*

Take several right-angled triangles (*see* Fig. 1), as B A C, D A E, F A G, H A K, etc., which have the same acute angle at A common to each triangle; it is certain that the triangles are not equal, since they differ in magnitude, one from another; but they are nevertheless in all respects *similar,* since, the acute angle A, appertaining equally to all the triangles, is due to the ratio between the sides being constant; the magnitude of the sides is not in question; the condition lies in this, that whatever part A C is of A E, the same part must C B be of E D, etc., and so forth; or, on the basis of ratios

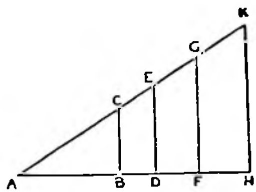


Fig. 1.

$$CB : CA = ED : EA = GF : GA = KH : KA$$

Or,
$$\frac{CB}{CA} = \frac{ED}{EA} = \frac{GF}{GA} = \frac{KH}{KA}$$

since the ratio is generally expressed in the form of a fraction.

To impress upon you more clearly still the idea of what is meant by the connection between the angles and the ratios of a triangle; in the above fig. right-angled at B, say we have $\angle$ C and $\angle$ A each equal to 45°; this would not arise from the opposite sides being each 10 feet, or each 50 feet, or each 100 feet, but from the *equality* of the lengths. So also if $\angle$C were 30°, as would happen if the hypotenuse were 100 feet, and the side opposite $\angle$C only 50 feet, then $\angle$C would be 30°, not owing to the

magnitude of the sides, but *because* the hypotenuse was *double* the side opposite $\angle$C. Hence—

The angles of a triangle depend not upon the absolute length of the sides of the triangle, but upon their RELATIVE *lengths*, that is, *upon the ratios existing between them.*

On page 41 the sines, cosines, tangents, &c., are taken to be lines connected with an arc, but in practice they are considered as *quantities* corresponding to certain *ratios*, called trigonometrical functions of an angle.

In the annexed figure (2) the three sides of the right-angled triangle, by taking the sides two and two, give six ratios as follow—

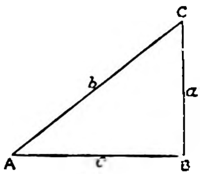


Fig. 2.

$$\frac{CB}{AC} \quad \frac{AB}{AC} \quad \frac{CB}{AB} \quad \frac{AB}{CB} \quad \frac{AC}{AB} \quad \frac{AC}{CB}$$

of which the last three are the inverse of the first three; and each of these ratios *measures* and *determines* the angle A, inasmuch as "one variable quantity which *measures* another will also *determine* it, if a determinate value of one necessarily corresponds to a determinate value of the other; and conversely."

The definitions that the ratios, taken in the above order, give to the angle A, are in succession,

sine cosine tangent cotangent secant cosecant ;

they also define the angle C as

cosine sine cotangent tangent cosecant secant,

since the right-angle B = 90°; and A + C = 90°; consequently C = (90° — A) being the complement of A; and the sine of an angle equals the cosine of its complement, etc., etc.

These remarks, set out in tabular order, are more clearly seen below—

$$\text{sin. } A = \frac{CB}{AC} = \text{cos. } C \qquad \text{cot. } A = \frac{AB}{CB} = \text{tan. } C$$

$$\text{cos. } A = \frac{AB}{AC} = \text{sin. } C \qquad \text{sec. } A = \frac{AC}{AB} = \text{cosec. } C$$

$$\text{tan. } A = \frac{CB}{AB} = \text{cot. } C \qquad \text{cosec. } A = \frac{AC}{CB} = \text{sec. } C$$

You should study these carefully, and be able to write them off rapidly, for on these six ratios depend the whole of the formulæ of Trigonometry.

It not unfrequently happens that, in cases where the angle A is taken as contained by the *base* and *hypotenuse*, and subtended by the *perpendicular*, the trigonometrical ratios are given as—

$$\sin. A = \frac{\text{perp.}}{\text{hyp.}} \qquad \tan. A = \frac{\text{perp.}}{\text{base}} \qquad \sec. A = \frac{\text{hyp.}}{\text{base}}$$

$$\cos. A = \frac{\text{base}}{\text{hyp.}} \qquad \cot. A = \frac{\text{base}}{\text{perp.}} \qquad \text{cosec.}\ A = \frac{\text{hyp.}}{\text{perp.}}$$

Now the side that subtends the right angle is invariably called the hypotenuse, but it is the more frequent and better method to consider the remaining sides as *sides*, without any characteristic distinction beyond the literal one.

From the trigonometrical ratios are readily deduced.

RULES

For computing the Angles and Sides of Right-Angled Triangles—

RELATIONS BETWEEN THE SIDES AND ANGLES.—As the six ratios determine the angle, they also give six equations through which the value of the respective sides may be found. In collating these (*see* Fig. 2, page 44), the angles are indicated by the capital letters A, B, C, and the corresponding sides opposite to them by the italic letters *a, b, c;* then we have—

	Ratios, as Definitions.		Equations, or Consequences.	C as the complement of A.
(1)	$\text{Sin. } A = \frac{a}{b} = \cos. C$;	hence,	$a = b \sin. A$	$= b \cos. C$
(2)	$\text{Cos. } A = \frac{c}{b} = \sin. C$;	,,	$c = b \cos. A$	$= b \sin. C$
(3)	$\text{Tan. } A = \frac{a}{c} = \cot. C$;	,,	$a = c \tan. A$	$= c \cot. C$
(4)	$\text{Cot. } A = \frac{c}{a} = \tan. C$;	,,	$c = a \cot. A$	$= a \tan. C$
(5)	$\text{Sec. } A = \frac{b}{c} = \text{cosec. } C$;	,,	$b = c \sec. A$	$= c\ \text{cosec. } C$
(6)	$\text{Cosec. } A = \frac{b}{a} = \sec. C$:	,,	$b = a\ \text{cosec. } A$	$= a \sec. C$

$$A = (90° - C), \text{ and } C = (90° - A).$$

Also by Euclid, Book I., prop. 47—

(7) Since $b^2 = a^2 + c^2$, therefore $b = \sqrt{a^2 + c^2}$

(8) Also, $a^2 = b^2 - c^2 = (b + c)(b - c)$

And $c^2 = b^2 - a^2 = (b + a)(b - a)$

But the methods of computation (7 and 8) give no saving of figures.

The foregoing include all cases of finding either angle or any side in a right-angled triangle and are adapted to logarithmic computation.

NOTE.—When two quantities are put together without any *sign* between them, they are multiplied together, thus in (1) b sin. A means b multiplied by sin. A. When quantities are within brackets without any sign between the brackets, the quantity within the one bracket is to be multiplied by the quantity within the other bracket, thus in (8) $(b+c)(b-c)$ means that $b+c$ is to be multiplied by $b-c$.

The sides and angles of a triangle make up the six parts of the triangle ; and if any three of these six parts, *excepting the three angles*, be given, the remaining three may be found by calculation. In a right-angled triangle the right angle is always known, hence it is sufficient for the determination that any two of the other five parts (*excepting the two acute angles*) be given.

It is obvious from the simplest principles of Geometry that the three angles alone cannot determine the other three parts, *viz.*, the three sides, since all *equiangular* triangles are alike, in respect to the equality of the angles, but the sides may differ ; hence an infinite variety of triangles may be constructed with the same three angles.

In a right-angled triangle, therefore, the given parts may be either

(1) A side and one of the acute angles ; or
(2) Two of the sides.

General rule to find a side.—In (1) to solve the triangle, we have to find the other two sides and the remaining acute angle. Let us first find one of the sides. Write down the side required, and underneath it put the given side so as to form a fraction ; we shall then on reference to the figure see what trigonometrical ratio of the given angle has been made. Now put the sign of equality between the fraction and the trigonometrical ratio; the resulting equation will give the formula by which the required side is found. To exemplify this, suppose in the Fig. 2, p. 44, the side b was given and also the angle A, and we want to find the side a. Write down a and underneath it put b, thus, $\frac{a}{b}$; on referring to the figure we see that this fraction represents the trigonometrical ratio sin. A. We now put the fraction equal to the trigonometrical ratio thus, $\frac{a}{b} = \text{sin. A}$, therefore, $a = b \times \text{sin. A}$. Hence the side a will be found by adding together the logarithm of b and the sin. A, rejecting 10 from the index and taking out the natural number corresponding to the sum of the logarithms. The other side of the triangle is found in a similar manner.

To find the remaining acute angle, subtract the given angle from 90°.

General rule to find an angle when two sides are given.—In (2) to solve the triangle, we have to find the two acute angles and the remaining side. When an angle is not given we must always first find one of the angles. To do this, write down in the form of a fraction the two given sides (it does not matter which is put on top), put the corresponding trigonometrical ratio equal to this fraction, and the formula is complete for finding the angle. To exemplify this, suppose in the Fig. 2, p. 44, a and b are given, and we want to find the angle A. Form a fraction with a and b thus, $\frac{a}{b}$; on referring to the figure we see that this fraction represents the trigonometrical ratio sin. A. We now put this trigonometrical ratio equal to the fraction, thus, $\text{sin. A} = \frac{a}{b}$. Hence the angle A will be found by subtracting the logarithm of b from the logarithm of a plus 10, and the degrees

and minutes corresponding to this difference in the sin. column will be the angle A required. The other parts will be found as in (1).

Given the Hypotenuse (b) *and Angle* A, *to find the Base* (c), *and Perpendicular* (a)

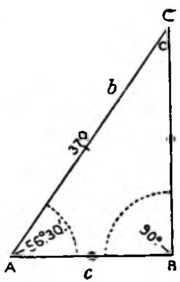
Example.—Let the hypotenuse = 370 yards, and the angle A = 56° 30′; required the angle C, also the base, and the perpendicular.

BY CONSTRUCTION

To find Angle C

$$\begin{array}{rr} & 90^\circ\ 00^\circ \\ \angle A & 56\quad 30 \\ \hline \angle C & 33\quad 30 \end{array}$$

Draw the line A B of any length, and make the angle at A 56° 30′ (Problem XII. Geometry); from A to C lay off 370, the length of the hypotenuse, taken from any convenient scale of equal parts, and from the point C let fall the perpendicular C B (Problem III. Geometry); then A B C is the triangle required: the base A B, measured on the same scale of equal parts by which the hypotenuse was measured, will be 204·2, and the perpendicular B C 308·5.



Calculation by Logarithms

To find the Base.

$\frac{c}{b}$ = cos. A ∴ $c = b$ cos. A.

log. c = log. b + L. cos. A — 10.

b 370	log.	2·568202
A 56° 30′	L. cos.	9·741889
c 204·22	log.	2·310091

To find the Perpendicular.

$\frac{a}{b}$ = sin. A ∴ $a = b$ sin. A.

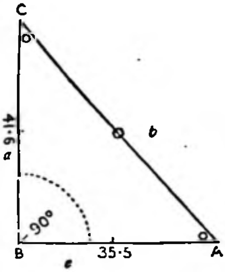
log. a = log b + L. sin. A — 10.

b 370	log.	2·568202
A 56° 30′	L. sin.	9·921107
a 308·54	log.	2·489309

Ans. C = 33° 30′; base = 204·22 yards; perpendicular = 308·54 yards.

Given the Base (c), *and Perpendicular* (a), *to find the Angles and the Hypotenuse* (b)

Example.—Let the base = 35·5, and the perpendicular = 41·6; required the angles A and C, and the hypotenuse.



BY CONSTRUCTION.

Draw the line B A, and from B erect the perpendicular B C; make B A equal to 35·5, and B C equal to 41·6, and draw the line A C; then the hypotenuse A C will measure 54·7, the angle A $49\frac{1}{2}°$, and the angle C $40\frac{1}{2}°$.

To find the Angles A *and* C

$$\text{tan. A} = \frac{a}{c}$$

L. tan. A = log. a + 10 — log. c.

a 41·6	log. (+ 10)	11·619093
c 35·5	log.	1·550228
A 49° $31\frac{1}{2}'$	L. tan.	10·068865
90		
C 40 $28\frac{1}{2}$		

A = 49° $31\frac{1}{2}'$ C = 40° $28\frac{1}{2}'$.

To find the Hypotenuse

$$\frac{b}{c} = \text{sec. A} \therefore b = c \text{ sec. A.}$$

log. b = log. c + L. sec. A — 10

c 35·5	log.	1·550228
A 49° $31\frac{1}{2}'$	L. sec.	10·187678
b 54° 69′	log.	1·737906

Hypotenuse = 54·69 yards.

Examp'es for Practice

1. Given the hypotenuse 108, and the angle opposite the perpendicular 25° 36′; required the base and perpendicular.
Ans. The base is 97·4, and the perpendicular 46·66.

2. Given the base 96, and its opposite angle 71° 45′; required the perpendicular and the hypotenuse.
Ans. The perpendicular is 31·66, and the hypotenuse 101·1.

3. Given the perpendicular 360, and its opposite angle 58° 20′; required the base and the hypotenuse.
Ans. The base is 222 and the hypotenuse 423.

4. Given the base 720, and the hypotenuse 980; required the angles and the perpendicular.
Ans. The angles are 47° 17′ and 42° 43′, and the perpendicular 664·8.

5. Given the perpendicular 110·3, and the hypotenuse 176·5; required the angles and the base.
Ans. The angles are 38° 41′ and 51° 19′, and the base 137·8.

6. Given the base 360, and the perpendicular 480; required the angles and the hypotenuse.
Ans. The angles are 53° 8′ and 36° 52′, and the hypotenuse 600.

7. Given the base 346·5, and the adjacent angle 35° 24′; required the perpendicular and the hypotenuse.
Ans. The perpendicular is 246·2, and the hypotenuse 425·1.

8. Given the hypotenuse 36·5, and the angle opposite the base 65° 15′; required the perpendicular and the base.

Ans. The perpendicular is 15·28, and the base 33·15.

9. Given the perpendicular 725, and the adjacent angle 21° 36′; required the base and the hypotenuse.

Ans. The base is 287·1, and the hypotenuse 779·8.

10. Given the base 32·76, and the hypotenuse 56·95; required the angles and the perpendicular.

Ans. The angles are 35° 7′ and 54° 53′, and the perpendicular 46·58.

11. Given the perpendicular 98·4, and the hypotenuse 1013; required the angles and the base.

Ans. The angles are 5° 34′ and 84° 26′, and the base 1009.

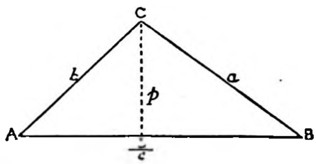
12. Given the base 4567, and the perpendicular 3251; required the angles and the hypotenuse.

Ans. The angles are 35° 27′ and 54° 33′, and the hypotenuse 5606.

OBLIQUE-ANGLED TRIGONOMETRY

NOTE.—If you have carefully studied the Ratios (p. 13) you can easily understand the following investigation.

It is required to find a relation between the sides of a plane triangle and the trigonometrical ratios of its angles.



In the triangles A B C, which has all its angles acute angles, let the side opposite angle A be a, that opposite angle B be b, and that opposite angle C be c. From C draw C D as a perpendicular, which call p; then in the right-angled triangle A C D we have—

$$\frac{p}{b} = \text{sin. A} \qquad p = \text{b sin. A}$$

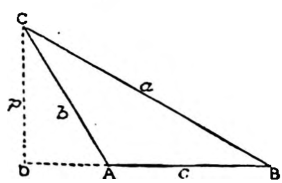
also, in the right-angled triangle B C D we have—

$$\frac{p}{a} = \text{sin. B} \qquad p = a \text{ sin. B}$$

hence it is evident that—

$$a \text{ sin. B} = b \text{ sin. A, since each} = p$$

If the angle A is obtuse, then the perpendicular C D will fall without the triangle A B C, and B A must be produced to meet it; then in the right-angled triangle B C D we have—



$$\frac{p}{a} = \text{sin. B} \qquad p = a \text{ sin. B}$$

and in the right-angled triangle A C D we have—

$$\frac{p}{b} = \text{sin. D A C} \qquad p = b \text{ sin. D A C}$$

but, since the angles D A C and A are supplemental their sines are equal. therefore—

$$p = b \text{ sin. A}$$
$$a \text{ sin. B} = b \text{ sin. A}$$

Hence $a : b = \text{sin. A} : \text{sin. B}$; or $\frac{a}{b} = \frac{\text{sin. A}}{\text{sin. B}}$

Similarly, by drawing perpendiculars from the other angles it can be shown that—

$$a : c = \text{sin. A} : \text{sin. C}; \text{ or } \frac{a}{c} = \frac{\text{sin. A}}{\text{sin. C}}$$

and

$$b : c = \text{sin. B} : \text{sin. C}; \text{ or } \frac{b}{c} = \frac{\text{sin. B}}{\text{sin. C}}$$

which is called the *Rule of Sines*, and may be expressed as follows—

The sides of a plane triangle are in the same ratio as the sines of the opposite angles.

(1) A triangle is determined by one side and two angles; and inasmuch as two angles determine the third angle, there remains only to find the two sides.

(2) A triangle is determined (in all cases but one) by two sides and an angle opposite to one of them.

NOTE.—The doubt or ambiguity arises in the one case when the given angle is not that appertaining to the greater side. There is no ambiguity when the given angle is opposite to the greater side.

(1) To solve the triangle when one side and two angles are given, we have to find the remaining angle and the other two sides.

The remaining angle is found by adding together the two given angles and subtracting the sum from 180°, because the sum of the three angles of a plane triangle is always equal to 180°.

GENERAL RULE TO FIND A SIDE.—Write down the side required, and underneath it put the given side so as to form a fraction. Now put this fraction equal to its corresponding fraction in the *Rule of Sines*, and multiply both sides of the equation by the denominator of the first formed fraction; the resulting equation will give the formula by which the required side is found. To exemplify this, suppose the side b was given, and also the angles A and B, and we want to find the side a. Write down a, and underneath it put b, thus, $\frac{a}{b}$, the corresponding fraction in the *Rule of Sines is* $\frac{\text{sin. A}}{\text{sin. B}}$. Putting these fractions equal to one another, we have $\frac{a}{b} = \frac{\text{sin. A}}{\text{sin. B}}$, then by multiplying by the denominator b, we have $a = b \times \frac{\text{sin. A}}{\text{sin. B}}$. Hence the side a will be found by adding together the logarithm of b and the sin. A, and from the sum subtracting the sin. B. The natural

number corresponding to the remainder will be the value of *a*. The side *c* will be found in a similar manner, but before it can be done the angle C must be found.

GENERAL RULE TO FIND AN ANGLE.—(2) To solve the triangle when two sides and an angle opposite one of them are given. Write down the sin. of the required angle (this must be always the angle opposite the other given side), and underneath it put the sin. of the given angle so as to form a fraction. Now put this fraction equal to its corresponding fraction in the *Rule of Sines*; the resulting equation will give the formula by which the required angle is found. To exemplify this, suppose the two sides *a* and *b* are given (of which *a* is greater than *b*) and the angle A. We must first find the angle B. Write down sin. B, and underneath it put sin. A, thus, $\frac{\text{sin. B}}{\text{sin. A}}$, the corresponding fraction in the *Rule of Sines* is $\frac{b}{a}$.

Putting these fractions equal to one another, we have $\frac{\text{sin. B}}{\text{sin. A}} = \frac{b}{a}$ and multiplying by the denominator sin. A we have sin. B $= \frac{b}{a} \times$ sin. A. Hence the angle B will be found by adding together the logarithm of *b* and sin. A, and from the sum subtracting the logarithm of *a*. The degree and minute corresponding to this remainder in the sin. column will be the value of the angle B. The other parts are found exactly as in (1).

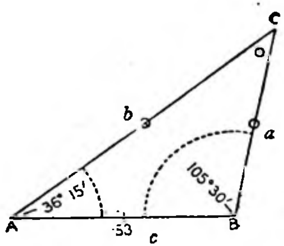
Given one Side and two Angles, to find the other Sides, and the remaining Angle.

Example.—Given the angle A = 36° 15′, the angle B = 105° 30′, and the side A B = 53; required the sides A C and B C, and the angle C.

BY CONSTRUCTION

To find the Angle C

∠ A	36°	15′
∠ B	105	30
Sum	141	45
	180	00
∠ C	38	15



Draw the line A B, and make it equal to 53; make the angle B A C 36° 15′, and the angle A B C 105° 30′, and draw the lines A C and B C till they meet in C; then A C will measure 82·5, and B C 50·62.

BY CALCULATION

To find the Side A C

$\frac{b}{c} = \frac{\sin. B}{\sin. C} \therefore b = \frac{c \sin. B}{\sin. C}$

$\log. b = \log. c + L.\sin. B - L.\sin. C.$

c 53	log.	1·724276
B 105° 30′	L. sin.	9·983911
		11·708187
C 38° 15′	L. sin.	9·791757
b 82·5	log.	1·916430

To find the Side B C

$\frac{a}{c} = \frac{\sin. A}{\sin. C} \therefore a = \frac{c \sin. A}{\sin. C}$

$\log. a \ \log. c + L.\sin. A - L.\sin. C.$

c 53	log.	1·724276
A 36° 15′	L. sin.	9·771815
		11·496091
C 38° 15′	L. sin.	9·791757
a 50·62	log.	1·704334

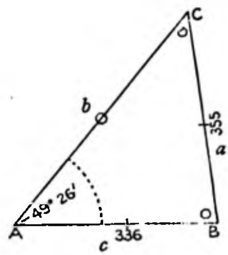
A C = 82·5 B C = 50·62 C = 38° 15′

Given two Sides and an Angle opposite one of them, to find the other Angles and the third Side

Example I.—Given the side A B = 336, the side B C = 355, and the angle A 49° 26′; required the angles B and C, and the side A C.

BY CONSTRUCTION

Draw the line A B, which make equal to 336; draw the line A C so as to make an angle of 49½° with A B; take the length of B C in the compasses, and setting one foot in B, let the other cut the line A C in C, and draw the line B C; then the angle B will measure 84½°, the angle C 46°, and the side A C 465·3.



BY CALCULATION

There is no ambiguity here, inasmuch as the given angle is opposite to the greater given side.

To find the Angle C

$\frac{\text{Sin. C}}{\text{Sin. A}} = \frac{c}{a}, \therefore \sin. C = \frac{c \sin. A}{a}$

$L.\sin. C = \log. c + L.\sin. A - \log. a$

c 336	log.	2·526339
A 49° 26′	L. sin.	9·880613
		12·406952
a 355	log.	2·550228
∠ C 45° 58′	L. sin.	9·856724
∠ A 49 26		
Sum 95 24		
180 00		
∠ B 84 36		

To find the Side A C

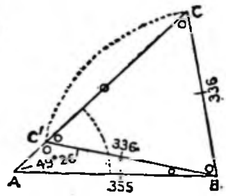
$\frac{b}{a} = \frac{\sin. B}{\sin. A} \therefore b = \frac{a \sin. B}{\sin. A}$

$\log. b = \log. a + L.\sin. B - L.\sin. A$

a 355	log.	2·550228
B 84° 36′	L. sin.	9·998068
		12·548296
A 49° 26′	L. sin.	9·880613
b 465·3	log.	2·667683

B = 84° 36′ C = 45° 58′ A C = 465·3

Example II.—Given the side A B = 355, the side B C = 336, and the angle A = 49° 26′; required the angles B and C to the nearest second, and the side A C.



NOTE.—The given angle being opposite to the less side, there are two *different* triangles which may possess the same data, because a circle described with B as centre and radius B C will cut C A in C′, hence B C′ has the same value as B C, so that both triangles C B A and C′ B A will satisfy the conditions of the question; it is therefore doubtful, or ambiguous, whether we must make C less than 90°, or greater than 90°.

This example constructed with the side A B = 355, and side B C = 336, the projection makes A C = 30·4 or 431·3, since the angle C may be either $53\frac{1}{2}°$ or $126\frac{1}{2}°$, *i.e.*, either acute or obtuse.

BY CALCULATION

To find the Angle C

$$\frac{\text{Sin. C}}{\text{Sin. A}} = \frac{\text{A B}}{\text{B C}} \therefore \text{sin. C} = \frac{\text{A B sin. A}}{\text{B C}}$$

$$\text{L. sin. C} = \text{log. A B} + \text{L. sin. A} - \text{log. B C}$$

A B 355	log.	2·550228	∠ A	49° 26′ 0″	∠ A	49° 26′ 0″
A 49° 26′	L. sin.	9·880613	∠ C	53 22 47	∠ C′	126 37 13
		12·430841		102 48 47		176 3 13
a 336	log.	2·526339		180		180
∠ C 53° 22′ 47″	L. sin.	9·904502	∠ B	77 11 13 or	∠ B	3 56 47
180						
∠ C′ 126 37 13						

To find the Sides A C *and* A C′

$$\frac{\text{A C}}{\text{B C}} = \frac{\text{sin. B}}{\text{sin. A}} \therefore \text{A C} = \frac{\text{B C sin. B}}{\text{sin. A}}$$

$$\text{log. A C} = \text{log. B C} + \text{L. sin. B} - \text{L. sin. A}$$

B C 336	log.	2·526339	....		2·526339
B 77° 11′ 13″	L. sin.	9·989049	B 3° 56′ 47″	L. sin.	8·837734
	Sum	12·515388			11·364073
A 49° 26′	L. sin.	9·880613	....		9·880613
A C 431·3	log.	2·634775	A C′ 30·44	log.	1·483460

Ans. B = 77° 11′ 13″ C = 53° 22′ 47″ A C = 431·3
or, B = 3° 56′ 47″ C′ = 126° 37′ 13″ A C′ = 30·44

To find the other two angles, the two sides and the included angle being given.—

First Method—

$$\text{Tan. } \tfrac{1}{2}(A \sim B) = \frac{a \sim b}{a+b} \times \text{tan. } \tfrac{1}{2}(A + B)$$

$$\text{L. tan. } \tfrac{1}{2}(A \sim B) = \log.(a - b) + \text{L. tan. } \tfrac{1}{2}\text{ L. }(A + B) - \log.(a + b).$$

The value of $\frac{1}{2}(A + B)$ being known, and that of $\frac{1}{2}(A \sim B)$ being found, their sum and difference give respectively the values of the angles A and B.

The third side may be found by the Rule of Sines.

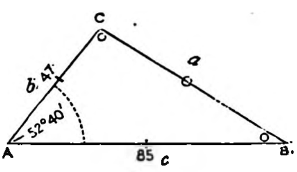
GENERAL RULE *for the half difference of the Angles.*—Find the sum and difference of the given sides, subtract the given angle from 180°, and take half the remainder, for half the sum of the unknown angles: to the log. of the diff. of the sides add the tan. of the half-sum of the unknown angles: from the sum of these logs. take log. of the sum of the sides; the remainder is the tan. of the half-diff. of unknown angles, which take out: add the half-diff. to the half-sum of unknown angles for the angle opposite the greater side, and subtract it to get the less angle.

Given two Sides and the included Angle, to find the other Angles and the third Side.

Example.—Given the side A B = 85, the side A C = 47, and the angle A = 52° 40′; required the angles C and B, and the side B C.

BY CONSTRUCTION

Draw the line A B, and make it equal to 85; at A make the angle B A C 52° 40′; from A to C lay off 47, and draw the line B C; then A B C is the triangle required; the angle B will measure $33\frac{1}{2}$°, the angle C 49° and the side B C 67·7.



To find the Angles and remaining Side.

A B or c =	85		180° 0′
A C or b =	47	∠ A	52 40
Sum or $c + b$ =	132	Sum of ∠ B and C	127 20 = C + B
Diff. or $c - b$ =	38	$\frac{1}{2}$ sum of ∠	63. 40 = $\frac{1}{2}$(C + B)

$$\text{L. tan. } \tfrac{1}{2}(C \sim B) = \log.(c - b) + \text{L. tan. } \tfrac{1}{2}(C + B) - \log.(c + b).$$

$(c - b)$	$= 38$	log.	1·579784
$\frac{1}{2}$ (C + B)	$= 63° 40'$	L. tan.	10·305434
			11·885218
$(c + b)$	$= 132$	log.	2·120574
$\frac{1}{2}$ (C − B)	$= 30° 11'$	L. tan.	9·764644
$\frac{1}{2}$ (C + B)	$= 63\ 40$		
Sum	$= \angle$ C 93 51		
Diff.	$= \angle$ B 33 29		

$$\frac{a}{b} = \frac{\sin. A}{\sin. B} \therefore a = \frac{b \sin. A}{\sin. B}$$

log. a = log. b + L. sin. A — L. sin. B.

b	47	log.	1·672098
A	52° 40′	L. sin.	9·900433
			11·572531
B	33° 29′	L. sin.	9·741699
a	67·74	log.	1·830832

The greater angle is opposite the greater side. A B is greater than A C, hence the angle C is greater than the angle B. The sum is therefore the angle C, and the difference is the angle B.

NOTE.—This triangle may be solved by letting fall a perpendicular from the angle C on the side A B, which will divide it into two right-angled triangles; then with the hypotenuse A C and angle A find the perpendicular and the base, which base subtracted from the side A B will leave the base of the other triangle; then, with the perpendicular and base find the angle B, which added to angle A, and their sum subtracted from 180°, will give the angle C; and, with one of the angles and its opposite side find the side B C.

The third side can be found direct by the following formula—

$$\theta = \sqrt{\text{Cos.}^2 \frac{A}{2} \ . \ c \ . \ b}$$

$$\frac{a}{2} = \sqrt{\left(\frac{c+b}{2} + \theta\right)\left(\frac{c+b}{2} - \theta\right)}$$

	$A = 52° 40'$			
L. Cos. $\frac{A}{2}$ 9·952914	$\frac{A}{2} = 26\ 20$	. . . . .	L. Cos.^{2}	9·904838
2				
L Cos^2 $\frac{A}{2}$ 9·904838	$c = 85$	. . . . .	Log.	1·929418
	$b = 47$	. . . . .	Log.	1·672098
	$c + b = 132$			2)3·506355
	$\frac{c+b}{2} = 66$	$\theta = 56·65$	Log.	1·753177
	$\frac{c+b}{2} + \theta = 122·65$	. . . . .	Log.	2·088666
	$\frac{c+b}{2} - \theta = 9·35$	. . . . .	Log.	0·970812
				2)3·059478
	$\frac{a}{2} = 33·864$		Log.	1·529739
	2			
	Side a = 67·728			

To find the Angles, the Sides being given

A triangle may be determined from its three sides, by the following formulæ—where $s = \frac{1}{2}$ sum of the sides—

(1) By $\cos. \frac{1}{2} A = \sqrt{\frac{s(s-a)}{b\,c}}$ Or, (2) By $\tan. \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

$\cos. \frac{1}{2} B = \sqrt{\frac{s(s-b)}{a\,c}}$ $\tan. \frac{1}{2} B = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$

$\cos. \frac{1}{2} C = \sqrt{\frac{s(s-c)}{a\,b}}$ $\tan. \frac{1}{2} C = \sqrt{\frac{(s-a(s-b)}{s(s-c)}}$

The usual method of finding the angles is through the *cosine* (1), except when the angle is very small, for then the change in log. cosines is very small, and consequently not conducive to accuracy. In this case the angle should be found through the tangent.

From (1)—

$$\text{Cos. } \tfrac{1}{2} A = \tfrac{1}{2} \{\log. s + \log. (s-a) + 20 - (\log. b + \log. c)\} \text{ where } s = \frac{a+b+c}{2}$$

Rule I. Cosine Method.—Find half the sum of the sides, and from it subtract the side opposite the angle sought, which call the *remainder.* Add together the logarithms of the half sum of the sides, and the remainder, and increase the index by 20; also add together the logarithms of the two sides containing the angle sought; subtract the latter sum from the former and divide by 2; this will give the cos. of half the angle sought.

But the method by the *tangent* is the readiest, when all the three angles are sought, as only four logarithms are required to be taken out of the Tables; and the rule may be simplified as follows—

From (2)—

$$\text{Tan. } \tfrac{1}{2} A = \tfrac{1}{2} \{\log. (s-6) + \log. (s-c) - \log. s - \log. (s-a)\} + 10$$

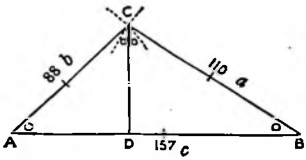
Rule II. Tangent Method.—Add together the three sides a, b, and c; take half their sum, which call s; from this half sum subtract each side in succession, and thus you have the value of s, $s-a$, $s-b$, and $s-c$; subtract the logarithm of s from 20·000000; to the remainder add the logarithms of $s-a$, $s-b$, and $s-c$; divide the sum by 2 for the half-sum, which becomes a *constant;* from this *constant* subtract in order the logarithm $(s-a)$, for tan. $\frac{1}{2}$ A; the logarithm $(s-b)$ for tan. $\frac{1}{2}$ B; and the log. $(s-c)$ for tan. $\frac{1}{2}$ C; multiply each angle by 2, and the three angles of the triangle are obtained, the sum of which should be 180°.

Given the three Sides, to find the Angles

Example.—Given the side A B = 157, the side B C = 110, and the side A C = 88, to find the angles A, B, and C.

BY CONSTRUCTION

Draw the line A B, which make equal to 157; take the length of A C 88 in the compasses, and with one foot on A describe the arc C; then, with the length of C B 110 in the compasses, and one foot in B, describe an arc cutting the former in C, to which draw the lines A C and B C; then A B C is the triangle required; the angle A will measure 42° 44′, the angle B 32° 53′, and the angle C 104° 23′.



By Rule I.

$$\text{Cos.}\ \tfrac{1}{2}A = \tfrac{1}{2}\ \{\log. s + \log. (s - a) + 20 - (\log. b + \log. c)\}\ \text{where}\ s = \frac{a+b+c}{2}$$

a	110		
b	88	log.	1·944483
c	157	log.	2·195900
	355		4·140383
s	177·5	log.	2·249198
a	110		
$s-a$	67·5	log.	1·829304
			24·078502
(log. b + log. c)			4·140383
			2)19·938119
½ A	21° 22′ 19″	cos.	9·969059
A	42 44 38		

And similarly for angles B and C, or by the Rule of Sines. If the latter is used, the greatest angle is the proper one to find first through the cosine.

By Rule II. *For* ∠ A

a =	110	
b =	88	
c =	157	
	2)355	
s =	177·5	
$s-a$ =	67·5	
$s-b$ =	89·5	
$s-c$ =	20·5	

		20·000000
s	177·5 log.	2·249198
	(ar. co.)	17·750802
$s-a$	67·5 log.	1·829304
$s-b$	89·5 log.	1·951823
$s-c$	20·5 log.	1·311754
		2)22·843683
	Constants	11·421841
$s-a$	67·5 log.	1·829304
½ A 21° 22′ 18″	L. tan.	9·592537

For ∠ B

....	11·421841
$s-b$ 89·5 log.	1·951823
½ B 16° 26′ 35″ tan.	9·470018
B 32 53 10	

For ∠ C

....	11·421841
$s-c$ 20·5 log.	1·311754
½ C 52° 11′ 7″ tan.	10·110087
C 104 22 14	

A	42	44	36
B	32	53	10
C	104	22	14
	180	0	0

The *three angles* of a triangle determine it in *species* only, and not in magnitude: they may define it as an acute-angled triangle, a right-angled triangle,

or an obtuse-angled triangle. Moreover, since the sum of the three angles of every triangle is equal to 180°, it is evident that an indefinite number of similar triangles may be constructed, the angles of which shall be equal respectively to three given angles the sum of which is 180°.

Examples for Practice

1. Given one side 129, an adjacent angle 56° 30′, and the opposite angle 81° 36′; required the third angle and the remaining sides.

Ans. The third angle is 41° 54′, and the remaining sides are 108·7 and 87·08.

2. Given one side 96·5, another side 59·7, and the angle opposite the latter side 31° 30′; required the remaining angles and the third side.

Ans. This question is ambiguous, the given side opposite the given angle being less than the other given side (*see* Example II., p. 53); hence, if the angle opposite the side 96·5 be acute, it will be 57° 38′, the remaining angle 90° 52′, and the third side 114·2; but if the angle opposite the side 96·5 be obtuse, it will be 122° 22′, the remaining angle 26° 8′, and the third side 50·32.

3. Given one side 110, another side 102, and the contained angle 113° 36′; required the remaining angles and the third side.

Ans. The remaining angles are 34° 37′ and 31° 47′; and the third side is 177·5.

4. Given the three sides respectively, 120·6, 125·5, and 146·7; required the angles.

Ans. The angles are 51° 53′, 54° 58′, and 73° 9′.

5. Given one side 684·5, another side 496·7, and the angle opposite the latter side 40° 58′; required the remaining angles and the third side.

Ans. If the angle opposite the former side be acute, the remaining angles will be 64° 37′ and 74° 25′, and the third side 729·8; but if obtuse, the angles will be 115° 23′ and 23° 39′, and the third side 303·9.

6. Given one side 117·8, another side 96·55, and the contained angle 67° 30′; required the remaining angles and the third side.

Ans. The remaining angles are 64° 41′ and 47° 49′, and the third side 120·4.

7. Given the three sides 87·6, 66·2, and 41·3; required the angles.

Ans. The angles are 26° 49′, 46° 20′, and 106° 51′.

APPLICATION OF TRIGONOMETRY

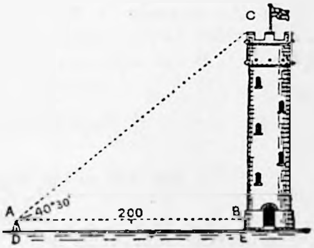
The following methods of ascertaining the height or distance of an object being frequently useful, they may properly be introduced here before the present subject is dismissed.

To find the Height of an Accessible Object

Measure the horizontal distance, between the eye and the object, of the point immediately under it, and observe the angle of elevation with a quadrant or sextant: thus will be obtained the base and angles of a right-angled triangle, the perpendicular of which being found will be the height of the object above the horizontal plane, to which add the height of the eye.

Or, by removing either towards or from the object, until the angle of elevation be 45°, the horizontal distance, added to the height of the eye, will give the height of the object.

Example.—From the bottom of a tower I measured 200 feet on a horizontal plane; I then took the angle of elevation, and found it 40° 30′, the height of my eye being 6 feet; required the height of the tower.



In the right-angled triangle A B C are given the side A B 200 feet, and the angle B A C 40° 30′, to find the perpendicular B C

$$\frac{BC}{AB} = \tan A \quad \therefore BC = AB \times \tan. A$$

$$\log. BC = \log AB + L. \tan. A - 10$$

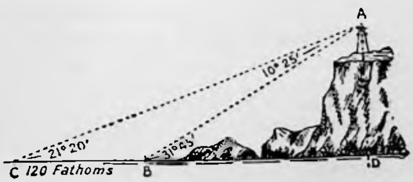
Distance A B or D E 200....		log.	2·301030
A 40° 30′		L. tan.	9·931499
Perpendicular B C	170·8	log.	2·232529
Height of the eye A D	6		
Height of the tower C E	176·8 feet.		

If the height of the object be known, and the angle of elevation observed, the horizontal distance of the eye may be found; for in this case there will be given the perpendicular and angles of a right-angled triangle to find the base or distance required.

$$AB = BC \cot. A.$$

To find the Height of an Inaccessible Object

Measure the angle of elevation at a convenient distance from the given object; then remove in a direct line from the object, and again observe the angle of elevation, the distance between the two stations being carefully measured; hence will be given one side and the angles of an oblique-angled triangle, with which, find either of the two other sides. Now that side will be the hypotenuse of a right-angled triangle, the perpendicular of which being found, and the height of the eye added to it, their sum will be the height of the object.



Example.—Wanting to know the height of a lighthouse above the level of the sea, and not being able to measure its horizontal distance, I took the

angle of elevation, and found it to be 31° 45′, and after removing from it 120 fathoms, I observed the angle of elevation to be 21° 20′; required the height of the lighthouse.

In the triangle A B C are given the angle A C B 21° 20′; the angle C A B, which is the difference between the angles A B D and A C D; and the side C B 120 fathoms, to find the side A B.

$$\frac{A\,B}{C\,B} = \frac{\text{sin. } A\,C\,B}{\text{sin. } C\,A\,B} \qquad \therefore A\,B = \frac{C\,B \text{ sin. } A\,C\,B}{\text{sin. } C\,A\,B}$$

$$\text{log. } A\,B = \text{log. } C\,B + \text{L. sin. } A\,C\,B - \text{L. sin. } C\,A\,B$$

∠ A B D ...... 31° 45′	Side C B 120	log.	2·079181
∠ A C B 21 20	∠ A C B 21° 20′	L. sin.	9·560855
∠ C A B 10 25			11·640036
	∠ C A B 10° 20′	L. sin.	9·257211
	Side A B 241·45		2·382825

In the right-angled triangle A B D are given the angle A B D 31° 45′, and the hypotenuse A B 241·45, to find the perpendicular A D.

$$\frac{A\,D}{A\,B} = \text{sin. } A\,B\,D \quad \therefore\ A\,D = A\,B \times \text{sin. } A\,B\,D$$

$$\text{log. } A\,D = \text{log. } A\,B + \text{L. sin. } A\,B\,D - 10$$

Hypotenuse A B 241·45	log.	2·382825
∠ A B D 31° 25′ 45″	L. sin.	9·721162
Perpendicular A D 127·05		2·103987

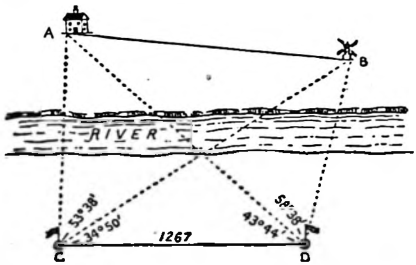
Hence the height of the lighthouse is 127·05 fathoms, or 762·3 feet above the level of the sea.

To find the Distance of Two Points on a Horizontal Plane, which are Inaccessible

Let A and B be the objects: from C, one extremity of a measured distance, observe the angles B C A and B C D; from D, the other extremity of the measured distance, observe the angles A D C and A D B; then by *The Rule of Sines*, p. 50, compute the sides D A and D B; and from the sides D A and D B, and included angle A D B, compute by Rule on p. 54 the required distance A B.

Example.—To find the distance between a windmill (B) and house (A), I took two stations, C and D, on the opposite side of a river, distant 1,267

yards from each other, and such, that from each of them the other station and the windmill (B) and house (A) were seen. At C the angles B C A



53° 38′, and B C D 34° 50′ were observed ; at D the angles A D C 43° 44′ and A D B 58° 38′ were observed. Required the distance between the house and mill.

Ans. Side A D = 1709·7 yards ; side D B = 1065·1 yards ; $\angle$A B D = 83° 9′ 26″. Dist. between house and mill = 1470·3 yards.

Examples for Practice

1. Find the height of a cliff in feet, the angle of elevation of its top at a distance of *one nautical* mile being 5°.

Ans. 531·9 feet.

2. The angle of elevation of a tower was 26° 30′, and 150 yards nearer to it the angle of elevation was 51° 30′ ; find the height of the tower, and the distance of its base from the last station.

Ans. 123·94 yards ; distance 98·59 yards.

3. Two ships are 950 yards apart ; an observer on each finds the angle between the other and a buoy to be 57° 30′ ; find the distance of the buoy from each ship.

Ans. 884·05 yards.

4. From a ship a point of land bore N. 20° W., and after sailing N. 60° W. 3 miles it bore N. 37° E. ; find the distance of the ship from the point of land when the second bearing was taken.

Ans. 2·299 miles.

5. Two landmarks are distant from a ship 1,550 yards and 975 yards, they bear respectively N. 73° E. and S. 30° E. ; find the distance between the landmarks.

Ans. 1,635 yards.

6. From the top of a ship's mast 75 feet above the water, the angle of depression of a floating mark was 12° 20′ ; required the distance of the top of the mast from the floating mark.

Ans. 351·1 feet.

7. Three ships not in one line are distant from each other 750 yards, 930 yards, and 1,006 yards ; find the greatest difference of bearing between them.

Ans. 72° 40½′.

SPHERICAL TRIGONOMETRY

The problems on Great Circle Sailing and Nautical Astronomy are solved by either right-angled or oblique-angled spherical trigonometry.

If a spherical triangle has one of its angles a right angle, it is called a right-angled triangle; if one of its sides be a quadrant (90°) it is called a quadrantal triangle; if two of the sides be equal it is called an isosceles triangle, etc., as in plane trigonometry.

The following properties relate to spherical triangles—

(a) Any side of a spherical triangle is less than a semicircle, and any angle is less than two right angles.

(b) The sum of the three angles is greater than two right angles and less than six right angles.

(c) If the three sides of a spherical triangle be equal, the three angles will also be equal, and *vice versâ*.

(d) If the sum of any two sides of a spherical triangle be equal to 180°, the sum of their opposite angles will also be equal to 180°, and *vice versâ*.

(e) If the three angles of a spherical triangle be acute, all right, or all obtuse, the three sides will be accordingly all less than 90°, all equal to 90°, or all greater than 90°, and *vice versâ*.

(f) The sum of any two sides is greater than the third side, and their difference is less than the third side.

(g) The sum of any two angles is greater than the supplement of the third angle.

(h) The sum of the three sides is less than the circumference of a great circle.

(i) If any two sides of a triangle be equal to each other, their opposite angles will be equal, and *vice versâ*.

NAPIER'S RULES FOR THE SOLUTION OF RIGHT-ANGLED SPHERICAL TRIANGLES

In every right-angled spherical triangle there are five "circular parts," exclusive of the right angle, which is not taken into consideration. These five parts consist of the two legs containing the right angle, the complement of the hypotenuse, and the complements of the two angles. They are called "circular parts" because they are each measured by the "arc" of a Great Circle.

Before proceeding to the use of Napier's Rules it is necessary to have a clear idea of what is meant by the "MIDDLE PART," and the following explanation should make it clear.

In the solution of every problem three parts are concerned, two of which are given and the one required to be found.

If the three parts follow each other in succession, that is, touch each other, that part which is in the middle is the "middle part" and the other two parts are the adjacent parts; if the three parts do not touch each

other, two of them must, and that part which is separated from them is the "middle part" and the other two parts are the opposite parts.

It is necessary to point out that the angles separate the hypotenuse from the base and perpendicular, but the perpendicular and base, which contain the right angle, touch each other, that is, the right angle does not separate them.

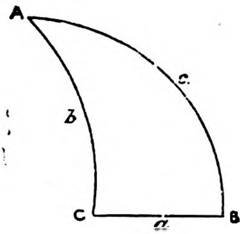
Another important point to remember is, when a complement comes into an equation the sin., cos., or tan. of the formula are to be changed into cos., sin., or cot., as the case may be, and the following cases will make these points clear.

"NAPIER'S RULES"—

(1) Sine of the Middle Part = Product of tangents of adjacent parts.
(2) Sine of the Middle Part = Product of cosines of opposite parts.

The above rules have reference to the natural sines, cosines, and tangents. In practise, however, log. sines, etc., are used in the ordinary way, facilitating the work of multiplication and division by addition and subtraction respectively.

If the student has carefully studied the foregoing the following examples will be clear to him.



In the right-angled spherical triangle A B C right-angled at B, given the side b, and $\angle$A, to find side c.

Now it is obvious from what has been said that $\angle$A is the middle part and the sides b and c the adjacent parts.

Napier's first rule says the sine of the middle part equals the product of the tangents of the adjacent parts. That is—

$$\text{Sine A} = \tan. b \times \tan. c$$

but as the complements of A and side b are to be taken the formula becomes—

$$\text{Cos. A} = \cot. b \times \tan. c$$

$$\therefore \text{Tan. } c = \frac{\cos. \text{A}}{\cot. b} = \cos. \text{A} \div \cot. b$$

which in the logarithmic form becomes—

$$\text{Log. tan. } c = \log. \cos. \text{A} + 10 - \log. \cot. b$$

and by substituting the reciprocal of cot. b we get the following—

$$\text{Log. tan. } c = \log. \cos. \text{A} + \log. \tan. b - 10$$

In the same triangle, given side b and $\angle$ C, to find side c.

Side c is the middle part, and side b and $\angle$ C the opposite parts:

Napier's second rule says the sine of the middle part equals the product of the cosines of opposite parts. That is—

$$\text{Sin. } c = \cos. b \times \cos. C$$

but as the complements of b and C are to be taken the formula becomes—

$$\text{Sin. } c = \sin. b \times \sin. C$$

$$\therefore \text{Log. sin. } c = \text{log. sin. } b + \text{log. sin. C} - 10$$

In the same triangle, given $\angle$ C and side a, to find side c.

Side a is the middle part, and $\angle$ C and side c the adjacent parts.

$$\text{Sin. } a = \tan. c \times \tan. C$$

but the complement of C is taken and the formula becomes—

$$\text{Sin. } a = \tan. c \times \cot. C$$

$$\therefore \text{Tan. } c = \frac{\sin. a}{\cot. C} \text{ or } \sin. a \times \tan. C$$

which in log. form becomes—

$$\text{Log. tan. } c = \text{log. sin. } a + \text{log. tan. C} - 10.$$

When an angle and the side opposite are given the case is ambiguous and there will be two answers—the part found and its supplement.

The rule of signs must be attended to, as like signs give plus and unlike signs give minus when multiplied together or divided by each other.

The sine and cosecant are + up to 180°, that is, in the first and second quadrants.

The cosine, secant, tangent, and co-tangent are plus in the first quadrant, that is, up to 90°, and change to minus in the second quadrant, that is, from 90° to 180°.

The sine and cosecant are minus in the third and fourth quadrants, that is, from 180° to 360°.

The cosine and secant are minus in the third quadrant and plus in the fourth quadrant.

The tangent and co-tangent are plus in the third quadrant and minus in the fourth quadrant.

The student should now be able to solve all the cases in right-angled spherical trigonometry without assistance and to arrive at the required equation at a glance.

Example 1.—In the right-angled spherical triangle A B C (Fig. 1) right-angled at C, given the side b 86° 10′ 00″ and the angle A 74° 45′ 15″, to find the angle B.

∠ B is the middle part and ∠ A and side b opposite parts.

Formula—

$$\text{Cos. B} = \text{sin. A} \times \text{cos. } b.$$

$$\text{Log. cos. B} = \text{log. sin. A} + \text{log. cos. } b - 10$$

A	74°	45′	15″	sin.	9·984441
b	86	10	00	cos.	8·825130
B	86°	18′	6″	cos.	8·809571

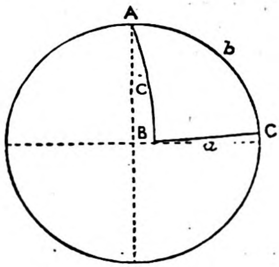


Fig. 1.

In the same triangle, given ∠ A 74° 45′ 15″ and side b 86° 10′, to find side a.

Side b is the middle part, and ∠ A and side a adjacent parts.

$$\text{Sin. } b = \text{tan. } a \times \text{cot. A}$$

$$\therefore \text{ Tan. } a = \frac{\text{sin. } b}{\text{cot. A}}$$

$$\text{Log. tan. } a = \text{log. sin. } b + 10 - \text{log. cot. A}$$

b	86°	10′	0″	sin.	9·999027
A	74	45	15	cot.	9·435451
a	74°	43′	17″	tan.	10·563576

By using the reciprocal of cot. A we get the same result by addition (which is really multiplication).

$$\text{Log. tan. } a = \text{log. sin. } b + \text{log. tan. A} - 10.$$

b	86°	10′	0″	sin.	9·999027
A	74	45	15	tan.	10·564549
a	74°	43′	17″	tan.	10·563576

Example 2.—In the right-angled triangle C A B (Fig. 2) right-angled at B, given ∠ C 46° and side c 40°, to find side a.

Side a is the middle part, ∠ C and side c adjacent parts.

$$\text{Sin. } a = \text{cot. C} \times \text{tan. } c.$$

$$\text{Log. sin. } a = \text{log. cot. C} + \text{log. tan. } c - 10$$

C 46°	cot.	9·984837
c 40	tan.	9·923814
a = 54° 7′ 34″	sin.	9·908651
or		
a' 125° 52′ 26″		

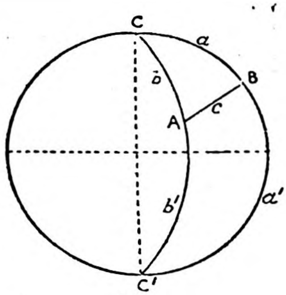


Fig. 2.

This is the ambiguous case, because the given side is opposite the given angle and there is nothing to indicate which arc is to be taken; as the

angle C is equal to angle C′ and c is common to both, therefore, each part found will have two values, that is, the part found and its supplement which is found by subtracting the arc or angle found from 180°.

In the same triangle Fig. 2, given ∠ C 46° and side c 40°, to find ∠ A.

∠ C is the middle part, ∠ A and side c opposite parts.

$$\text{Cos. C} = \text{sin. A} \times \text{cos. } c$$

$$\therefore \text{Sin. A} = \frac{\text{cos. C}}{\text{cos. } c}$$

$$\text{Log. sin. A} = \text{log. cos. C} + 10 - \text{log. cos. } c$$

C 46°		cos.	9·841771
c 40		cos.	9·884254
A = 65°	4′ 6″	sin.	9·957517
or			
114°	55′ 54″		

Example 3.—In the right-angled spherical triangle C A B (Fig. 3) right-angled at B, given ∠ C 46° and side a 125° 52′ 26″, to find side b.

∠ C is the middle part, sides a and b adjacent parts.

$$\text{Cos. C} = \text{tan. } a \times \text{cot. } b$$

$$\therefore \text{Cot. } b = \frac{\text{cos. C}}{\text{tan. } a}$$

$$\text{Log. cot. } b = \text{log. cos. C} + 10 - \text{log. tan. } a$$

∠ C 46° +	cos.	9·841771
a 125° 52′ 26″ −	tan.	10·140751
63° 19′ 35″ −	cot.	9·701020
180 0 0		
b = 116 40 25		

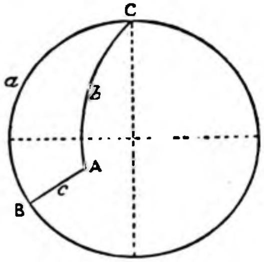


Fig. 3.

In this case the cosine is plus and the tangent minus, therefore, the cot. is minus and the arc just found is to be subtracted from 180°, as the minus cot. lies in the second quadrant.

Exercises.—Right-angled Spherical Triangles.

1. In the right-angled spherical triangle A B C, right-angled at C, given A C 65° and B C 51° 36′, to find ∠ A, ∠ B, and the side A B.
Ans. ∠ A 54° 15′ 52″; ∠ B 70° 5′ 36″ and A B 74° 53′ 52″.

2. In the right-angled spherical triangle A B C, right-angled at C, given B C 32° 51′ and B 56° 17′, to find the other parts.
Ans. ∠ A 45° 40′ 14″; A C 39° 6′ 20″ and A B 49° 18′ 54″.

3. In the right-angled spherical triangle A B C, right-angled at B, given A B 110° 17′ and B C 98° 46′, to find the other parts.
Ans. ∠ C 110° 3′ 55″; ∠ A 98° 13′ 51″ and A C 86° 58′ 17″.

4. In the right-angled spherical triangle A B C, right-angled at A, given A B 97° 20′ and B C 94° 13′, to find the other parts.

Ans. ∠ B 55° 2′ 52″; ∠ C 96° 0′ 20″ and A C 54° 49′ 36″.

5. In the right-angled spherical triangle A B C, right-angled at A, given A B 114° 22′ and ∠ C 108° 19′, to find the other parts.

Ans. B 49° 37′ or 130° 23′; A C 46° 57′ 45″ or 123° 2′ 15″ and B C 106° 21′ 15″ or 73° 38′ 45″.

6. In the right-angled spherical triangle A B C, right-angled at A, given A B 95° 52′, B 76° 13′, to find the other parts.

Ans. ∠ C 95° 41′ 50″; A C 76° 8′ 49″ and B C 91° 24′ 9″.

QUADRANTAL SPHERICAL TRIANGLES

A quadrantal spherical triangle has one of its sides 90°, hence its name. "Napier's Rules" for circular parts are used in the solution of these triangles and in considering the "Circular Parts" the quadrantal side is omitted in the same manner as the right angle in right-angled spherical trigonometry, and the "Circular Parts" are the two angles adjacent to the quadrantal side, which does not separate them, the complements of the other angle and the two remaining sides. The amplitude is the commonest example in nautical astronomy of the quadrantal triangle.

In the quadrantal triangle P Z X (Fig. 4), the quadrantal side being Z X, given P X 70°, and P Z 40°, to find ∠ Z.

P X is the middle part and ∠ Z and P Z the opposite parts.

$$\text{Cos. } PX = \cos. Z \times \sin. PZ$$

$$\therefore \text{Cos. } Z = \frac{\cos. PX}{\sin. PZ}$$

P X 70°	0′	0″	cos.	9·534052
P Z 40	0	0	sin.	9·808067
Z 57	51	12	cos.	9·725985

The complement of the angle just found is the amplitude and can be taken from the top of the table.

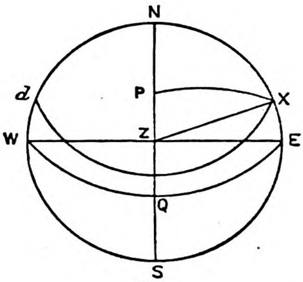


Fig. 4.

Exercises in Quadrantal Triangles.

1. In the quadrantal triangle A B C, given the quadrantal side (a) and ∠ B = 85° 20′, and side c = 81° 36′, to find the other parts.

Ans. ∠ A 90° 41′, ∠ C 81° 34′, and side b 85° 23′.

2. Given side b = 90°, ∠ B = 80° 32′, and ∠ C = 100° 14′, to find the other parts.

Ans. a 22° 31′, c 86° 6′, ∠ A 22° 13.

3. Given a = 90°, b = 101° 15′, ∠ C = 79° 36′, to find the other parts.

Ans. ∠ A 87° 57′, ∠ B 101° 26′, C 79° 47′.

FIGURE DRAWING

The diagrams used to demonstrate the problems in Nautical Astronomy are generally drawn on the Stereographic Projection of the sphere on the plane of the rational horizon, as it is generally agreed that this projection shows the positions of the celestial objects, as viewed by an observer at sea, in the most natural way.

The first circle drawn is the rational horizon and is called the primitive, and this circle contains the projection.

Right circles are great circles seen edgewise and appear as straight lines, such as the meridian of the observer and the prime vertical. These right circles are always a diameter of the primitive.

Oblique circles are great circles which lie between the right circles and the primitive, such as hour circles, the equinoctial and parallels of declination.

The six o'clock hour circle is that great circle which passes through the pole and cuts the rational horizon in the true east and west points. It is, therefore, obvious that when the latitude is 0° the rational horizon is also the six o'clock hour circle. The radius of the six o'clock hour circle increases as the secant of the latitude.

The poles of the meridian of the observer are at the east and west points; and the poles of the prime vertical are at the north and south points.

The poles of all other circles vary in position according to the position of the observer.

Celestial bodies are said to be circumpolar when they neither rise nor set, but circle round the pole of the observer always above his horizon. This is always the case when the latitude (the elevation of the pole) is greater than the polar distance of the object.

We shall now construct a figure for latitude 40°; declination 10° N.; easterly hour-angle 4 hours, using any scale of chords, tangents, semi-tangents and secants.

Arrange the data thus:

Lat. 40° N.	Declination 10° N.	Co-lat.	50°	Hour angle	4 hrs.
Co-lat. 50	Polar distance 80	Polar dist	80		6
		Sum =	130	Comp. of H.A.	2
		Diff. =	30		

FIG. 1.—With the chord of 60° in the compasses and from Z as centre, describe the circle N W S E, which represents the rational horizon. Draw the prime vertical W Z E, bisect the prime vertical, and draw N Z S, the meridian of the observer, and extend it both ways.

To locate the Pole (P), take the semi-tangent of co-latitude (50°) in the

dividers and place one foot on Z and the other foot will find P, the pole, on the meridian N Z S.

To draw the Equinoctial W Q E

Take from the scale the secant of the co-latitude (50°) and with one foot of the compasses on W or E, the other foot will find C, the centre of the equinoctial, on the meridian produced, and C W or C E is the radius; draw the circle W Q E, and it will be the equinoctial.

To draw the Parallel of Declination

Take from the scale the semi-tangent of 130°, and with one foot of the compasses on Z the other will find the point *d′* in the direction of N; also take the semi-tangent of 30° and with one foot on Z the other will find *d″* in the direction of S on the meridian N Z S. If the declination were greater than the latitude it would be laid off in the direction of N; bisect *d′ d″* in *f*, and *f* is the centre of the parallel of declination and *f d′* its radius; draw the circle *d d″ d* and it is the required parallel.

Explanation of Figure I

N W S E	= Rational Horizon.	P Z	= The Co-latitude.
W Q E	= The Equinoctial.	P X	= Object's Polar Distance.
P	= The Pole.	Z X	= ,, Zenith ,,
N Z S	= Meridian of Observer.	*d d″ d*	= Parallel of Declination.
W Z E	= The Prime Vertical.		

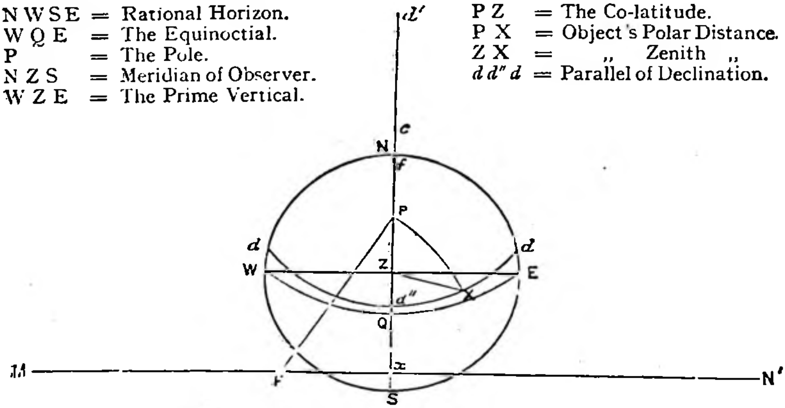


Fig. 1.

To draw the four-hours Easterly Hour Circle

Take from the scale the secant of the latitude and with one foot of the compasses on P the other foot will find *x* on the meridian N Z S; through *x* draw a line parallel to W Z E, which mark M N′; on this line will be found the centres of all the hour circles for that latitude. The point *x* is the centre of the six o'clock hour circle.

Lay off from P, with a protractor, the complement of the hour angle (30°) and draw P F, then F is the centre of the four o'clock easterly hour circle

and P F is its radius; draw the circle P X, and it will be the required hour circle.

Now draw Z X, the zenith distance, and the figure is complete.

If the 1, 2, 3, 4, 5 and 6 o'clock easterly hour circles be continued through the pole till they meet the primitive they will become the 11, 10, 9, 8, 7 and 6 o'clock westerly hour circles respectively.

It, therefore, follows that the centres of the easterly hour circles from 1 to 6 and the westerly hour circles from 6 p.m. to midnight lie to the west of the meridian on the line M x N'; and the centres of the westerly hour circles from 1 to 6 and the easterly hour circles from midnight to 6 a.m. lie to the east of the meridian on the line M x N'.

It is practically impossible to draw the figures for latitude by ex-meridian altitude and latitude by the pole star to scale, owing to the arcs being so small, and it is usual to exaggerate the figures in these cases.

Construct a figure for latitude 0°; declination 40° N.; hour-angle 4 hours East.

Arrange the data as before.

Latitude	0°	Declination	40° N.	Co-lat.	90°	Hour-angle	4 hrs.
Co-latitude	90	Polar Distance	50	Polar Dist.	50		6
				Sum =	140	Comp. H.A.	2
				Diff. =	40		

FIG. 2.—With the chord of 60° describe the circle N W S E, the rational horizon. Draw W Z E, the prime vertical and equinoctial, and N Z S, the observer's meridian, and produce it to d'.

N.B.—The rational horizon is also the six o'clock hour circle in this case.

To draw the four o'clock Hour Circle

From Z towards R lay off the semi-tangent of 60° = 4 hours and with the secant of the complement of the hour-angle 30° = 2 hours, in the compasses, place one foot on N and the other foot will find the centre, C, of the four o'clock hour circle on W Z E, and P c is its radius. Now draw N R, the required hour circle.

To draw the Parallel of Declination

From Z lay off the semi-tangent of 140° towards N on the meridian produced, which mark d', also lay off the semi-tangent of 40° and mark d''; bisect d' d'' in f and with the radius f d' draw the parallel of declination d d'' d.

Draw Z X, the zenith distance, and the figure is complete.

In practice the arc of the hour circle from X to R would be omitted.

The centre of the hour circle, Fig. 2, can also be found in the same way

as in Fig. 1. Lay off from the meridian at N, by means of a protractor, the angle Z N C, and C is the required centre as before.

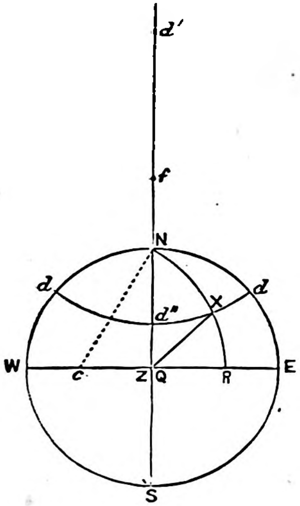


Fig. 2.

Construct a figure for the Double Altitude Problem

				H.	M.		H.	M.
Lat. 45° N.	Dec. 15° N.	Co-lat. 45°	1st H.A.	2	36 E.	2nd H.A.	1	54 W.
Co-lat. 45	Co-dec. 75	Co-dec. 75		6	00		6	0
		Sum = 120	Comp. H.A	3	24	Comp. H.A.	4	6
		Diff. = 30						

Fig. 3 is the figure constructed from the above data in the same manner as Fig. 1. But it is necessary to remark that the arc X′ X in the figure is an arc of a small circle, whereas it should be an arc of a great circle, as shown in Fig. 3A, which, however, is not the same triangle as in Fig. 3, and is simply put in to show the method of finding the centre and radius of the great circle X′ X.

Proceed as follows: Join the positions X′ X, Fig. 3A, by a straight line; bisect it in the point C, and draw C C′ at right angles to X′ X. With the chord of 60° in the dividers and one leg on X′ or X, the other leg will find the centre of the required circle on the line C′ C; draw the arc X′ N X and it will be an arc of a great circle.

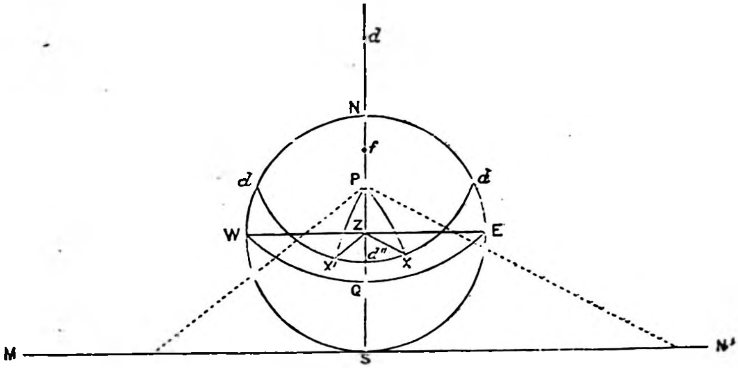


Fig. 3

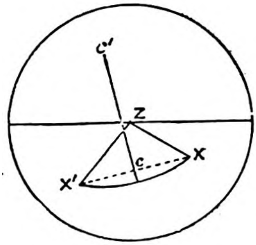


Fig. 3a.

Draw the amplitude figure for latitude 50° S. and declination 10° N.

				H.	M.
Lat. 50° S.	Declination 10° N.	Co-lat. 40°	Time of rising	6	49
Co-lat. 40	Polar Distance 100	Polar Dist. 100	or Hour-angle =	5	11 E.
		Sum = 140		6	0
		Diff. = 60	Comp. of H.A.	0	49

FIG. 4. is constructed from above data. The angle N P R is the complement of the easterly hour-angle and the point R is the centre of the 5h. 11m. easterly hour circle; C is the centre of the equinoctial W Q E; *f* is the centre of the parallel of declination, *d d''*, and X the celestial object rising, and arc E X the rising amplitude.

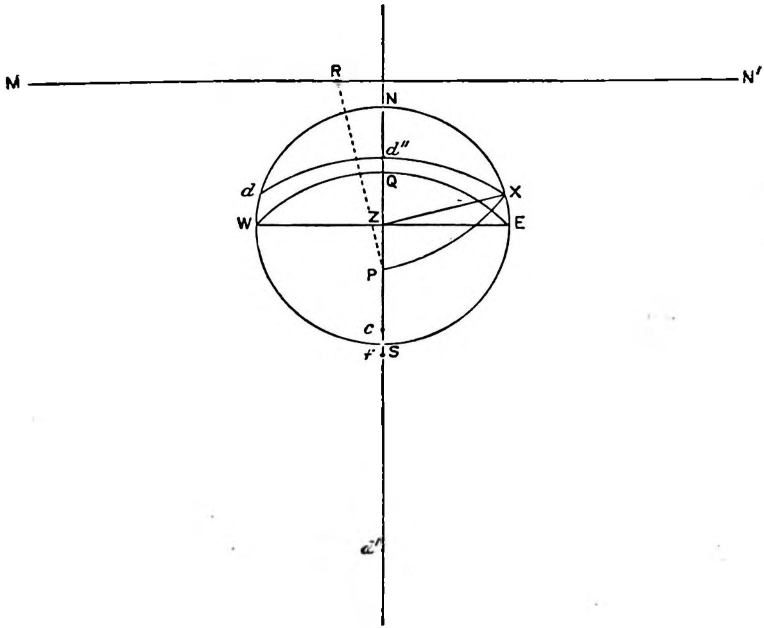


Fig. 4.

N.B.—If a scale of secants is not handy, lay off from E. or W., on the primitive, the chord of the complement of the latitude, and from Z draw a line through the arc, measured by the chord, until it meets the tangent drawn from E. or W.; this line will be the secant required and is the radius of the equinoctial. It is obvious that as the latitude decreases to 0° the radius of the equinoctial increases to infinity and the equinoctial would, for all practical purposes, be a straight line.

GEOGRAPHY AND NAVIGATION

It is necessary to begin by introducing a few preliminary *definitions* connected with the sphere.

A SPHERE is a solid body, every part of the surface of which is equally distant from a fixed point within it which is called the *centre*.

A SPHEROID is a solid body, having the form of a sphere, but not quite round,—being *flattened* in some particular direction.

Every section of a sphere made by a plane is a CIRCLE of the sphere.

A DIAMETER of a sphere is any straight line passing through its centre, and extending in both directions to the surface.

The POLES of a circle on the sphere are those points on the surface of the sphere equally distant from the circumference of that circle. Every circle on a sphere has two poles, one on each side of its plane, and they are at the extremities of a diameter perpendicular to that plane.

A GREAT CIRCLE is any circle whose plane passes through the centre of the sphere; and every point on the great circle is 90 degrees from its pole on the surface of the sphere.

The centre of the sphere is the common centre of all its great circles, and no two great circles can have a common pole.

GREAT CIRCLES of the sphere are equal, and when they intersect they cut one another into two equal parts.

A SMALL CIRCLE is unequally distant from its two poles, since its plane does not pass through the centre of the sphere. Every point on the small circle is less than 90 degrees from one pole, and more than 90 degrees from the other pole of the sphere.

A sphere is divided into two equal parts by the plane of every great circle: and into two unequal parts by the plane of every small circle.

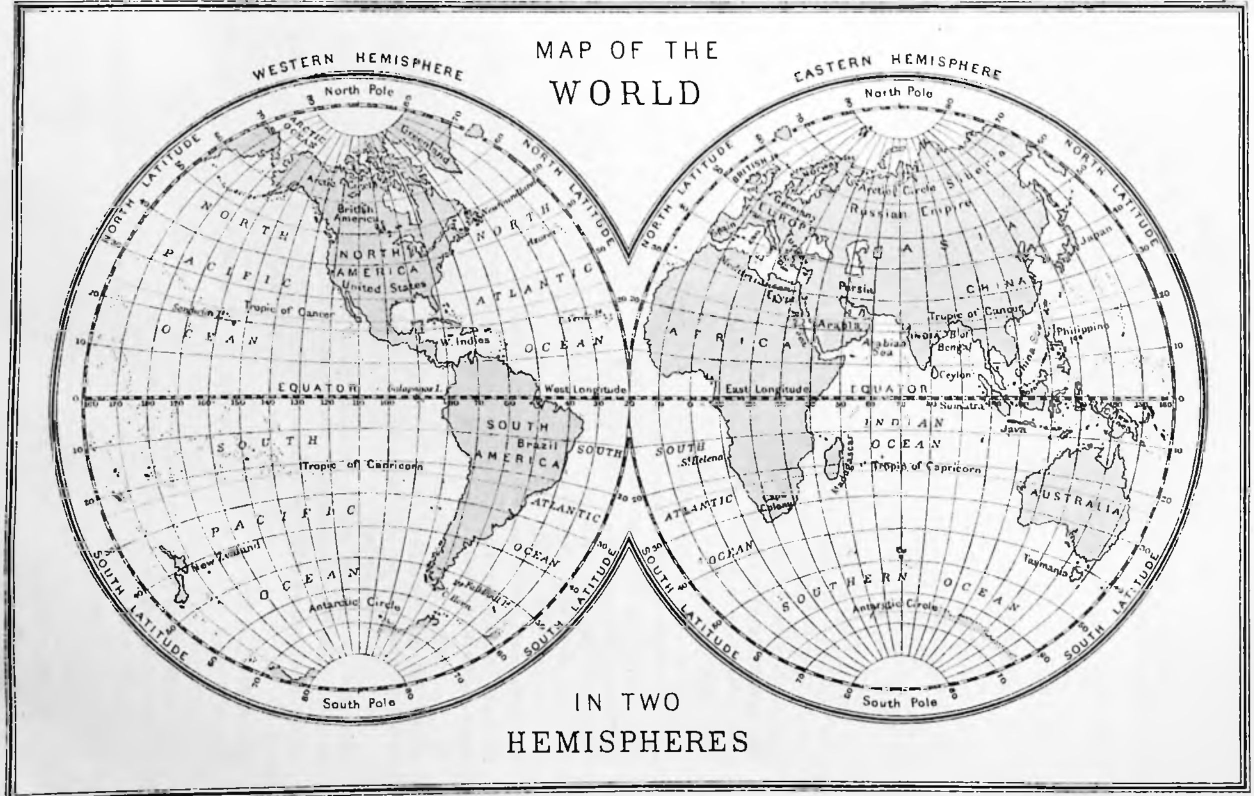
PARALLEL CIRCLES of the sphere are those circles the planes of which are parallel to the plane of some great circle; and all these circles have the same pole.

The ARC of a circle is any part (small or large) of the circumference of a circle—it may be a degree or less, or many degrees, in length.

The *shortest* DISTANCE between two points on the surface of a sphere is the arc of a great circle passing through those points.

A SPHERICAL ANGLE is the inclination of two great circles of the sphere meeting one another.

A SPHERICAL TRIANGLE is a figure formed on the surface of a sphere by the mutual inter sections of three great circles.



Imray, Laurie, Norie & Wilson, London.

Any two sides of a spherical triangle are together greater than the third side; and the three sides of a spherical triangle are together less than the circumference of a great circle.

In a spherical triangle the greater side is opposite the greater angle, and the greater angle opposite the greater side.

THE FIGURE AND MAGNITUDE OF THE EARTH

The earth, consisting of land and water, is a body of a spherical or globular form, but not a perfect sphere, since it is flattened towards the poles. Hence it is a *spheroid*, or more properly an oblate spheroid.

The *earth* is certainly *not flat*, as may be proved by various facts and phenomena.

(1.) The most obvious of the several arguments which prove the sphericity of the earth, and what must particularly strike every mariner, are, that when approaching the shores of countries, the points of high rocks, lighthouses, steeples of churches, and other thin but lofty objects, come into view much sooner than houses or other buildings of greater magnitude, but less height; in like manner, when ships are approaching each other at sea, the masts and rigging are discerned some time before the hull and lower parts of the vessel, though much larger, come into view.

(2.) Seamen, it is well known, frequently discover distant lands from the tops of a ship's masts, long before they are visible to those who stand upon deck. These circumstances prove that the surface of the earth is convex; and as the same appearances happen wherever the observer is situated, this convexity must be approximately uniform: hence we conclude that the earth is globular.

(3.) The sphericity of the earth is likewise demonstrated by navigators who have sailed quite round it, by constantly going westward and arriving home from the eastward, or going eastward and arriving home from westward, which could not be effected were the earth a plane.

(4.) During an eclipse of the moon the shadow of the earth is thrown by the sun on the moon's surface; the shadow thus projected is invariably found to be circular, and such as could only be given by a spherical body.

(5.) The actual measurements of an arc of a meridian in various parts of the world, together with a comparison of the results of pendulum experiments in high latitudes and at the equator, all tend to prove the same fact—that the earth is an oblate spheroid.

DIMENSIONS OF THE EARTH.—The Astronomer Bessel gave the following constants as admeasurements of the earth:—

Equatorial radius in feet	20923600
Polar radius ,,	20853657
Degree of longitude at equator	365186
Degree of latitude at equator	362750
,, ,, at lat. 45°	364566
,, ,, at lat. 52°	365000
,, ,, at the Pole	366396

The approximate dimensions of the earth considered as a spheroid of revolution may be taken to be—

Equatorial radius	in feet, 20926202 :	in English miles,	3963·296
Polar radius	„ 20854895 :	„ „	3949·791

whence the equatorial diameter exceeds the polar diameter by 27 miles. Sir G. B. Airy makes it $26\frac{1}{2}$ miles, and the compression $\frac{1}{300}$ nearly.

But from recent observations, and the result of actual surveying, the earth is not a spheroid of revolution ; the determinations of its various dimensions are given in a work on Geodesy by Colonel A. R. Clarke, C.B., as follow :—" If it be supposed that the earth is an ellipsoid with three unequal axes (diameters), then—

Equatorial radius..............	longest, in feet	20926629
	shortest, in feet	20925105
Polar radius	„	20854477

and the longest equatorial diameter meets the surface in long. 8° 15′ W. of Greenwich ; and the shortest in Ceylon. But it is necessary to guard against an impression that the figure of the equator is thus definitely fixed, for the available data are far too slender to warrant such a conclusion."

Thus the equatorial circumference may be roughly estimated at 25,000 English miles, and the ellipticity or earth's compression at about $\frac{1}{293}$.

DEFINITIONS RELATING TO THE EARTH

To point out the relative situation of places on the surface of the globe, geographers imagine certain points, lines, and circles belonging thereto ; of these it will be necessary here to explain such of them as appertain more immediately to Navigation.

AXIS.—The *axis* of the earth is that diameter around which the earth daily revolves from west to east ; the revolution is completed in 24 hours.

POLES.—The *poles* are the two ends of the diameter (or axis) around which the earth's daily revolution is performed. The pole which is nearest to us, in Europe, is called the *North Pole*, and the other is the South Pole, and as the poles are the ends of a diameter they are 180° apart.

EQUATOR.—The *equator* is a great circle on the earth equally distant from the two poles ; it divides the earth into two equal parts, called *hemispheres ;* the part having the North Pole for its centre is the *Northern Hemisphere ;* and the part having the South Pole for its centre is the *Southern Hemisphere.* Every point on the equator is 90° (of a great circle) from each pole.

MERIDIANS are great circles on the earth passing through both poles, and therefore perpendicular to the equator. Every place or point on the earth's surface has its meridian, and in this sense the meridian is a semi-circle extending from one pole to the other, and passes through the given place.

Most of the principal maritime nations have a national observatory ; and some of these nations, for the purposes of Geography and Navigation, adopt a *first meridian* whence longitude is reckoned ; thus the French adopt Paris

the Spaniards adopt San Fernando (Cadiz), and we (in the British Empire) select Greenwich, as the first meridian. Some such arrangement is necessary, for, while the starting point for latitude among all nations is the equator—that well-defined great circle midway between the poles, and which divides the globe into two equal parts—there is no such well-defined great circle on the globe whence to reckon the longitude. A congress has recently been held at Washington for the purpose of adopting a *universal prime meridian*, to be used by all nations, and Greenwich has been selected for this purpose.

LATITUDE.—The distance of a place from the equator, measured (in degrees and parts of a degree) on the meridian of that place, is its latitude. As latitude begins at the equator, where it is 0°; so it ends at the poles where it is greatest, or 90°. The latitude is N. when the place is situated in the Northern Hemisphere, and S. when it is situated in the Southern Hemisphere.

PARALLELS OF LATITUDE are small circles on the globe parallel to the equator; every place on the earth's surface has its parallel of latitude, and any two or more places or points on the circumference of any one of these small circles, being equally distant from the equator, have the same latitude.

The parallel of latitude 23° 28′ north of the equator is the *Tropic of Cancer;* and the parallel of latitude 23° 28′ south of the equator is the *Tropic of Capricorn.* These parallels mark the limits of the sun's N. and S. declination.

Difference of Latitude (Diff. Lat.) is an arc of a meridian, or the least distance, between the parallels of latitude of two places, indicating how far one of them is to the northward, or southward, of the other. If two places have latitudes both north, or both south, their Diff. Lat. is found by subtracting the less latitude from the greater: but when one place is in north latitude, and the other in south latitude, the Diff. Lat. is found by taking the sum of the two latitudes. The Diff. Lat. can never exceed 180°.

LONGITUDE.—The longitude of any place on the earth's surface is expressed as an *arc of the equator* contained between the meridian passing through that place and the *first meridian* whence longitude is reckoned to begin. Longitude, like latitude, is estimated in degrees and parts of a degree, and may be Eastward or Westward of the first meridian, reckoned from 0° (when a place is on that meridian) to 180°,—the meridian on the opposite side of the globe.

Longitude is also the measure of the angle at the pole between the first meridian and the meridian passing through a place.

Difference of Longitude (Diff. Long.) is the arc of the equator, or the angle at the pole included between the meridians passing through any two places. If two places have longitudes both east or both west, their Diff. Long. is found by subtracting the less longitude from the greater; but when one place is in east longitude and the other in west longitude, the Diff. Long. is found by taking the sum of the two longitudes; and in the latter case, since the Diff. Long. of two places can never exceed 180°, when that sum is greater than 180°, subtract it from 360°, and the remainder is the Diff. Long.

Note.—If we know the latitude of any place and not the longitude, or, on the other hand, the longitude and not the latitude, its position on the earth is not determined, because, if we say it is in lat. 42° N., it may be anywhere on the parallel of 42° N. in E. or W. long., in the Pacific or in the Atlantic Ocean; similarly, if we say a place is in long. 32° E., it may be in Europe, in Africa, in the Mediterranean, or in the seas south of the Cape of Good Hope. Hence both latitude and longitude are required to fix the position; and when we say that a *ship* is in lat. 46° 20′ N., long. 32° 40′ W., we have its PLACE AT SEA accurately noted—a spot in the Atlantic—and a course can be shaped therefrom towards the port of destination. It is part of the routine of a voyage to determine the ship's place day by day, approximately *by dead reckoning,* and more accurately *by observation* of the heavenly bodies.

The HORIZON of any place is that apparent circle which limits or bounds the view of a spectator on the sea, the eye being always supposed in the centre of the horizon. This circle is divided into parts similar to the divisions on the Mariner's Compass.

The ZONES.—The earth is considered to be divided by certain circles, parallel to the equator, into broad spaces, called ZONES; of these there are five, viz., one torrid, two frigid, and two temperate, in allusion to the general temperature which prevails in each of the regions.

The TORRID ZONE is that region of the earth over some part of which the sun is vertical at some time of the year. This zone is 47 degrees in breadth, extending to about 23° 28′ on each side of the equator; the parallel of latitude bounding it in the northern hemisphere is called the *Tropic of Cancer;* and in the southern hemisphere the limiting parallel is called the *Tropic of Capricorn.*

The FRIGID ZONES are those regions of the earth around the Poles where the sun at certain times of the year does not rise or set for some days or weeks; they extend round the Poles to the distance of 23° 28′. The parallel which bounds this limit in our northern hemisphere is called the *Arctic Circle,* and that portion of the globe included within it is the *North Frigid Zone* or *Arctic Regions.* The parallel which is at the same distance from the South Pole, in the southern hemisphere, is called the *Antarctic Circle,* and the space included within it is the *South Frigid Zone,* or *Antarctic Regions.*

The TEMPERATE ZONES are those portions of the earth comprehended between the Torrid and the Frigid Zones; they are distinguished respectively as the *North Temperate* and *South Temperate Zone.*

I. *To find the Difference of Latitude between two Places*

RULE.—When the latitudes are both of the same name, that is, both North or both South, subtract the less from the greater, and the remainder will be the *difference* of latitude. But when one is North, and the other South, their sum will be the *difference* of latitude.

EXAMPLE I. What is the difference of latitude between the Lizard and Cape Finisterre?

Latitude of the Lizard ..	49°	58′	N.
Lat. of Cape Finisterre ..	42	53	N.
Diff. of latitude	7	5	
	60		
In miles	425		

EXAMPLE II. A ship from latitude 3° 10′ S. arrives in latitude 2° 26′ N.: required the difference of latitude made good.

Latitude left	3°	10′	S.
Latitude in	2	26	N.
Diff. of latitude	5	36	
	60		
In miles	336		

II. *With the Latitude left and the Difference of Latitude, to find the Latitude in*

RULE.—When the latitude left and difference of latitude are of the same name, their sum gives the latitude in; but when they are of different names, their difference is the latitude in, of the same name with the greater.

EXAMPLE I. A ship from the West end of the Island of Madeira, in latitude 32° 48′ N., sails North 520 miles*: what latitude is she in?

Latitude of Madeira	32°	48′	N.
Diff. of latitude 520 m. =	8	40	N.
Latitude in	41	28	N.

EXAMPLE II. A ship three days ago was in latitude 2° 48′ N., and has since then sailed South 426 miles: required her present latitude.

Latitude left	2°	48′	N.
Diff. of latitude 426 m. =	7	6	S.
Latitude in	4	18	S.

LAT. LEFT is the latitude from which the ship has *departed.*

DIFF. LAT. is the *change of latitude* in any interval.

LAT. IN is the latitude at which the ship *arrives.*

The method of obtaining the Lat. in, from the Lat. left and Diff. Lat., is shown in the following examples and explanations—

	(1)	(2)	(3)	(4)	(5)	(6)
Lat. left ..	49° 3′ N.	49° 3′ N.	1° 3′ N.	49° 3′ S.	49° 3′ S.	1° 3′ S.
Diff. lat. ..	2 5 N.	2 5 S.	2 5 S.	2 5 S.	2 5 N.	2 5 N.
Lat. in	51 8 N.	46 58 N.	1 2 S.	51 8 S.	46 58 S.	1 2 N.

Understanding that latitude is reckoned from the equator towards the pole:—

(1) If you are in *north* latitude and sail *north*, Lat. left must be increased by Diff. Lat.

(2) If you are in *north* latitude and sail *south*, Lat. left must be decreased by Diff. Lat.

(3) If you are in *north* latitude and sail *south*, and the Diff. Lat. exceeds the Lat. left, take the less from the greater, and the remainder will be the Lat. in, *south.*

(4) If you are in *south* latitude and sail *south*, Lat. left must be increased by Diff. Lat.

* When the difference of latitude or longitude is given in miles or minutes (′) it is to be divided by 60, to reduce it to degrees and minutes.

(5) If you are in *south* latitude and sail *north*, Lat. left must be decreased by Diff. Lat.

(6) If you are in *south* latitude and sail *north*, and the Diff. Lat. exceeds the Lat. left, take the less from the greater, and the remainder will be the Lat. in, *north*.

The Lat. in, when it has been obtained by *dead reckoning*, is indifferently called the Latitude by Account, or Latitude by Dead Reckoning, and written in short Lat. by Acc., or Lat. by D. R.

III. *To find the Middle Latitude*

The *Middle Latitude* is the parallel of latitude *midway* between two places, hence it is half the sum of the two latitudes when they have the same name (N. or S.); half the difference when they have different names —one N. and the other S:—

Lat. of A..	46°	20′	N.	Lat. of C..	4°	10′	N.
Lat. of B..	39	20	N.	Lat. of D..	6	50	S.
	2)85	40			2)2	40	
Mid. Lat...	42	50	N.	Mid. Lat...	1	20	S.

IV. *To find the Difference of Longitude between two Places*

RULE.—If the longitudes of the given places be both East or both West, subtract the less from the greater; but if one be East and the other West, add them together, and the sum or remainder will be the *difference* of longitude. When the sum of the two longitudes exceeds 180 degrees, subtract it from 360 degrees, and the remainder will be the *difference* of longitude.

EXAMPLE I. What is the difference of longitude between the Lizard and St. Mary's, one of the Western Islands?

Longitude of the Lizard	5°	12′	W.
Longitude of St. Mary's	25	10	W.
Diff. of longitude	19	58	
	60		
In minutes (′) ..	1198′		

EXAMPLE II. A ship sailing westward from North Cape, New Zealand, arrives in longitude 164° 47′ W.: required the diff. of longitude made good.

Long. of North Cape ..	173°	5′	E.
Long. of ship	164	47	W.
Sum	337	52	
	360	0	
Diff. of longitude	22	8	
	60		
In minutes (′)	1328′		

V. *With the Longitude left, and Difference of Longitude, to find the Longitude in*

RULE.—If the longitude left and difference of longitude are of different names, subtract the less from the greater, and the remainder will be the longitude in, of the same name with the greater; but if the longitude left and difference of longitude are of the same name, their sum will be the longitude in, of the same name with the longitude left. If, however, this sum exceed 180°, subtract it from 360°, and the remainder will be the longitude in, of a contrary name to the longitude left.

EXAMPLE I. Suppose a ship from St. Helena sail eastward until her difference of longitude be 220′: required her longitude in.

Long. of St. Helena	5°	44′	W.
Diff. of longitude 220′ =	3	40	E.
Longitude in	2	4	W.

EXAMPLE II. If a ship from longitude 176° 49′ W. sail westward until her difference of longitude be 10° 14′, what is her present longitude?

Longitude left	176°	49′	W.
Diff. of longitude	10	14	W.
Sum	187	3	W.
	360	0	
Longitude in	172	57	E.

LONG. LEFT is the longitude from which the ship has *departed*.

DIFF. LONG. is the *change* of longitude in any interval.

LONG. IN is the longitude at which the ship *arrives*.

The method of obtaining the Long. in, from the Long. left and Diff. Long., is shown in the following examples and explanations:—

	(1)	(2)	(3)	(4)
Long. left	40° E.	40° E.	3° E.	34° W.
Diff. Long.	2 E.	2 W.	5 W.	4 W.
Long. in..............	42 E.	38 E.	2 W.	38 W.

	(5)	(6)	(7)	(8)
Long. left	34° W.	2° W.	178° E.	179° W.
Diff. Long.	4 E.	6 E.	6 E.	5 W.
Long. in..............	30 W.	4 E.	184 E.	184 W.
			360	360
		Long. in	176 W.	176 E.

Understanding that Longitude is reckoned E. or W. from the meridian of Greenwich to 180°:—

(1) If you are in *east* long. and sail *east*, Long. left must be increased by Diff. Long.

(2) If you are in *east* long. and sail *west*, Long. left must be decreased by Diff. Long.

(3) If you are in *east* long. and sail *west*, and Diff. Long. exceeds Long. left, take the less from the greater, and the remainder will be the Long. in, *west*.

(4) If you are in *west* long. and sail *west*, Long. left must be increased by Diff. Long.

(5) If you are in *west* long. and sail *east*, Long. left must be decreased by Diff. Long.

(6) If you are in *west* long. and sail *east*, and Diff. Long. exceeds Long. left, take the less from the greater, and the remainder will be Long. in, *east*.

(7 and 8) If the sum of the Long. left and Diff. Long. exceeds 180°, take the sum from 360° for the Long. in, which will have a different name from the Long. left.

Examples for Practice

1. Required the difference of latitude and longitude between Ushant in lat. 48° 27½′ N., long. 5° 7½′ W., and Cape Ortegal in lat. 43° 45′ N., long 7° 56′ W.

Ans. Diff. of latitude 282½ miles; diff. of longitude 168½′.

2. What is the difference of latitude and longitude between the Cape of Good Hope in lat. 34° 21′ S., long. 18° 30′ E., and St. Helena in lat. 15° 55′ S., long. 5° 44′ W.?

Ans. Diff. of latitude 1106 miles; diff. of longitude 1454′.

3. Required the difference of latitude and longitude between Cape Verde (paps) in lat. 14° 43′ N., long. 17° 34′ W., and Cape St. Roque in lat. 5° 29′ S., long. 35° 15′ W.

Ans. Diff. of latitude 1212 miles; diff. of longitude 1061′.

4. Required the difference of latitude and longitude between Cape Clear (Ireland) in lat. 51° 26′ N., long. 9° 29′ W., and St. Agnes Light (Scilly Islands) in lat. 49° 53½′ N., long. 6° 20½′ W.

Ans. Diff. of latitude 92½ miles; diff. of longitude 188½′.

5. A ship from Funchal, in Madeira, in lat. 32° 38′ N., long. 16° 55′ W., sails in the S.E. quarter until her difference of latitude is 326 miles, and difference of longitude 425′; required her present latitude and longitude.

Ans. Latitude 27° 12′ N.; longitude 9° 50′ W.

6. A ship from latitude 2° 56′ S., and longitude 5° 14′ E., sails north-westerly until her difference of latitude is 352 miles, and difference of longitude 628′; required her present latitude and longitude.

Ans. Latitude 2° 56′ N.; longitude 5° 14′ W.

7. A ship from the Equator, and longitude 89° 17′ E., sails south-westerly until her difference of latitude is 370 miles, and difference of longitude 118′; required her present latitude and longitude.

Ans. Latitude 6° 10′ S.; longitude 87° 19′ E.

8. A ship from Cape East (New Zealand) in lat. 37° 40′ S., long. 178° 36′ E., sails in the N.E. quarter until her difference of latitude is 114 miles, and difference of longitude 297′; required her present latitude and longitude.

Ans. Latitude 35° 46′ S.; longitude 176° 27′ W.

9. Required the difference of latitude and longitude between Valparaiso in lat. 33° 2′ S., long. 71° 41′ W., and Gutzlaff Island in lat. 30° 47′ N., long. 122° 11′ E.

Ans. Diff. lat. 3829 miles; diff. long. 9968′.

MARINE SURVEYING

About one hundred and ten years ago John William Norie brought out the first edition of his "Epitome of Navigation." In the chapter on Marine Surveying he said—

"The art of surveying coasts and harbours being very essential to those who visit unknown parts is treated in a manner which it is hoped will make its acquisition and practice perfectly easy. Notwithstanding the great importance of accurate surveys of various coasts and harbours frequented by mariners, it must be confessed that this branch of the nautical art has been little attended to, and that the opportunity which so frequently occurs to seamen is almost entirely neglected. We therefore think it proper to lay down a few general directions showing how a coast or harbour may be easily surveyed with such instruments as are used at sea."

At the present time there are few places that have not been surveyed, but there are many places that have not been thoroughly surveyed, many places that were surveyed so long ago that alterations, sometimes of serious extent, have taken place, or the navigator may come upon an uncharted bank or rock, or obtain other information he desires to record. He should, therefore, know how to fix a position accurately upon a plan or chart, or if necessary construct a chart for himself and put in anything he desires. So, repeating Norie's words—

We therefore think it proper to lay down a few general directions how to make a survey of a harbour, fix the position of a rock, shoal, or wreck.

Instruments

Chronometer whose rate on G.M.T. is known.
Sextant properly adjusted, and whose centering error is known.
Artificial Horizon for use on shore.
A Hack Watch.
Azimuth and Portable Compasses.
Aneroid Barometer marked in feet (mountain use).
Camera for Photographic observations.
A Reliable Tape Measure and other measures.
Field's Parallel Rulers.
Station Pointer.
Protractor.
Drawing Board and T Square.
A number of pointed staves to mark off distances when measuring the base line.
Several long staffs with bunting at the end to mark special points.
A bucket or two of whitewash to help to distinguish a station.
Morse and Semaphore Flags for communication from a distance

For Work on the Water—

Patent Log.
Lead Line marked in fathoms and feet.

A painted pole marked in feet and half-feet for shallow water and, if there is time, for tidal observations.

A tidal gauge or pole with alternate feet painted black and white also showing half-feet or tenths of a foot.

Book to record angles, distances, depths of water, height of hills, and any other notes for plotting on the chart.

TO SURVEY A BAY OR HARBOUR.

Take a general view of the place either by walking or sailing round it in a boat or steam launch or by both.

During the journey make notes and rough sketches, or, better still, take photographs of the most prominent objects, headlands, bays, hills, or mountains, particularly when they come in line, and of anything conspicuous or remarkable.

Select a place suitable for a base line. A long level stretch of sand at sea level is the best, but it must not be land-locked, as it is necessary that as many objects, headlands, and points should be seen from both ends of the line as possible.

It is possible that suitable marks will have to be made by the use of staffs or otherwise. All the information should be marked on a rough sketch and the objects observed named, lettered, or numbered, in order to distinguish them when wanted.

The Base Line.

The fixing of the base line must be done with all possible accuracy, for an error in the base line will throw out all the observations referred to it or derived from it. The position should be selected with care either on the land, or when it is not possible to get a land base, then on the water. If not all, then most of the stations should be visible from each end of the line. Its length and direction should be such that the angle contained between it and any of the stations taken from one end of the base line may differ at least ten degrees from the same object taken from the other end thereof. Set up the two station marks as far apart as the ground permits, and accurately measure the distance between them by

Direct measurement,
Velocity of sound,
Astronomical observation,
Masthead angles,
Patent Log while under steam; or by a combination of more than one method.

Direct Measurement.

Having marked the observation station and called it A, select position of B, then stick into the ground at intervals flagstaffs sighted in exact line between them so that the measurers may get the true direction.

Measure with the tape 100 feet of well-used and stretched deep-sea lead-line or sounding wire. Splice a ring one or two inches in diameter into both ends; let that length be 100 feet over all. Have a quantity of wooden

staves one inch square, two feet long, pointed at one end. Unship the staff at station A and drive in a peg; slip the ring at one end of the line over it, take the other end towards the flagstaff and station B, haul it well taut, drive in a peg, and ship the other ring over it. The line should be sighted to see that it is in the right direction, then measured to see that it is right in length. The first end is then slid off the first peg and carried forward and the station staff replaced.

When the line is again taut another peg is driven in and the ring shipped on to it. This process is repeated until station B is reached. The number of pegs represent the number of hundreds of feet or any other agreed unit of measurement. The line should occasionally be remeasured with the tape.

Measuring on the water between two buoys, boats, or rocks the wire should be attached to floats to keep it from sinking, and as long a stretch as possible made. A boat or buoy moored takes the end. Such a measurement should be made when there is no tide or currents. Such measurements are only possible for short bases. When the base is a mile or more in length and cannot be measured as above the distance can be measured by sound.

Sound Measurements.

A small gun is mounted at each end of the base line. When all is ready the gun is fired and the interval between the flash and the report is accurately noted by hack watch. A gun is then fired from the other end and the operation repeated several times. The mean of all the times is taken as the true time. The distance is then found from the formula. Distance = Time × velocity of sound at the given temperature. Sound travels at the rate of 1,091 feet per second at the temperature of 32° F. and increases 10 feet per second for an increase of 9° in temperature.

Example.—The mean of the times of the observations was 11·5 seconds, temperature 60°. Required the distance.

Formula.—$D = T \times (v + x)$; where D = distance; T = time; v = velocity at standard temp. and x = correction for difference of temperature above freezing point.

v	1091 feet	Observed temp.	60°
x	+31	Standard ..	32°
$(v+x)$	1122	Diff.	28°
	11·5		
	5610		
	12342		
	12903·0 feet	Log.	4·110691
	6080 feet	Log.	3·783904
	D = 2·122 m.	Log.	0·326787

$9 : 28^\circ :: 10 \text{ feet} : x$

$$\therefore x = \frac{28 \times 10}{9} = \frac{280}{9} = 31 \text{ feet.}$$

If the observed temperature were below the freezing point the formula would be modified. *i.e.* $(v + x)$ would become $(v - x)$.

As an error of ·1 second causes an error in the measurement of 100 feet this method is not suitable for short base lines.

Masthead Angles.

When from any reason a base cannot be fixed on shore, then a base must be measured on the water. The vessel forms one station and the other station may be fixed on shore preferably, or by a boat moored at a suitable distance and direction. The distance is obtained by measuring

the angle made by the masthead and the waterline. The height of the masthead being known and the angle measured, the distance is found by the formula D=M cot. A.

Where D=distance, M=height, and A=the angle subtended.

Example.—From masthead to waterline measured 200 feet.

The angle subtended from the shore station was 5°. What is the distance from the ship?

$$\begin{array}{lll} M = 200 \text{ feet} & \log & = 2{\cdot}301030 \\ A = 5^\circ & \text{cot.} & = 11{\cdot}058048 \\ \hline D = 2286 \text{ feet} & \log & = 13{\cdot}359078 \end{array}$$

Patent Log

The two positions being determined and marked by boats, buoys, or in any other suitable way, take up a position some distance from the first buoy; when well under weigh stream the log. Note the log when the buoy is a beam and note again when the second buoy is abeam, also note the time elapsed. If possible this should be repeated, steering the opposite course, and the mean taken as the distance. It should be done at slack water where there is a tide.

In all cases the true bearing of the base line should be found by the sun's bearing in transit across the two stations, or if a light is fixed at each end of the base, bearings can be taken frequently by stars, or the bearing may be found by azimuth and sextant angle combined.

Compasses cannot be depended upon when they are landed on strange ground; there may be magnetic ore in the neighbourhood.

Tidal Observations, Soundings, etc.

The most important information on the chart, to the navigator, are the soundings. There are therefore two prime objects to be kept in view—the depth of water and the nature of the bottom. The latter information is indispensable to the seaman in foggy weather. He should also gather all the information he can as to the set and drift of the tide at flood and ebb and at spring and neap tides, and carefully note any local peculiarities.

While operations on shore are going forward a "tide gauge" should be set up, fixed in a well-sheltered place easy of access and in such a manner that the zero of the gauge is below the lowest low-water mark. The gauge should be firmly fixed, able to withstand any weather. Observations should be continued through a lunar month, if possible. The mean of all the high and low-water readings will give the half-mean spring range or what would be the mean level of the sea if there were no tidal rise and fall. The half-mean spring range is the true scientific level of reference in all matters relative to the tides. This data should be marked in a permanent manner by being cut in the rock or on a spot or in some other suitable way that would ensure permanency.

Soundings.

Determine upon what plan and in what direction it is intended to run out lines of soundings. Note any objects in transit, especially if well

distant from one another. If there are no natural objects in line use flag staffs, keeping them one behind the other as far apart as possible, shifting them to suit each new line. Fix the boat's position at starting by sextant angles and station pointer, and fix anywhere along the line of soundings when suitable objects present themselves. Make a good fix at the end of the line.

The boat should be pulled on these lines at a uniform speed and casts of the lead made at regular intervals; the time, the depth, and the nature of the bottom should be carefully noted.

In a harbour where the tidal range is not known an observer should be stationed at the tide gauge to note the height every half-hour, but at more frequent intervals at about the time of high and low water. The watches used at the tide gauge and by the boat parties should be compared before and after operations.

The lead line should be wetted, stretched, and marked in fathoms and feet. For shallow water a staff or pole is used, marked in feet and tenths of a foot; the pole is shod with a square piece of wood to prevent the pole sinking into the mud.

All soundings must be reduced to mean low-water spring tides before they are inserted on a chart. This information is obtained by means of the tide gauge, and is the result of the observations carried through a full month.

To fix the Position of a Rock or Shoal when out of sight of land.

Required: Position, extent, depth of water on the rock or shoal.

In bad weather no fixing of any value can be made. In fine weather and smooth water the fix will depend on the accuracy of the vessel's position. Assuming that to be correct, and a shoal patch discovered, send a boat away to anchor if possible in the least water. The observer in the boat should be provided with a lead line, a sextant, a reliable watch, note-book and pencil, and Morse or semaphore signalling flags. When the boat is in position the vessel steams or sails round the boat, sounding at intervals and asking the boat to take masthead angles as they are required, at the same instant compass bearings of the boat are taken from the ship. This is repeated until the circuit of the shoal is made.

With the data obtained the boat's position can be plotted with reference to the ship, and ship's position with reference to the boat, by working out the masthead angles for the distance and the direction by the compass bearing corrected for compass error. At one, or more than one, position the ship is fixed by observation, from which the position of the shoal can be worked out.

It is hoped that the following examples will enable the student to grasp the principles involved in carrying out a survey and inculcate in him a desire to master this much-neglected branch of useful knowledge to the mariner.

EXAMPLE I.

Let it be required to survey the harbour shown in Plate I. by observations made on the water.

Having sailed round the harbour and fixed upon the several stations on the coast, let the two buoys, A and B, be moored so that all the points or

stations selected may be seen from both A and B; the bearing of B from A is 45°, that is, N. 45° E. true, the variation 23° W., and the distance one mile; then, having taken the boat to the stations A and B, assume the following bearings to have been taken. The bearings are given both true and magnetic, but it is advisable to construct plans from true bearings, as they will not be affected by lapse of time.

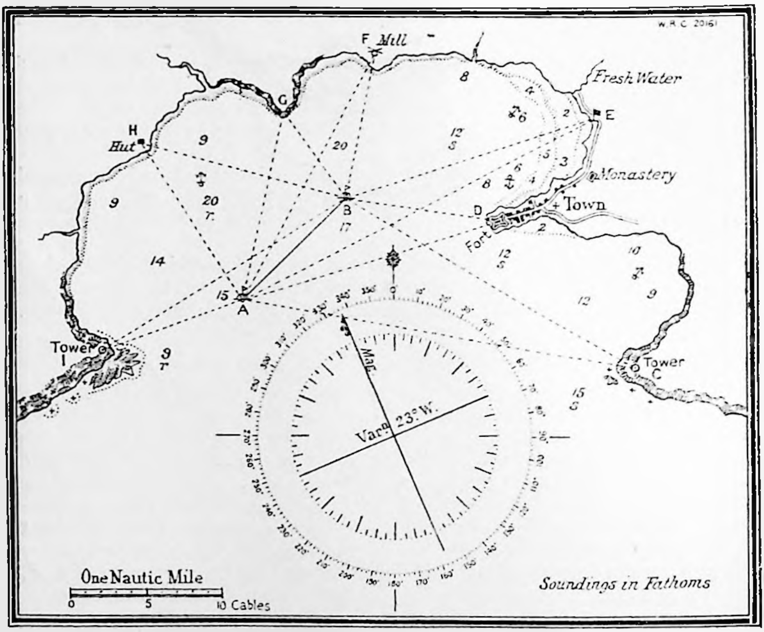


Plate 1.

From Station A

C	bore	99°	true	= S. 58° E.	mag.
D	,,	72°	,,	= S. 85° E.	,,
E	,,	62°	,,	= N. 85° E.	,,
F	,,	28°	,,	= N. 51° E.	,,
G	,,	12°	,,	= N. 35° E.	,,
H	,,	328°	,,	= N 9° W.	,,
I	,,	249°	,,	= N. 88° W.	,,

From Station B

C	bore	120°	true	= S. 37° E.	mag.
D	,,	98°	,,	= S. 59° E.	,,
E	,,	72°	,,	= S. 85° E.	,,
F	,,	11°	,,	= N. 34° E.	,,
G	,,	324°	,,	= N. 13° W.	,,
H	,,	285°	,,	= N. 52° W.	,,
I	,,	237°	,,	= S. 80° W.	,,

These bearings were checked by sextant angles from A and B, and the positions of A and B as found by the station pointer were in agreement with the positions of A and B as laid down on the plan.

Sextant angles from A	Sextant angles from B
I 79° H 44° G	G 47° F 87° D

Example II.

Wanting to survey a coast whilst sailing along it, I ran from A to B (see Plate II.), 248°, 6 miles; from B to C, 270°, 4 miles; from C to D, 331°, 3·5 miles, taking the following bearings and angles at each station. All courses and bearings are true.

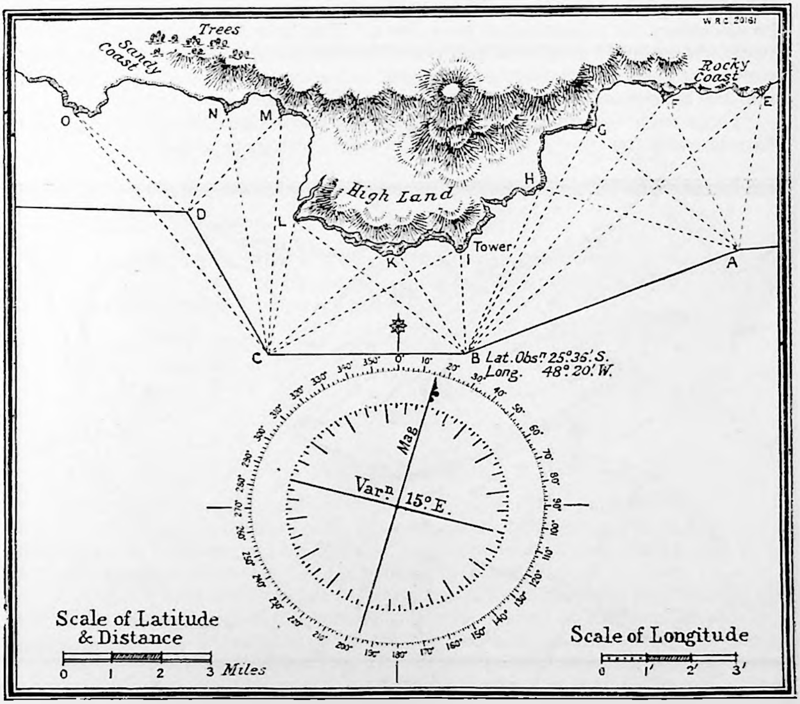


Plate II.

From A	From B	From C
G bore 310°	K bore 328°	K bore 46°
Angle G A E = 58°	Angle K B L = 18°	Angle K C I = 13¼°
,, G A F = 25°	,, K B I = 29°	,, K C L = 36°
,, G A H = 21° 30′	,, K B H = 55¼°	,, K C M = 43° 50′
	,, K B G = 60°	,, K C N = 55½°
	,, K B F = 69°	,, K C O = 83⅓°
	,, K B E = 79½°	

From D
N bore 20°
Angle N D O = 67°
,, N D M = 22°

Example III.

Let it be required to make an accurate survey of the harbour and adjacent island and make a plan (see Plate III.).

Sail round the coast to be surveyed, and fix station staves on the principal points where there are no remarkable objects to distinguish them ; at the same time make a rough sketch of the harbour, on which denote the positions of the objects and station staves by the letters *a*, *b*, *c*, *d*, etc. ; seek for a proper place on the shore on which a base line may be measured ; and as there is no part of the coast that commands a view of all the stations, it will be necessary to measure two base lines. The base line A B is first fixed upon, the ground being level and a considerable number of the stations being visible from each extremity ; its length, as measured by a chain, is 8 cables, and the bearing of B from A is 50° (N. 50° E.) true.

From each end of the base measure the angles contained between the base line and the several stations within sight, which are as follows—

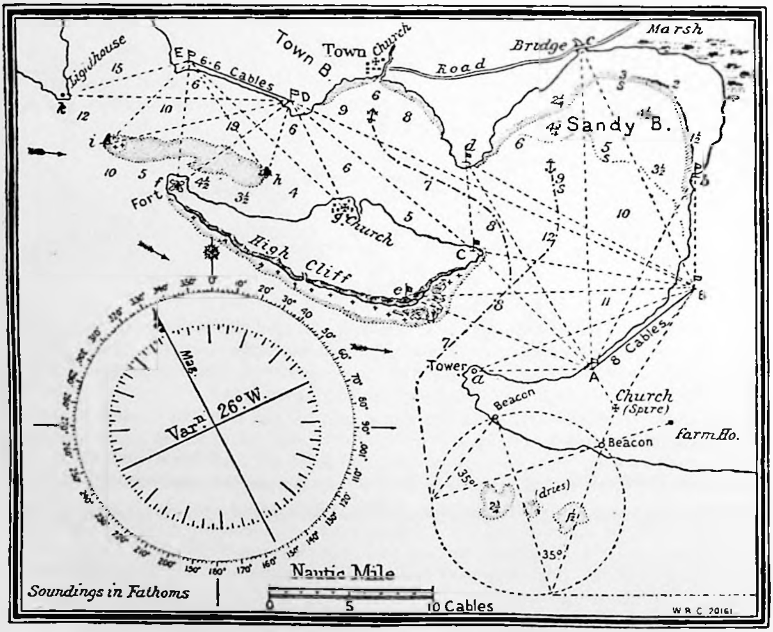


Plate III.

From Station A

Angle B A *b*	= 23°	15′
,, B A C	= 51	50
,, B A *d*	= 80	0
,, B A *c*	= 93	45
,, B A church spire	= 86	0
,, C A *e*	= 24	15
,, C A *a*	= 47	55

From Station B

Angle A B *a*	= 18°	0′
,, A B *e*	= 37	30
,, A B C	= 49	15
,, A B D	= 65	10
,, A B *d*	= 69	30
,, *d* B *c*	= 36	0
,, *d* B *b*	= 60	0
,, A B church spire	= 15	0

The above are all the stations that are visible from each of the stations A and B, hence it is necessary to fix upon a place where another base line may be measured from each end of which the remaining stations can be seen. The most convenient spot is between D and E; let D E be the second base line, its length being 6·6 cables, and the bearing of E from D 293° (N. 67° W.) true; but as its southern extremity D can be seen only from one end of the first base line, the $\angle\, d$ C D is to be observed from station C (on the island) in order to ascertain the position of the second base with regard to the first; this angle is found to be 45° 40′. Now observe the angles formed by lines drawn from each end of this base line to each of the station staves or other objects, which are as follow—

From Station D			From Station E		
Angle E D *k*	= 20°	55′	Angle D E *g*	= 22°	0′
,, E D *f*	= 60	50	,, D E *f*	= 72	15
,, *f* D *g*	= 77	0	,, *f* E *k*	= 69	10

In sounding for depths of water, a shoal was discovered in the western entrance, and in order to fix its extremities, buoys were placed at *i* and *h*, and the following true bearings taken—

From Station D	From Station E
i bore S. 77° W.	*i* bore S. 44° E.
h ,, S. 16° W.	*h* ,, S. 34° E.

ANCHORAGES

When *c* (on the mainland) bears N. 14½° E. and the angle between *c* and *b* is 81°, the vessel is on the anchorage ground for large vessels in Sandy Bay; and when D bears N. 76° W. and the angle between Town church and D is 82°, the vessel is on the anchorage ground in Town Bay.

The above angles and bearings being laid off will give the positions of all the stations relative to the base lines. If it be desired to fix the geographical positions, it will be necessary to fix the latitude and longitude of some prominent place, as in Example II., by astronomical observations, from whence the geographical position of any other point can be found. The latitude will be determined by meridian and ex-meridian altitudes of sun and stars, and the longitude by sun and star chronometers taken on or near the prime vertical, and observed in an artificial horizon. The true direction of the base line must be found by reference to the true meridian, the direction of which will have been determined from the sun's true bearing, either computed or taken from the azimuth tables.

In order to plot the observations in the three examples just given, proceed as follows:

Mount the paper on a drawing-board. Select a convenient position on the paper and make a dot and enclose it in a circle; this will represent the observatory station. Through the observatory station draw the true meridian. The spot chosen should admit of all the stations being got on the plan.

From the observatory station draw the base line in the true direction on the required scale, and at the other extremity make a clear mark.

From each end of the base line lay off the angles or bearings observed by means of a protractor or Field's parallel rulers, and where the several lines intersect each other will be the position of each station relative to the base line.

In these examples only the prominent points have been fixed, so that the work could be easily followed and the method of carrying out the work clearly seen.

For the purpose of cutting in the coast line a sufficient number of stations must be selected to enable the work to be done with accuracy, and, in all probability, it will be necessary to use other base lines for this purpose, remembering that it is better to cut inwards from a long base than outwards from a short base. Any of the sides of the triangles can be computed and used as new base lines, and the work of fixing the stations carried out exactly in the same manner as explained above.

Insert as much topographical detail as is necessary, also lines of soundings with the nature of the bottom, using for this purpose the conventional signs and abbreviations. The scales of latitude and distance, and longitude, should always be drawn on the plan, so that any alteration, owing to atmospheric changes, will affect each in the same manner

In Plate III there are some shallow patches at the eastern entrance, some of which dry at low water. A vessel entering or leaving the harbour will clear the danger by keeping an angle of 35° on the sextant between the two beacons at the eastern entrance. For example: the dotted line at the eastern extremity represents a vessel approaching from the eastward; at the point of contact with the circle the church and eastern beacon are in transit, and the horizontal angle between the beacons is 35°; by maintaining this angle on the sextant the vessel cannot get into danger, and when the eastern beacon and the farmhouse are in transit the vessel is clear of all danger, and can shape courses for the anchorage. A vessel leaving the harbour and bound to the eastward would be clear of all danger when the eastern beacon and the church were in transit and the angle between the beacons was 35°.

Any student wishing to prove that the ship is on the circle so long as the angle is 35°, can do so by taking a piece of tracing paper and drawing an angle of 35° on it; then place the angular point on any part of the circle seaward of the two beacons, and the legs forming the angle will lie over the two beacons; now move the angular point round the circle, and the legs will continue to lie over the two beacons, but it will be observed that one leg shortens as the other lengthens, the angle remaining the same. A vessel striking any part of the circle seaward of the two beacons would have an angle of 35° between them.

CHART CONSTRUCTION

Before commencing chart construction it is necessary, for the proper understanding of a chart, to have a thorough knowledge of what is meant by the expression " Natural Scale," which is found in the title of a chart or plan expressed as a fraction. The " Natural Scale," when properly understood, conveys an idea of how much detail can be shown on a chart or plan.

DEF.—The natural scale is the ratio which the length of a certain unit on the chart bears to the real length of that unit on the earth's surface; for example, natural scale $\frac{1}{72960}$ means that 1 inch on the chart represents 72,960 inches on the earth's surface, or 1 nautical mile of 6,080 feet equal to 72,960 inches. In the following examples the mean length of a minute of latitude is used, but it must be distinctly understood that in constructing a plan the real length of a mile for that particular geographical position for which the plan is constructed must always be used.

The following natural scales, fully explained, will make the subject clear. The numerator represents 1 inch and the denominator the number of inches on the earth's surface equivalent to it.

Natural Scale	$\frac{1}{72960}$	= 1 inch to 1 mile, because 1 inch represents 72,960 inches on the earth's surface, equal to a mile of 6,080 feet.			
,, ,,	$\frac{1}{145920}$	= 1 inch to 2 miles, because 145,920 inches = 2 miles.			
,, ,,	$\frac{1}{218880}$	= 1 ,, ,, 3 ,,	,,	218,880	,, = 3 ,,
,, ,,	$\frac{1}{24320}$	= 3 inches to 1 mile	,,	24,320	,, = $\frac{1}{3}$ of a mile.
,, ,,	$\frac{1}{12160}$	= 6 ,, ,, 1 ,,	,,	12,160	,, = $\frac{1}{6}$,, ,,

It will be observed that as the denominator increases the scale of the chart or plan decreases, and as the denominator decreases the scale increases

If the scale of inches per mile be given to find the natural scale, divide the number of inches in a mile by the scale of inches per mile, and the result is the natural scale required.

Example.—If 3 inches represent one mile, what is the natural scale?

$$\overset{\text{in.}}{72960} \div 3 = 24320$$

Ans. Natural scale $= \frac{1}{24320}$

If the natural scale be given to find the number of inches which represent 1 mile, divide the number of inches in a mile by the given natural scale and the result is the number of inches required.

Example.—Given the natural scale $\frac{1}{12160}$ to find the number of inches representing 1 mile.

$$72960 \div 12160 = 6 \text{ in.}$$

Ans. 6 inches represent 1 mile.

If the natural scale be given to find the number of miles to 1 inch, divide the given natural scale by the number of inches in a mile and the result is the number of miles to 1 inch.

Example I.—Given the natural scale $\frac{1}{12160}$ to find the number of miles represented by 1 inch.

$$12160 \div 72960 = \cdot 16$$

Ans. ·16 of a mile.

Example II.—Given the natural scale $\frac{1}{145920}$ to find the number of miles represented by 1 inch.

$$145920 \div 72960 = 2$$

Ans. 2 miles to 1 inch.

Given the longitude scale 6 inches to one degree, find the length between 47° N. and 48° N.; that is, the latitude scale.

Lat. scale = long. scale × sec. of middle latitude.

Long scale 6 inches	log.	0·778151
Middle lat. 47° 30′	sec.	10·170317
8·881	log.	0·948468

Ans. Latitude scale = 8 881 inches.

Given 6 inches to a degree of middle latitude, to find the longitude scale, the middle latitude being 47° 30′ N.

Long. scale = middle lat. scale × cosine middle lat.

Middle lat. scale 6 inches	log.	0·778151
„ „ 47° 30′	cos.	9·829683
4·054	log.	0·607834

Ans. Longitude scale = 4·054 inches.

Let it be required to find the length of the meridian between lat. 50° N. and 50° 40′ N., when the longitude scale is 6 inches to 1 degree.

Lat. 50° mer. pts.	3474·47
Lat. 50° 40′ mer. pts.	3537·14
Mer. parts between 50° and 50° 40′ =	62·67

The mer. parts between lat. 50° and lat. 50° 40′ N. are 62′·67, and this distance, taken in the compasses, will reach from lat. 50° to 50° 40′ N.

Or the length can be found by multiplying the number of inches representing 40′ of longitude by the secant of the middle latitude. To find the number of inches for 40′ of Long.

$$60' : 6'' : : 40' : x$$

$$\therefore \frac{40 \times 6}{60} = 4 \text{ inches, the length for } 40' \text{ of Long.}$$

To find the number of inches between 50° N. and 50° 40′ N.

4 in.	log.	0·602060	
Middle lat. 50° 20′	sec.	10·194961	Length between 50° N. and 50° 40′ N. is 6·27 inches.
6·27	log.	0·797021	

METHOD OF CONSTRUCTING A MERCATOR'S CHART

For the sake of the younger students it may be advisable to notice the following points. On the sphere the distance between the meridians decreases from the equator towards the pole, where they all meet, whilst the parallels of latitude are everywhere practically the same distance apart. Now on a Mercator's Chart the meridians are the same distance apart in all latitudes, but the distance between the parallels increases from the equator towards the pole in the same ratio as the distance between the meridians has been increased ; that is, as the secant of the latitude. The meridional parts between any two parallels are found by multiplying a degree of longitude at the equator by the secant of the middle latitude. For example : find the meridional parts between 47° N. and 48° N. The middle latitude is $47\frac{1}{2}$°, and 60′ × sec. $47\frac{1}{2}$° = 88·82 minutes of longitude. It would serve no useful purpose to multiply 1 minute of longitude by 47° 1′, 47° 2′, etc., up to 48°, and taking the sum of all the products, because by using the secant of the middle latitude the result is, for all practical purposes, the same. It is obvious from what has been said that all distances measured on a chart on Mercator's Projection must be measured on the graduated meridian in the latitude in which the ship is.

On a Mercator's Chart the mile of latitude
= the minute of long. × sec. lat.
∴ minute of long = mile of lat. × cos. lat.

One minute of longitude at the equator is called a geographical mile and contains 6,086 feet. It is very nearly equal to the average length of a minute of latitude. The length of a minute of latitude at the equator is 6,043·4 feet and at the poles 6,128·6 feet, and the mean length 6,080 feet.

Construct a Mercator's Chart on a scale of 6 inches to a degree of longitude extending from lat. 47° N. to lat. 50° N. and from long. 20° W. to long. 25° W.

Method I, using Meridional Parts

Lat. 48° mer. pts.	3291·53
„ 47° „ „	3202·71
Mer. pts. between 47° and 48° =	88·82

Lat. 49° mer. pts.	3382·08
„ 48° „ „	3291·53
Mer. pts. between 48° and 49° =	90·55

Lat. 50° mer. pts.	3474·47
„ 49° „ „	3382·09
Mer. pts. between 49° and 50° =	92·38

Meridional parts are minutes of longitude at the equator.

Between lat. 47° and lat. 48°	the parallels of lat. are	88′·82 of longitude apart.
,, ,, 48° ,, ,, 49°	,, ,,	90′·55 ,, ,, ,,
,, ,, 49° ,, ,, 50°	,, ,,	92′·38 ,, ,, ,,

Or the length between the parallels can be found by multiplying the longitude scale by the secant of the middle latitude.

Formula—

Lat. scale = long. scale × sec. middle lat.

Long. scale 6″	log. 0·778151		6″ log.	0·778151		6″ log.	0·778151
Middle lat. $47\frac{1}{2}$°	sec. 10·170317		$48\frac{1}{2}$° sec.	10·178735		$49\frac{1}{2}$° sec.	10·187456
8·881 ins.	log. 0·948468		9·055 ins. log.	0·956886		9·238 ins. log.	0·965607

The length of the chart equals 8·881 ins. + 9·055 ins. + 9·238 ins. = 27·174 inches, and the breadth equals 30 inches, and would be shown thus: 27·174 × 30.

The natural scale is found as follows—

The latitude scale is 9·055 inches to a degree of middle latitude, and using the mean length of a minute of latitude the natural scale would equal the number of inches in one degree divided by the number of inches in the scale degree of middle latitude.

4377600 inches ÷ 9·055 = 483444, therefore the natural scale is $\frac{1}{483444}$

Test the accuracy of the calculations for the meridional length of the chart by finding the meridional difference between the two extreme latitudes and multiplying it by the longitude scale and dividing by 60; the result should be equal to the sum of the separate calculations.

Lat. 47° mer. pts.	3202·71	$\frac{271 \cdot 76 \times 6}{60} = 27 \cdot 176$
,, 50° ,, ,,	3474·47	
	271·76	

The meridional length is 27·176 inches, practically the same as above.

To draw the Chart.

Draw a horizontal line near the lower edge of the paper to represent the parallel of 47° N.; divide this parallel into five equal parts of six inches each to represent the degrees of longitude, and number them 20°, 21°, 22°, 23°, 24°, and 25°. Commencing on the right, at 20° and 25° erect, very carefully, perpendiculars 27·176 inches long and draw through the two points reached a line parallel to the parallel of 47° N., and it will be the parallel of 50° N.

To put in the parallel of 48° take 8·88 inches or 88·8 minutes of longitude from the graduated parallel, and with one leg on the parallel of 47° N. and the meridian of 20° W., the other leg will find the position of the parallel of 48° N.

This distance can be laid off on each meridian and the parallel drawn with the parallel rulers. The position of the parallel of 49° N. is found in a similar manner.

Draw the meridians of 22°, 23°, and 24° W. and graduate the meridian for latitude and distance and the parallels at the top and bottom for longitude and meridional parts. For the purpose of graduating the latitude and longitude scales the proportional compasses are most useful. The inner margin of the chart is called the frame. At a distance of about $\frac{1}{15}$ of an inch from the frame, and outside of it, draw another line parallel to the frame; the graduations come between this outer line and the frame. Draw a compass on the magnetic or true meridian, subdividing it to degrees or points as required, and finish according to individual taste.

The framework of the chart can be tested as follows: Measure the diagonals and if they are equal the framework is correct, as a rectangular parallelogram alone has its two diagonals equal in length, and a Mercator's Chart is a rectangular parallelogram.

DESCRIPTION AND USE OF CHARTS

Charts are *marine maps*, representing the whole or part of the surface of the water and adjoining coasts; exhibiting islands. rocks, shoals, banks, depths of water, the variation of the compass, and whatever other particulars may serve to direct the mariner on his voyage, or point out the dangers to be avoided.

If you take a globe and try to find the course and distance between any two places, you will experience considerable difficulty in ascertaining what you require, and you would find it impossible to do so on a map constructed on the globular projection. The early navigators had to contend with this difficulty, and projected what they called the *plane chart*, on which the parallels and meridians were equidistant straight lines. Gerard Kauffman (better known as Mercator) devised the method to which his name has been given. The system was brought to perfection by Edward Wright of Garveston, Norfolk, in 1594. The Mercator Chart is now universally used for general navigation; the parallels and meridians are straight lines; the meridians are equidistant, but the parallels are graduated. The construction is such that rhumbs are also represented by straight lines.

The parallels of latitude on the surface of a globe are everywhere equidistant, but the distance between the meridians lessens towards the poles. It may be seen that if on a plane surface the parallels are retained equidistant while the meridians are spread at the poles to the distance they have at the equator, there must be considerable distortion. But the difficulty is got over, and the relation between the different parts preserved by widening the distance between the parallels of latitude to the same proportionate extent that the meridians have been widened.

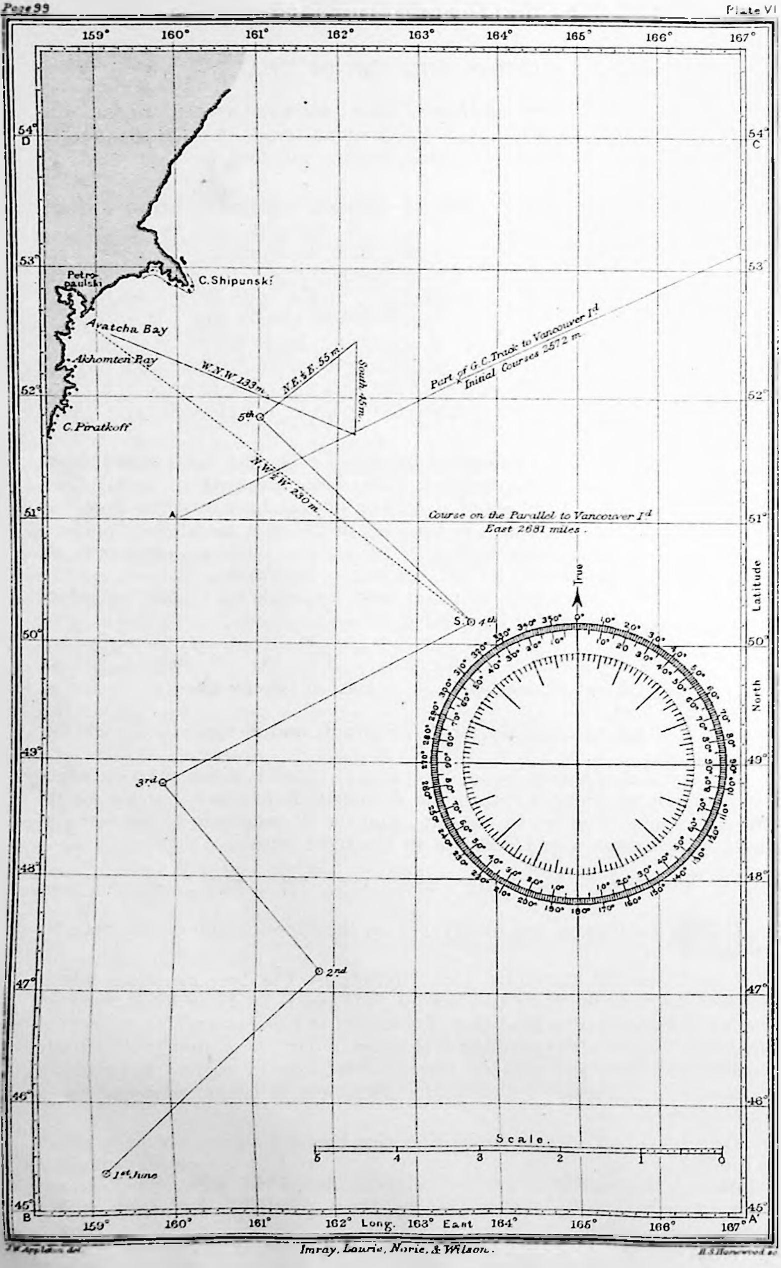
It matters not that the extent of land and water in the higher latitudes is out of proportion with the equatorial regions; the shape is still approximately preserved, and—what is of most importance in navigation—*the relative direction from one part to another*, and hence *the track of a ship steering the same course can be drawn as a straight line.*

Charts are drawn on a large or small scale. It will be a *large scale* if a small part of the coast is delineated or on a *small scale* if a large part of the coast is delineated.

A Plan is a chart that comprises a detached portion of a general chart on a large scale, as a harbour, roadstead, small bay, the entrance to a river, channels leading to a port, a small part of a sea where the navigation is intricate. Such a chart only occasionally shows parallels and meridians; when these are absent, a scale of miles and longitude is given.

Use of Mercator's Chart

When a chart is properly spread out before you, so that you can read it like the page of a book, the top is the north, the bottom is the south, the side to the right is the east, and the side to the left is the west. The


Petropaulski
C. Shipunski
Avatcha Bay
Akhomten Bay
C. Piratkoff
W.N.W. 133m.
N.E.½E. 55m.
South 45m.
5th
N.W.½W. 230m.
Part of G.C. Track to Vancouver Id
Initial Courses 2572 m.
Course on the Parallel to Vancouver Id
East 2681 miles.
True
S. 4th
3rd
2nd
1st June
Scale.
162° Long. 163° East 164°
North Latitude

only exception to this arrangement is when, for some special purpose, the chart is drawn on what is called the diagonal plan. A reference to the compass will at once show the north, south, east, and west directions of the chart.

The PARALLELS are represented by straight lines drawn across a chart from east to west; their distance apart is unequal, and the inequality increases more and more in proceeding from the equator. If the increase is in the direction of the north part of the chart, then the chart represents some portion of the northern hemisphere; if the increase is towards the south, it embraces some part of the southern hemisphere. If the chart embraces a considerable extent of ocean and land, the equator (lat. 0°) may be represented on it, and the distance between the parallels will increase northward and southward from the equator. The latitude of a place or of a ship, being its distance from the equator, is indicated on a chart by the parallels.

The MERIDIANS are represented by straight parallel lines drawn from north to south, the space between them being everywhere equal. The meridians mark the longitude of a place or ship, and are reckoned from the meridian of Greenwich, which is long. 0°, or the first meridian. If, then, the meridian *increases to the right*, as 5° 10° 15° 20°, the longitude is *east*; if it *increases to the left*, as 25° 20° 15°, the longitude is *west*.

But you may have both east and west longitude on a chart, as when the meridian of Greenwich is included, thus—

W. 20° 15° 10° 5° 0° 5° 10° 15° 20° E.
West of Greenwich. East of Greenwich,

showing increase to the right for E. longitude, and increase to the left for W. longitude.

There would *appear* to be an exception to this rule if the chart included the meridian of 180°; for then the E. longitude to the *left* of the 180th meridian would decrease to the left; and the W. longitude to the *right* of the 180th meridian would decrease to the right, thus—

E. 160° 165° 170° 175° 180° 175° 170° 165° 160° W.

This occurs because the observer is on the opposite side of the globe to Greenwich.

THE SCALE OF LATITUDE AND DISTANCE.—The two meridians which bound a chart on the right and left (as A C and B D, Plate VI.) are called *graduated meridians*, because they are *marked to degrees*—and to minutes if the width between the parallels is sufficient. On these meridians latitude is measured from the equator, towards the pole, in degrees and minutes according to the scale of the chart. DISTANCE *is always measured on a graduated meridian.*

THE SCALE OF LONGITUDE.—The two parallels which bound a chart at the bottom and top (as A B and C D, Plate VI.) are called *graduated parallels*, being *marked to degrees*—and minutes if the scale is sufficiently large; longitude only is measured on these parallels. DISTANCE *is not measured on the parallels.*

To find the Latitude of a Place on the Chart

With the dividers place one leg on the position, stretch the other to the nearest parallel, refer this to the graduated meridian, placing one leg of the dividers on the same parallel, and the other upwards or downwards as required; read this off as the latitude.

Or, lay the edge of a parallel ruler on the nearest parallel of latitude, and work it to the given place, then note the degree and minute at which the edge of the ruler cuts the graduated meridian.

Example.—By Chart (Plate V.) Cape St. Vincent is in lat. 37° N.; and by Plate VI. C. Shipunski is in lat. 52° 50′ N.

To find the Longitude of a Place on the Chart

Place one leg of the dividers on the position, stretch the other to the nearest meridian, refer this distance to the graduated parallel, placing one point of the dividers on the same meridian, and the other point to the right or left of it as required; read off the longitude.

Or by parallel ruler as in latitude.

Example.—By Chart (Plate V.) Cape St. Vincent is in long. 9° W.; and by Plate VI. C. Shipunski is in long. 160° 10′ E.

To mark the Ship's Place on a Chart

With the dividers take from the graduated meridian the given latitude; mark this on the meridian nearest to the given longitude; lay the edge of the parallel ruler on a near parallel, and work one side to the exact latitude you have marked on the meridian; then, with the dividers, take the given longitude from the graduated parallel; lay this off from the meridian along the edge of the parallel ruler which already marks the latitude, and you have the ship's place.

In this manner a ship's track is usually pricked off at sea, her latitude and longitude being laid down every day at noon, or at any other required time, and the ship's places connected by pencil lines drawn between the points.

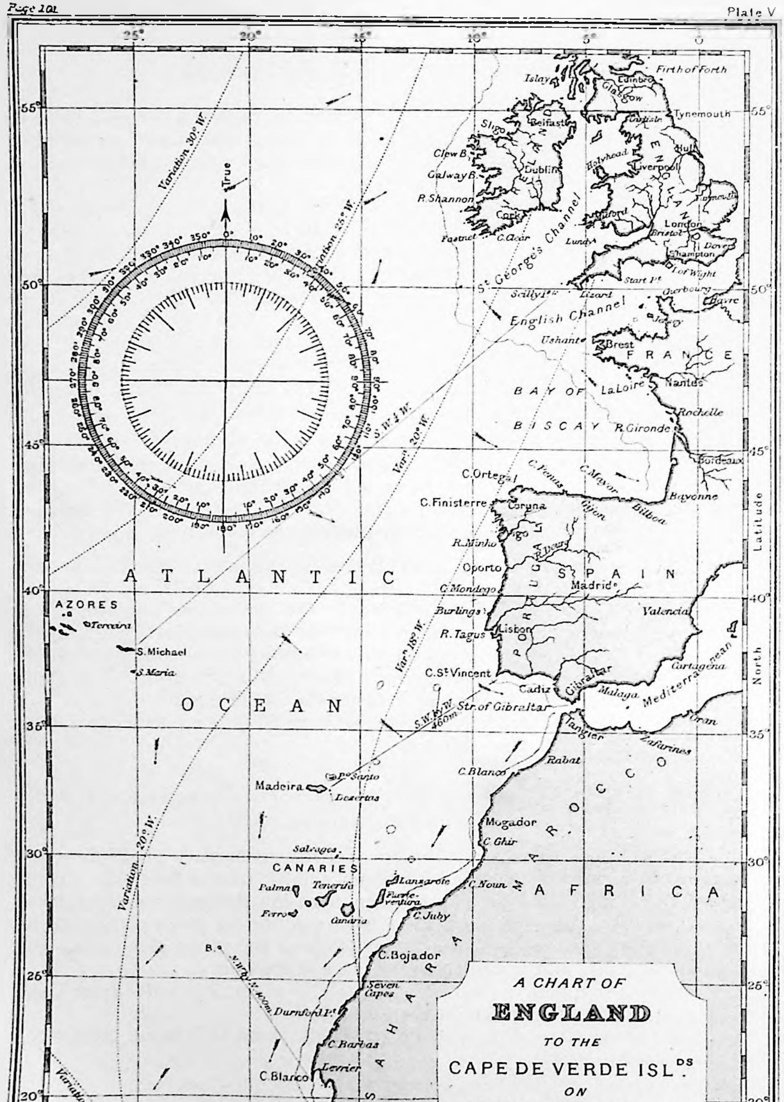
To find the Course or Bearing between two Places on the Chart

With a parallel ruler, lay one of its edges over both the places; then move the two parts of the ruler in succession until the edge of one of them passes through the centre of a compass on the chart, and that edge will point out the course, true or magnetic according as the compass is true or magnetic.

On the latest charts a double compass is engraved; the outer one is marked from 0 (N.) right round 90° at E., 180° at S., 270° at W., and 360° at N., and always *true*. The inner compass is also marked in degrees in the usual way, but *magnetic*. The difference between the two compasses is the variation for that place.

To find the Distance between any two Places on the Chart

1. If the given places lie under the same meridian, find their latitudes on the chart; and the difference or sum of these, according as the places lie on the same, or on different sides of the equator, will give the distance.



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2. If the given places lie on the same parallel take the distance between them in the dividers and place one leg as far above the parallel as the other is below; the distance contained between the legs on the graduated meridian is the required distance.

3. But if the given places differ both in latitude and longitude, take the distance between them, and apply it to either of the graduated meridians, so that one leg of the dividers may be as much above one place as the other leg is below the other place; then the degrees, or degrees and minutes, contained between the points of the dividers will be the distance required, which may be reduced to miles.

But if the places lie near to a parallel, and their distance be considerable, the middle latitude between the two places should be found; then half their distance being applied alternately above and below the middle latitude and the results added together will give the distance.

Example.—From Scilly to St. Michael's the true course is 48° W.; half the distance is opposite middle latitude 44° N.; one point of the dividers in this latitude and the other extended upwards reaches lat. 52° 20′ N., then extended downwards reaches lat. 34° 20′ N.; difference between 52° 20′ and 34° 20′ is 18°, which multiplied by 60 gives 1,080 miles.

Example.—Required from Chart Plate V. the true course and distance from Cape St. Vincent to the east end of the Island of Madeira.

Lay the edge of the parallel rulers over the two places, and then slide them to the nearest compass, when it will be seen that the true course is S. 56° W. The extent between the two places being taken with the dividers, and applied to one of the graduated meridians, will reach from 31° to about 38° 40′, being an interval of 7° 40′: hence the distance is 460 miles.

Also, by Chart Plate VI., the course from S. ⊙ 4 to Avatcha Bay is N. 48° W., and the distance 230 miles.

The Course and Distance run from any given Place being known, to find the Ship's Place on the Chart

Lay the edge of the parallel rulers over the given place in the direction of the ship's course by compass corrected to give true or magnetic according to the compass on the chart; then take the distance run from that part of one of the graduated meridians opposite the given place and the supposed place of the ship, which lay off from the given place along the edge of the ruler, and it will show the place of the ship.

Example.—Suppose a ship sail N. 34° N. (true) 400 miles from Cape Blanco. Required her place on the Chart Plate V.

By the above method the ship's place will be found at B in lat. 26° 10′ N., and in long. 21° 20′ W.

Fixing the Ship's Position on the Chart

There are two methods of fixing a ship's position relative to the shore. The most accurate is by angles measured by the sextant and used in conjunction with the station pointer (*see* Station Pointer); the other method is by compass bearings.

In making a fix by compass two bearings only are liable to error, therefore.

a third, or check, bearing of some other object should be taken, especially when near the shore or any danger. The coincidence of three bearing lines will eliminate any error.

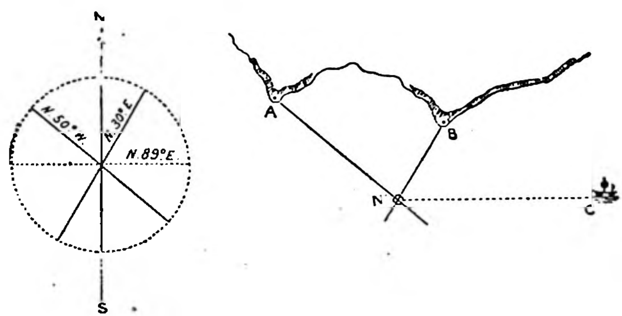
The compass is deflected from the meridian by two causes, variation and deviation. If the compass is corrected for deviation, then the magnetic compass or magnetic meridian is used. If the compass is corrected for both deviation and variation then the true compass or true meridian is used.

It is the common practice at sea to use the algebraic sum of the variation and deviation and to call it the *compass error*. If this error is used in connection with Field's rulers no reference need be made to any compass on the chart. The angle required is made between the ruler and the true meridian.

If a magnetic meridian is drawn by prolonging the N. and S. line of the magnetic compass or by drawing a line parallel to it, the ruler can be used in the same manner for magnetic courses.

Cross bearings.—If two known objects or points are visible a bearing of each is taken, then corrected for deviation or compass error for direction of ship's head.

The rulers are made to represent first one bearing then the other, by placing it on the compass or by making the angle with the meridian; the rulers are then carefully moved so as to cover the object observed and a line drawn from the object towards the ship. This is repeated with the other bearing. Where the lines intersect is the ship's position. (*See* following Fig.)



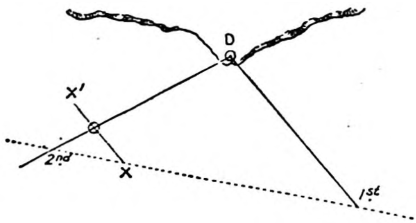
Example.—Let N S be any meridian on the chart either engraved or drawn in pencil. Now the bearings of the two points, A and B, are N. 50° W. and N. 30° E. respectively. With Field's rulers make the above angles with a meridian, then transfer them to the points, draw lines towards the ship; the intersection will be the position of the ship.

If a third bearing C, N. 89° E. is taken it would correct any error in A or B.

Generally only two bearings are used, but for exact work a third, or more, is necessary.

Two Bearings of one Point with a Run between.—A careful compass bearing is taken of an object on the bow, then the course and distance is made as correctly as possible until a second bearing of the same object is taken; there should be an alteration of not less than 60° in the two bearings. The first line of bearing is laid down on the chart from the object towards the ship; from any point on this line the course and distance run in the interval is laid down. The first bearing is transferred to the end of the distance run and where the second line of bearing and the transferred bearing cut is the ship's position.

Example.—A vessel takes a bearing of D, N. 40° W., and after running N. 80° W. 10 miles D bears N. 60° E

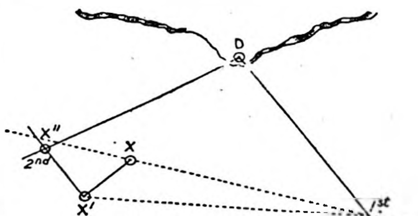


The course and distance between the bearings can be laid off from any point of the first bearing. Let the dotted line represent the course and x the position arrived at; transfer the first bearing to x and draw x x′, and where x x′ cuts the second line of bearing is the ship's position. The accuracy of the fix depends on the accuracy of the course and distance made good in the interval. Norie's Tables now contain a table for solving this problem by Inspection; Table, Distance off by Two Bearings and Distance run between Them.

In the foregoing no allowance is made for tide or current.

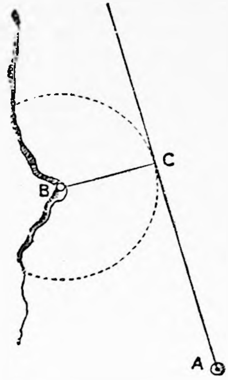
The correction for set and drift of the current is made by laying off at the end of the course and distance the set and drift and making the cut from the corrected position as follows—

Using data as in previous figure, suppose in the interval a current set S. 45° W. 2½ miles: where would the ship be?



From x, which is the end of the run N. 80° W. 10 miles, set off the current S. 45° W. 2½ miles; this will lie at x′, therefore the ship did not arrive at x but at x′. From x′ lay off the 1st bearing, and where this bearing cuts the second bearing is the ship's position, x″.

To pass a Point at a given Distance when abeam on the approaching Course



With the point as centre and the given distance taken from the graduated meridian, as radius, describe a circle. Then draw a line from the point of departure tangential to the circle, and this line will be the course to make good; at right angles to this line and from the centre of the circle draw another line until it cuts the course, and where the two lines cut will be the position of the ship when abeam, provided she has made that course.

In the diagram A is the point of departure; B the position of the light; C the position of the ship when B is abeam on the approaching course; A C the distance sailed measured on a graduated meridian.

To maintain a given Distance on two Courses when rounding a Point.

With the given distance taken from the graduated meridian in that latitude, and the given point as centre, describe a circle. Then from the point of departure draw a line tangential to the circle and from the point of destination draw another line also tangential to the circle, and where these two lines cut is the point where the course is to be altered. The distance on each course is to be measured from the point of intersection of the two courses to the points of departure and destination respectively.

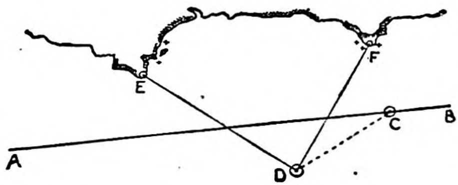
In the diagram B is the point of departure; A the point of destination; C the point to be rounded; D the point where the course is altered; B D the direction of first course and also the distance; D A the direction of the second course and also the distance measured on a graduated meridian.

N.B.—If two courses were forming an acute angle at their point of intersection it would become necessary to put in a third course in order to save time and distance.

To find the Set and Drift of the Current

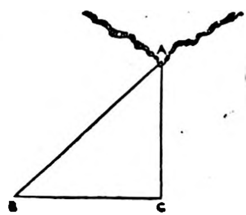
This is done by laying off the course of the ship and the distance sailed from a known position and then finding her actual position by cross bearings. The course and distance from the assumed position to the actual position will be the set and drift.

In the figure the ship sailed from A in the direction of B until, by dead reckoning, she was at C, when bearings of the points E and F were taken simultaneously, which placed the ship at D. Now it is obvious,



if the dead reckoning be correct, that the current has set the ship from C to D; the set is, therefore, the direction from C to D, and the drift is the length of C D measured on the graduated meridian in that latitude.

The Four-Point Bearing.—This problem is useful when taking a departure or in determining the distance of an object when abeam. Observe the object when four points on the bow and again when abeam; the distance run + or — the effect of the current will equal the distance off A. The figure forms a right-angled triangle, with equal angles at A and B, therefore B C = A C. In all cases where a run is made between bearings the set of tide or current should be taken into account.



Doubling the Angle on the Bow.—In passing *near* a point of land the method of fixing by doubling the angle on the bow should be used.

By taking bearings two points or four points on the bow a very good position is obtained before the object is passed.

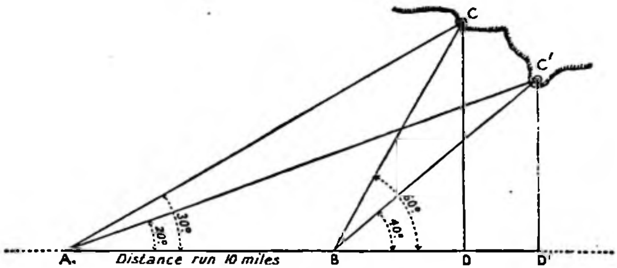
To get reliable results the difference between the first bearing and the course made good should not be less than 20°.

The principle is that the sides form an isosceles triangle, A B = B C or A B = B C′; and angles A and C are equal, as also are angles A and C′; then a perpendicular dropped upon A D′, as C D and C′ D′, will equal the distance the object will be when brought abeam, assuming that the ship continues on the same course.

If the figure is drawn on the chart the distance off when abeam and the distance the ship would have to sail before bringing it abeam can be measured on the graduated meridian.

These can also be found from the traverse table as follows: 60° as course, and C B, 10 miles, in the distance column give 8·7 in the departure

column as the distance in miles C is off when abeam, and in the difference of latitude column will be found 5 miles, the distance the ship will have to sail from B before bringing it abeam.



Doubling the angle on the Bow.

The same applies to C′.

40° as course and C′ B, 10 miles, in distance column give 6·4 miles in departure column as the distance C′ is off when abeam, and in the difference of latitude column will be found 7·7 miles the distance the ship would have to sail from B before bringing it abeam.

Vertical and Horizontal Danger Angles.

Neither of these methods gives a fix, but they are of great service in keeping the navigator outside the danger zone.

Vertical angles.—The heights of lighthouses, headlands, hills, and mountains are to be found in the List of Lighthouses and on charts. If the altitude is measured on and off the arc with a sextant the mean will be the angle required. By taking the angle off and on the arc any index error is eliminated.

The formula is—

$$\text{Dist.} = \text{height of object} \times \text{cot. of angle.}$$

If the height of the object is in feet the distance will be in feet. To get miles, subtract the log. of 6,080 feet from the log. of the distance in feet.

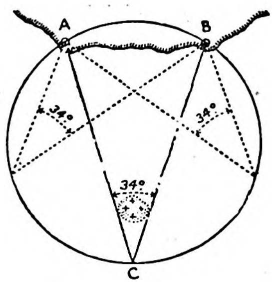
The distance can be found by inspection from "Distance by Vertical Angle" Table in Norie's Tables.

The formula for finding the angle is—

$$\text{Tan. } \theta = \frac{\text{height in feet}}{\text{distance in feet}}, \text{ where } \theta = \text{the Vertical Angle.}$$

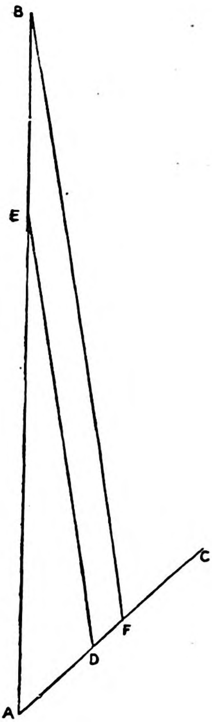
The angle can also be found by inspection from the above Table.

Horizontal Angles.—For this method *two* objects in a horizontal plane are required whose position and distance apart are known. The problem depends upon the fact that any chord of any arc subtends the same angle from any point on the circumference of the circle (Euclid III. 21). In practice the distance outside of which the danger is to be passed is marked on the chart as at C. A and B are prominent objects on shore marked on the chart; the angle between A and B subtended at C is measured by the compass, protractor, or rulers; in this case it is 34°.



The angle is placed on the sextant to the nearest half-degree. As the ship approaches the danger zone, steering towards the east in this case, A will be seen to approach B. When A is upon B the circle is reached at some point on the arc A C B, and a perfect arc of a circle can be maintained by conning the ship so that A is kept upon B; or if steering the opposite way, then by keeping B upon A. It is the finest, safest way of navigating an outlying danger we have, as it takes into account any set of tide or current and it is impossible under any circumstances to touch the danger so long as the data is correct and the objects are not at any time allowed to open.

A fix by Station Pointer is explained under *Station Pointer*. In all cases the greatest accuracy is obtained by using the compass error on the course steered, converting the compass bearings into true bearings and using the meridian and Field's rulers to make the desired angles. You are then independent of the magnetic compass, the small figures of which are mixed with the soundings on the chart, and in a bad light are difficult to read. You are also independent of the alteration due to change in the variation.



Find the course to steer from A to B to counteract the effect of a current which set N.E. magnetic at the rate of two miles per hour, ship steaming 9 knots, and supposing the ship to have left A at 4 p.m.; find the time of arrival, the distance steamed, and distance made good.

Join A and B, and from A draw A C of any convenient length in a north-easterly direction; on A C lay off A D equal to 4 miles, the drift in 2 hours; take from the graduated meridian in the compasses 18 miles, the distance the ship steams in 2 hours, and with one foot on D cut the line A B in E and draw E D, then will the direction of E D represent the true or magnetic course to be made good, which turn into a compass course in the usual way.

Draw B F parallel to E D, then will B F represent the distance steamed and A B the distance made good.

The number of miles in B F divided by the speed of the ship will give the time taken on the passage, which, added to 4 p.m., will give the time of arrival.

In this example B F measures $25\frac{1}{2}$ miles, and this divided by 9 knots gives 2h. $46\frac{2}{3}$m., which added to 4 p.m. gives 6h. $46\frac{2}{3}$m. p.m. as the time of arrival; the distance made good is 29·3 miles.

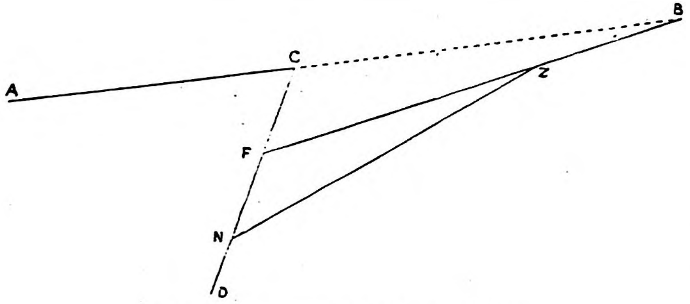
If the scale of the chart be small, and the current makes an acute angle

with the line joining the points of departure and destination, it is better to lay off the current for a longer period than it would take to perform the passage, when finding the course to make good, as it gives longer lines to work with, which is a great advantage and gives better results than using short lines.

Making allowance for "set and drift" experienced in the earlier part of the run.

While steaming from A to B it was found that after steaming for 2 hours at the rate of 10 knots the position was found to be at F; from this position find the compass course to steer to counteract the effect of a current similar to that experienced in the earlier part of the run, and give the distance made good in two hours towards B, the ship steaming at the rate of 12 knots.

Draw A B and lay off from A towards B 20 miles, let this point be marked C; from C draw the line C D of any convenient length through F, then will C F represent the set and drift of the current in the earlier part of the run. Now rub out the dotted portion of A B represented by C B, and draw F B, then F B will represent the direct course from F to B; from F, in the direction of D, lay off the current for 2 hours; this will equal F N. From the graduated meridian take 24 miles in the dividers, and with one leg on N cut the line F B in Z with the other leg, then N Z will be the course to make good, which find in the usual way, and F Z will be the distance made good in 2 hours towards B, and N Z measured on the graduated meridian will be the distance steamed.



The Deviation Card and its Use

The compass on board ship is always more or less affected by the iron and steel in the ship's construction, and in a much less degree by iron in the cargo; so the ship must be swung and a table of deviations made, one with the ship's head by compass, the other with the ship's head magnetic.

The card is used in two ways—to correct bearings, and correct the course.

The first case is simple. The deviation is sought for the direction of the ship's head when the bearing was taken, and it is applied in the usual way, as when correcting courses in the Day's Work; but to set a course the process is inverted, since the true or magnetic course is first found from the chart or by calculation. Here again there would be no difficulty if the deviation with ship's head magnetic were used, but as the first way is most commonly employed (when deviation card is used) a small calculation is necessary.

To find the Deviation to apply to the Magnetic Course from Chart in order to find the Compass Course to steer

Take a compass course from the deviation card and apply the deviation opposite to it in the same way as in correcting courses in a day's work. If this should agree with the magnetic course from the chart, that will be the required compass course and also the deviation ; if it does not agree with the magnetic course, select a compass course which will give a greater magnetic course and also one that will give a lesser magnetic course, and proceed as in the following examples, using deviation card for compass direction of ship's head.

1st compass course	N. 20° E.	2nd compass course	N. 30° E.
Dev.	+ 6 E.	Dev.	+ 11 E.
Lesser magnetic course	N. 26° E.	Greater magnetic course	N. 41° E.

Mag. course from chart	N. 36° E.	Greater mag. course	N. 41° E.	Dev.	11° E.
Lesser mag. course	N. 26 E.	Lesser mag. course	N. 26 E.	Dev.	6 E.
diff.	10°	diff.	15°	diff.	5°

Now, if 15° give a change of deviation of 5°, what will 10° give ?

$$\begin{array}{r} 5^\circ \\ \underline{10} \\ 15^\circ)50(3^\circ \\ \underline{45} \\ 5 \end{array}$$

Dev. at N. 26° E. mag. = 6° E.
Dev. at N. 36° E. mag. = 6° + 3° = 9° E.

Mag. Course	N. 36° E.
Dev.	9 E.
Comp. course	N. 27° E.

Example 2.—What is the deviation and the course to steer by compass, the magnetic course being north ?

1st comp. course	N. 0°	2nd comp. course	N. 10° E.
Dev.	7 W.	Dev.	1 E.
Lesser mag. course	N. 7° W.	Greater mag. course	N. 11° E.

Mag. course from chart	N. 0°	Greater mag. course	N. 11° E.	Dev.	1° E.
Lesser mag. course	N. 7 W.	Lesser mag. course	N. 7 W.	Dev.	7 W.
diff.	7°	diff.	18°	diff.	8°

Now, if 18° give a difference of 8°, what will 7° give ?

$$\begin{array}{r} 8^\circ \\ \underline{7} \\ 18^\circ)56(3^\circ \\ \underline{54} \end{array}$$

Dev. at N. 7° W. = 7° W.
Dev. at N. 0° = 7° — 3° = 4° W.

Mag. course from chart	N. 0°
Dev.	4 W.
Comp. course required	N. 4° E.

Example 3.—Find the deviation and compass course to steer in order to make good a magnetic course of S. 60° W.

1st comp. course S. 80° W.	2nd comp. course S. 90° W.
Dev. 22 W.	Dev. 26 W.
Lesser mag. course S. 58° W.	Greater mag. course S. 64° W.

Mag. course from chart S. 60° W.	Greater mag. course S. 64° W.	Dev. 26° W.
Lesser mag. course S. 58 W.	Lesser mag. course S. 58 W.	Dev. 22 W.
diff. 2°	diff. 6°	diff. 4°

Now, if 6° give a change of deviation of 4°, what will 2° give?

$$\begin{array}{r} 4^\circ \\ 2 \\ \hline 6^\circ)8(1^\circ \\ 6 \\ \hline 2 \end{array}$$

Dev. at S. 58° W. = 22° W.
Dev. at S. 60° W. = 22° + 1° = 23° W.

Mag. course from chart	S. 60° W.
Dev.	23° W.
Compass course required	S. 83° W.

Deviation Card for Magnetic Direction of Ship's Head

Example 4.—What is the deviation and the course to steer by compass in order to make good a magnetic course of S. 17° W.?

Mag. course from chart S. 17° W.	Lesser mag. course S. 13° W.	Dev. 3° E.
Lesser mag. course S. 13 W.	Greater mag. course S. 19 W.	Dev. 1 W.
4°	6°	diff. 4°

Now, if 6° give a change of 4°, what will 4° give?

$$\begin{array}{r} 4^\circ \\ 4 \\ \hline 6^\circ)16(3^\circ \text{ nearly} \end{array}$$

Dev. at S. 13° W. = 3°
Dev. at S. 17° W. = 3° — 3° = 0°
Dev. 0
Comp. co. S. 17° W.

Example 5.—What is the deviation and the course to steer by compass in order to make good a magnetic course of N. 7° W.?

Mag. course	N. 7° W.
Deviation as per card	7 W.
	0° = Compass course to steer, North.

Example 6.—What is the deviation and course to steer by compass in order to make good a magnetic course of N. 80° E. ?

Mag. course N. 80° E.	Lesser mag. co. N. 69° E.	Dev. 19° E.
Lesser mag. co. N. 69 E.	Greater mag. co. N. 82 E.	Dev. 22 E.
diff. 11°	diff. 13°	diff. 3°

Now, if 13° give a change of 3°, what will 11° give ?

$$\begin{array}{r} 3^\circ \\ 11 \\ \hline 13^\circ)33(2\tfrac{1}{2}^\circ \\ 26 \\ \hline 7 \end{array}$$

Dev. at N. 69° E. = 19° E.

Dev. at N. 80° E. = 19° + $2\frac{1}{2}$° = $21\frac{1}{2}$° E.

May. Course N. 80° E.
Dev. $21\frac{1}{2}$ E.
Comp. course N. $58\frac{1}{2}$° E.

N.B.—To make good a magnetic course from chart if a vessel is making leeway, the leeway should be eliminated by applying it to the magnetic course before finding the deviation by allowing the leeway to windward of the magnetic course taken from the chart.

Example.—Mag. course N. 20° E., ship making 20° of leeway, wind N.W. Apply the leeway to windward, that is, the mag. course will be North ; now find the deviation for that mag. course.

CLOSING REMARKS ON CHART WORK

It must be obvious to the thoughtful navigator that some localities are easier to navigate than others. Amongst the most difficult must be placed the seas bordering the coasts of the British Isles and Northern Europe, because the waters in these seas are ever swinging to and fro under the influence of strong tidal streams of varying direction and velocity, rendering navigation a difficult and far from exact science.

Whenever a course is to be set in these seas reference should always be made to the " Atlas of Tidal Streams," which shows the probable set of the tidal stream in all localities for one to six hours before and after high water at Dover, thus enabling the navigator to make a due allowance on the course for the state of the tide at the time the course is being set, and also what changes may be expected as the vessel proceeds on her way.

It follows, then, from what has been said, that too much care cannot be taken in setting courses, and when a course has been carefully and judiciously set from a known position to any other fixed position it should be rigorously followed until the distance between the two points has been made good ; but no opportunity of finding the ship's position should be lost, and a fresh course set, when found necessary.

If a vessel, from any cause, be stopped when amongst strong tidal streams, allowance should be made for the set and drift in the interval, and should the water be shallow the deep-sea lead should be dropped over the side and the line allowed to run out ; the direction of the line would, if there were no wind, show the direction of the set.

When sailing on a course which has been set to counteract the effect of a current it must be distinctly understood that the ship is supposed to be always on the straight line joining the point of departure with the point of destination, but with her head on the course that counteracts the effect of the current.

In low-lying localities with no prominent points suitable for getting a good fix recourse must be had to the use of the lead in order to avoid getting into too shoal water.

In shallow water and strong currents such as are experienced outside Rangoon and the entrance to the River Plate the ground log is the only reliable speed indicator and should always be used.

In rounding points never adopt the "rule of thumb" method of conning the ship so as to keep the point abeam as she comes round, because distance cannot be correctly estimated by the eye which never takes into account the effect of refraction, which varies in its effect according to the state of the atmosphere. The "Vertical Angle" should be used in rounding points to guard against getting into danger; and when it is impracticable to use the "Vertical Angle," owing to the object being too far inland, the horizontal angle should be used, as the distance of the objects inland in no way vitiates the excellent results obtained by this method.

In foggy weather it is well to remember that the distinctness or faintness of sound cannot be relied on as a guide to distance off, but the direction from whence it comes, coupled with the depth of water and the nature of the bottom obtained by means of the lead, would, in some localities, give a fair fix. It is a wise practice to take a few casts of the lead, in fine weather when the ship's position is known, as the results obtained might give confidence when the ship's position is uncertain.

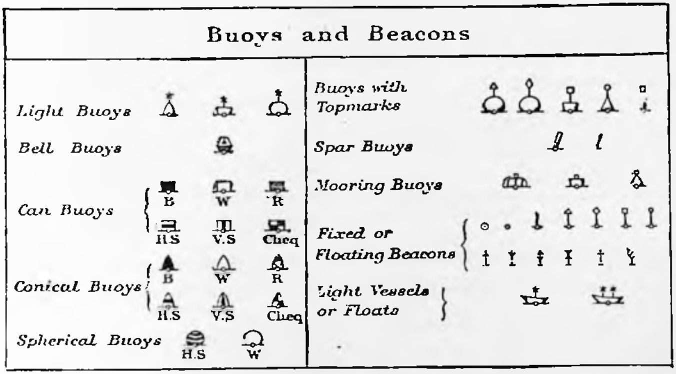
Never pass headlands, lighthouses, or light-vessels without getting a good fix by one of the foregoing methods.

SIGNS AND SYMBOLS

Quality of the Bottom

b	= blue	for	= foraminifera (3)	man	= manganese (6)	shin	= shingle
blk	= black	g	= gravel	ml	= marl (7)	sm	= small
br	= brown	gl	= globigerina (4)	mus	= mussels	sp	= sponge
brk	= broken	gn	= green	oys	= oysters	spk	= specks, speckled
c	= coarse	grd	= ground	oz	= ooze	st	= stones
cal	= calcareous (1)	gy	= gray	peb	= pebbles	stf	= stiff
chk	= chalk	h	= hard	pt	= pteropod (8)	stk	= sticky
choc	= chocolate	l	= large	pum	= pumice	t	= tufa (11)
cin	= cinders	lv	= lava	r	= rock	vol	= volcanic
cl	= clay	lt	= light	rad	= radiolaria (9)	w	= white
crl	= coral	m	= mud	s	= sand	wd	= weed
d	= dark	mad	= madrepore (5)	sc	= scoriæ (10)	y	= yellow
di	= diatom (2)			sft	= soft		
f	= fine			sh	= shells		

1. A substance containing lime. 2. A group of very minute organisms having a hard flinty outer skin. 3. Many-celled organisms. 4. Deep sea foraminifera. 5. A certain branching form of coral. 6. Black oxide of a certain metal. 7. A mixture of lime and clay. 8. A class of mollusc with small wing-like appendages. 9. Ooze containing small shells. 10. Volcanic ashes. 11. Soft sandy stone.



The position of the buoy or beacon is the Centre of the Base, and is usually indicated by a small circle.

B., Blk. ....Black
Cheq. ....Chequered
G.............Green
Gy. ....Gray
H.S. ......Horizontal Stripes
No. ....Number
R. ....Red
S.B. ....Submarine bell
V.S. ....Vertical Stripes
Y. ....Yellow
W., Wh. ....White

The Heights given against Beacons or marks forming beacons (such as Chimneys) represent the Height of the Top of the object above High Water Ordinary Spring Tides, or above the Sea Level where there is no tide.

LIGHTS

• ● ....lights, position of
Lt., Lts. ....light, lights
†Lt. Alt. ....light alternating
Lt. F. ...... ,, fixed
Lt. Fl. ...... ,, flashing
Lt. Occ. .... ,, occulting
Lt. Rev. .. ,, revolving
Lt. F. Fl. .... ,, fixed and flashing

*Lt. Gp. Fl. (3) ,, group flashing
*Lt.F.Gp.Fl. (4) ,, fixed and group flashing
*Lt. Gp. Occ. (2) ,, group occulting
†alt. ....alternating
ev. ....every
fl.fls. ......flash, flashes

G., Gn. ......green
Gp. ....group
horl. ....horizontal (Lights placed horizontally)
irreg. ....irregular
m. ....miles
min. ....minute or minutes
obscd. ....obscured
‡ occasl. ......occasional
R. ....red
sec. ....second or seconds
(U) ....unwatched
vertl. ....vertical (Lights placed vertically)
vis. ....visible
W., Wh. ......white

† Alt. (Alternating) signifies a Light which alters in colour.

* The number in brackets, after the description of Group Flashing or Group Occulting Lights, denotes the number of flashes or eclipses in each group.

‡ Occasional Fog Signal means a signal which is only given in answer to vessels' signals.

The height given against a light is the height of the focal plane of the light above high water. Ordinary spring tides, or above the sea level in cases where there is no tide.

The visibility of lights is given in nautical miles, assuming the eye of the observer to be 15 feet above the sea.

Bearings of lights are given from seaward.

TIDES

H.W.F. & C. IXh. 25m······high water full and change. The hours are expressed in Roman figures, except 2h.

Equinl. ....equinoctial	m. ....minutes
Fl., fl. ....flood	Np. ....neap tides
*H.W. ....high water	ord. ....ordinary
H.W.O.S. ..high water ordinary springs	Qr. ....quarter
h. ....hour, hours	Sp. Spr. ......spring tides
kn. ......knot, knots	..current
*L.W. ....low water	....flood tide stream
L.W.O.S. ..low water ordinary springs	....ebb tide stream

*H.W. or L.W. always refers to high water or low water of ordinary spring tides, unless otherwise stated.

The period of the tide, at which the streams are running in the direction of the arrows, is denoted as follows—

(1) 1st Qr., 2nd Qr., etc., for the quarters of each tide .... 2nd Qr. 2nd Qr.

(2) Ih., IIh., IIIh., etc., for 1st, 2nd, 3rd hours after high or low water. IIh. IIIh

(3) Black dots on the arrows, the number of hours after high or low water. (The reference being to high or low water in the locality, unless otherwise stated on the chart.)

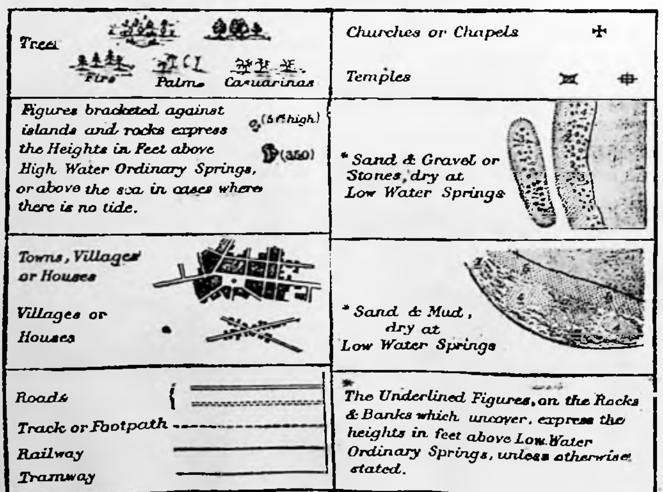
3 hours after high water or 3 hours ebb is indicated by

4 hours after low water or 4 hours flood is indicated by

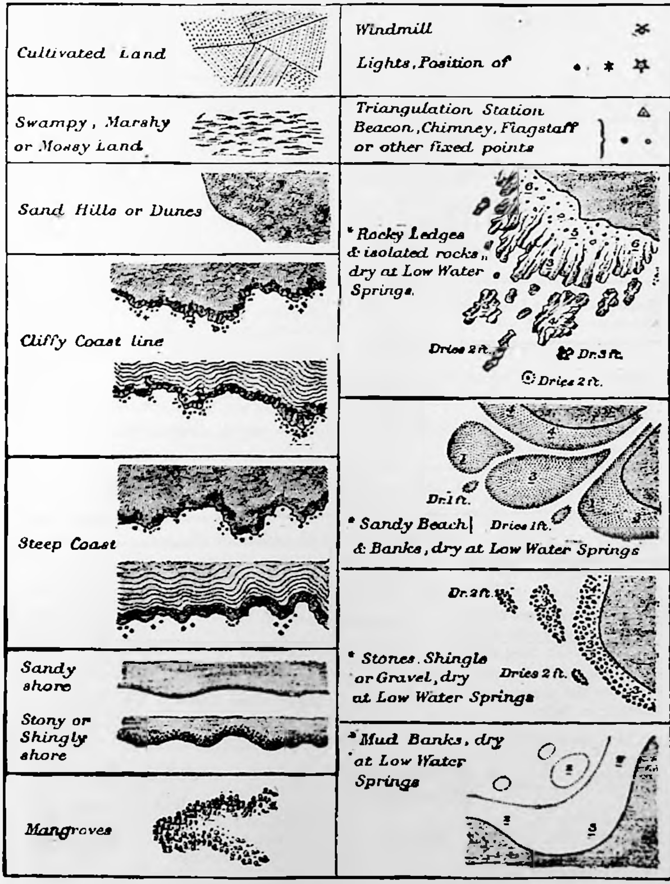
The velocity of currents and tidal streams is expressed in knots per hour, thus : .... 1 kn 5 kn

The rise of tide is measured from mean low water of ordinary spring tides, unless otherwise stated.

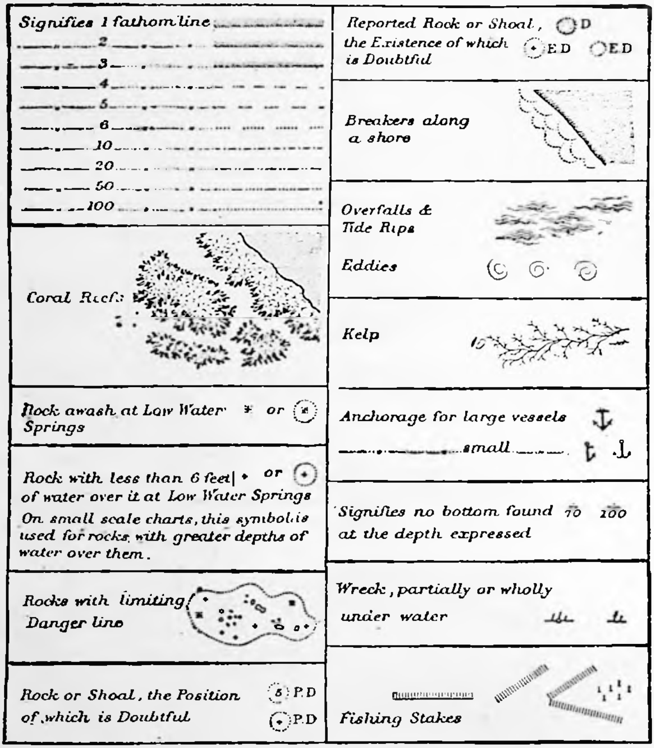
Conventional Signs



Conventional Signs *(continued)*



Conventional Signs (*continued*).



GENERAL REMARKS

Charts are generally drawn on the true meridian; if otherwise, a true meridian is given on the chart.

The datum to which the soundings are reduced is mean low water spring tides, unless otherwise stated in the title of the chart.

Soundings are expressed in feet or fathoms, as stated in the title of the chart.

1 fathom = 6 feet = 1·828766 metres.

Underlined figures, on rocks and banks which uncover, express the heights in feet above low water of ordinary springs, unless otherwise stated.

All heights (except those expressed in underlined figures) are given in feet above high water springs, or, in places where there is no tide, above the level of the sea.

The natural scale is the proportion which the scale of the chart bears to the actual distance represented, and is shown thus— $\frac{1}{12,160}$.

A sea mile is the length of a minute of latitude at the place, and a cable is assumed to be a tenth part of a sea mile.

The figures in brackets in the bottom right-hand corner of a chart, thus—(38·43 × 25·49) are the dimensions of the plate in inches between the innermost graduation or border lines.

All longitudes are referred to the meridian of Greenwich.

THE STATION POINTER

The theory of the Station Pointer is founded on the twenty-first proposition of the Third Book of Euclid, which proves that the angle subtended by any chord will be the same from any part of the same segment of a circle.

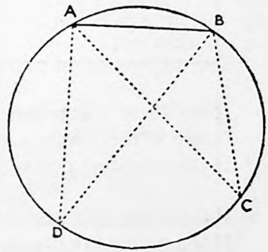
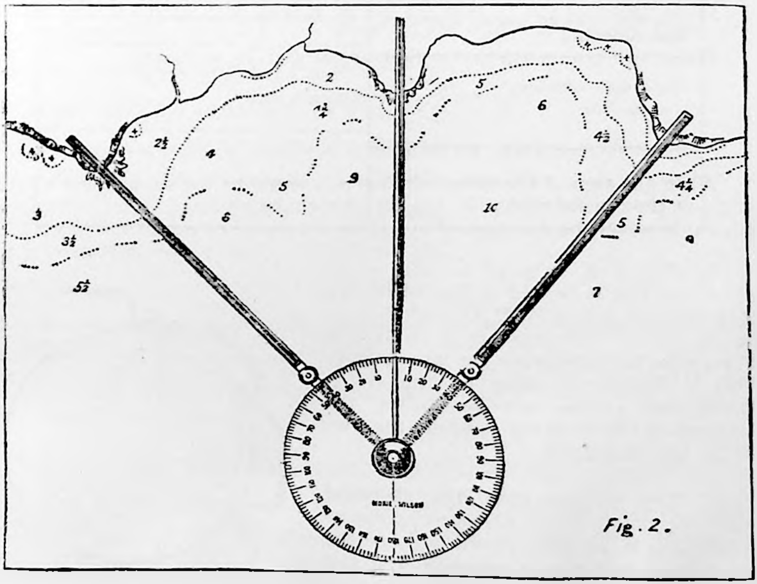


Fig. 1.

In Fig. 1, if the line A B subtend an angle of, say, 30° at C, it will subtend the same angle at D or anywhere on the arc A D C B. Therefore it follows that the position of observer can only be found by taking a bearing of one of the points.

The Station Pointer is useful in fixing a point by means of the "three-point problem" or "two points" in transit and one angle.

The instrument (*see* Fig. 2) consists of a disc about 6 inches in diameter. From a central ring proceed three arms about 12 inches long. The central arm is fixed and its bevelled edge coincides with the zero of the graduations. The other two arms can be moved to any required angle and can be fixed by a clamping screw. The



STATION POINTER

instrument is graduated in degrees to the right and left of the fixed arm, which is the zero of the instrument.

Method of using.—The legs of the instrument are clamped at the observed angles and the bevelled edge of the central leg is placed over the middle object, then the instrument is moved about, still keeping the central leg on the centre object, until the bevelled edges of the other two legs coincide with the other two objects or points observed. When the three legs lie over the three points (as shown in Fig. 2), the pricking point at the centre of the instrument being pressed down will mark the position of the observer on the chart.

The fix by station pointer is good—

If the three points are in the same straight line.

If two of the points are in transit and the observed angle is not small.

If the "central point" is the nearest to the observer and the angles not too small.

If the observer's position is within the triangle formed by the points.

If the points are nearly equidistant from the observer and the angles not less than 70°.

If one angle is large and the other small, and the small angle is made with an outer object far behind the central object.

The angles should be sensitive on a change of position of observer.

If the centre of the instrument can be moved without displacing one of the objects the "fix" is bad.

When two points are in line they are said to be *in transit.*

THE INDETERMINATE CASE

When the sum of the observed angles is equal to the supplement of the angle at the central object, B, the position of the observer is indeterminate. It can, however, be determined by taking a bearing of one of the objects.

In Fig. 3 if $(x + y) = (180° - B) = (A + C)$, it is proved in Euclid III. 22 that the four points A. B. C. and P must be on the circumference of a circle and P may be anywhere on the arc A P C. This is known as being on the circle. When two circles can be drawn the position of the observer is where the two circles cut each other.

If, then, in any case, the observed angles be nearly equal to the supplement of the angle at the central object, the two circles will nearly coincide and the "fix" will be a bad one.

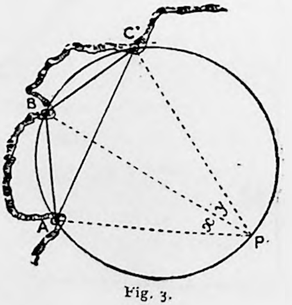


Fig. 3.

If no station pointer is at hand the centres of the circles on which the observer is situated can be found in the following manner :—

At each end of the line joining the two points subtending the angle lay off the complement of the angle, and where the two lines intersect will be the centre of the required circle.

There are two cases, one when the angle is acute, the other when it is obtuse.

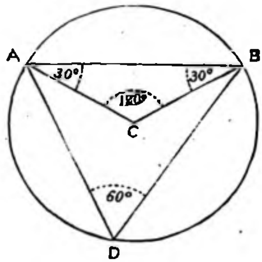


Fig. 4.

When the angle is acute.—In Fig. 4 let A B subtend an angle of 60° ; from each end of the line A B lay off an angle of 30°, and at C, their point of intersection, is the centre of the required circle.

The angle at D being less than a right angle, the segment A D B is greater than a semicircle (Euc. III. 31), therefore the centre of the circle and the observer are on the same side of the line A B.

When the angle is obtuse.—In Fig. 5 let A B subtend an angle of 120° at D. Lay off from each end of A B an angle equal to the excess of D above 90°, and at C, their point of intersection, is the centre of the required circle.

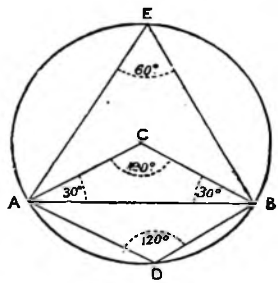


Fig. 5.

Now, $180° - D = E$ (Euc. III. 22).

$ACB = 2\,AEB$.

$\therefore 2\,CAB + 2\,(180° - D) = 180°$.

Also D being obtuse the segment A D B is less than a semicircle (Euc. III. 31), therefore the centre of the circle is on the side of A B remote from D, and the arc on which the observer is situated is the lesser segment, A D B.

The accuracy of the graduations of the station pointer can be tested in the following manner—

Let A B C (Fig. 6) represent any angle θ Take B A equal to the given radius. Describe the arc A C. Draw the chord A C. Bisect the angle A B C by the straight line B E; this will also bisect the chord A C at right angles (Euc. I. 26).

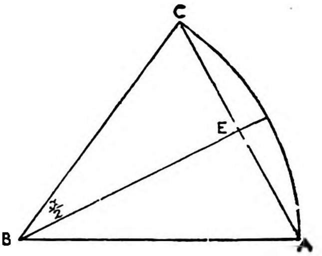


Fig. 6.

$$\text{Now by plane trigonometry, E C} = \text{B C} \times \sin. \frac{x}{2}$$

$$\text{And A C} = 2 \text{ E C}$$

$$\therefore \text{A C} = 2 \text{ B C} \times \sin. \frac{x}{2}$$

Therefore chord $= 2$ radius $\times$ sin. of half the given angle.

Example.—Test the angle of 45°, using a radius of 24 inches.

24×2 in. $= 48$ in. $= 2$ radius		48	log. 1·681241
$\frac{45^\circ}{2}$	$= 22\frac{1}{2}^\circ = $ half θ	$22\frac{1}{2}^\circ$	sin. 9·582840
		18·37	log. 1·264081

For a radius of 24 inches the length of the chord is 18·37 inches. The length can also be found by multiplying 48 in. by the nat. sin. of $22\frac{1}{2}^\circ$.

THE CHRONOMETER

The British Government in 1713 offered a reward of £20,000 for any method by which the longitude could at all times be determined at sea ; the whole reward would be given if the method, when tested by a voyage to the West Indies, were found to be within 30 miles of the truth, £15,000 if within 40 miles, and £10,000 if within 60 miles of the truth.

John Harrison (1693—1776), a Yorkshire carpenter, came to London in 1728 with drawings of a watch which he thought would earn the reward. In 1735 he presented his first watch to be tested by the Board of Longitude. He was sent to Lisbon with it, when he corrected the dead reckoning about 1½ degrees.

The master of H.M.S. *Oxford* on the homeward voyage reported the landfall to be the Start, but Harrison, trusting to his watch, insisted it was the Lizard, and he was found to be right.

In 1739 he completed his second watch, which was not tried at sea.

In 1749 he presented his third watch, which gained the gold medal of the Royal Society. It was less complicated and more accurate than the second, having an error of only three or four seconds a week. He then constructed his fourth watch, and applied for the full reward.

The tests were very severe. The chronometer was compared in the Observatory at Greenwich, sealed up and sent to Portsmouth, where Robertson the mathematician found its error by equal altitudes, which he reported to the Admiralty.

The watch was put on board H.M.S. *Deptford*, Capt. Diggs. It was secured by four separate locks, the keys of which were held by Governor Lyttleton of Jamaica, Captain Diggs, the first Lieutenant, and Harrison's son.

The *Deptford* sailed 18th November, 1761. During the voyage to Madeira the chronometer corrected the dead reckoning 1½ degrees. The ship arrived at Madeira three days before H.M.S. *Beaver*, which had sailed ten days before the *Deptford*—caused by trusting to dead reckoning. From Madeira to Jamaica the watch corrected the longitude to the extent of three degrees, while some ships differed from the correct longitude five degrees.

When the ship arrived at Port Royal the watch was found to be in error only five seconds. On the return voyage to Portsmouth the error in longitude was less than 18 miles.

In 1764 the same watch was sent to Barbados. He gave with the chronometer a temperature scale which if taken into consideration would have shown an error in 156 days of only 15 seconds. But it was not until 1773, after the King had personally interposed on his behalf, that he received the full reward of £20,000. He also received a further sum from the East India Company.

The two chief features are the "fusee," the escapement, and the balance. The fusee has the effect of rendering what would be a variable force uniform.

The escapement regulates the beat.

The balance expands or contracts with heat or cold. By an arrangement

in two segments, each composed of two metals, steel on the inside and brass on the outside, the unequal contraction or expansion is nullified and the chronometer is not unduly affected by temperature.

Although every care and device are used in the construction of chronometers it is not possible to make a perfect instrument ; but every care should be used in handling them. They should be wound carefully at about the same hour each day. The key should always be turned until it is felt to " butt."

They should be kept in well-padded boxes, or, as the latest method is, with springs fitted to fillet-pieces. They should not be subject to any unusual movement, as a chronometer is easily stopped by a rotary motion ; and care should be taken that they do not suffer any greater changes of temperature than are unavoidable.

A good stop-watch is a most valuable adjunct to the chronometer. The watch should be compared both before and after use with the chronometer.

The chronometer is merely a very perfect watch, in which the balance-wheel is so constructed that changes of temperature have the least possible effect upon the time of its oscillation ; such a balance is called a *compensation* balance. A chronometer may be well compensated for temperature and yet its *rate* may be gaining or losing on the time it is intended to keep ; the compensation is good when changes of temperature do not affect the rate.

It is not necessary that a chronometer's rate should be zero (or even very small, except that a small rate is practically *convenient*); it is sufficient if the rate, whatever it is, remains constant. The indications of a chronometer at any instant require a correction for the whole accumulated error up to that instant. If the correction is known for any given time, together with the rate, the correction for any subsequent time is known.

Winding.—Most chronometers are now made to run either eight days or two days. The former are wound every seventh day, the latter daily, so that in case the winding should be forgotten for twenty-four hours the chronometers will still be found running. But it is of importance that they should be wound regularly at stated intervals ; otherwise an unused part of the spring comes into action, and an irregularity in the rate may result.

Chronometers are wound with a given number of half-turns of the key. It is well to know this number, and to count in winding, in order to avoid a sudden jerk at the last turn : still, the chronometer should always be wound *as far as it will go*, that is, until it resists further winding. This resistance is produced not by the end of the chain, but by a catch provided to act at the proper time and thus protect the chain.

When a chronometer has stopped it does not again start immediately after being wound up. It is necessary to give the whole instrument a quick rotatory movement, by which the balance-wheel is set in motion. This must be done with care, however, and with little more force than is necessary to produce the result ; afterwards the chronometer must be guarded from all sudden motions. The hands of a chronometer can be moved without injury to the instrument, so that it may be set proximately to the true time. It is, however, not advisable to do this often.

Transporting.—Chronometers transported on board ship should be placed as near the centre of motion as possible, and allowed to swing freely in their gimbals, so that they may preserve a horizontal position. They should also be kept as nearly as possible in a uniform temperature.

When transported by land, the chronometer should no longer be allowed to swing in its gimbals, but is to be fastened by a clamp provided for the purpose; for the sudden motions which it is then liable to receive would set it in violent oscillation in the gimbals, and produce more effect than if allowed to act directly.

It has been found that the rates of chronometers have been affected by masses of iron in their vicinity, indicating a magnetic polarity of their balances. Such polarity may exist in the balance when it first comes from the hands of the maker, or it may be acquired by the chronometer standing a long time in the same position with respect to the magnetic meridian. In order to avoid any error that might result from this polarity (whether known or unknown) it will be well to keep the chronometers always in the same position; and they should not be removed from the ship to be *rated;* but their rates should be found after they are placed in the position they are to occupy.

Pocket Chronometers should be kept at all times in the same position: consequently, if actually carried in the pocket during the day, they should be suspended vertically at night.

Comparison of Chronometers.—One (supposed to be the best) instrument is selected as the *standard*, and with this all others are compared *after* winding. A record should be made and retained of the comparisons, which will furnish a graphic history of the performance of each instrument. For convenience, the standard may be distinguished by the letter R (*reference*), and the others by the letters A, B, etc., as far as they extend in number; thus—

Date.	Letter and No.	(R) Standard.	Chronometers.	Differences.	Second Differences.
		H. M. S.	H. M. S.	H. M. S.	
Friday, March 2	(A) 609	2 40 0	2 36 0·0	0 4 0·0	
	(B) 972	30	2 56 3·5	0 15 33·5	
					S.
Saturday, March 3	(A) 609	2 32 0	2 28 1·5	0 3 58·5	— 1·5
	(B) 972	30	2 48 6·0	0 15 36·0	+ 2·5

The second differences, in the last column, are the daily rates, if the standard does not change..

The practical benefit of a system of daily comparisons is that a guard may be kept on the steadiness of the rates of the instruments.

Comparison by Astronomical Observations.—When one or more chronometers have to be regulated by means of astronomical observations, these observations are made with but one of them, and the corrections of all the others are found by comparing them with this. On board ship the chronometers are never brought on deck; but the observations are made with a watch (often called a "hack watch"), or it may be a pocket chronometer, which is compared with the chronometer either before or after, or both before and after, the observations. The double comparison is necessary where extreme precision is required, in order to eliminate any difference of the rates of the watch and chronometer.

Comparison by Coincident Beats.—When two chronometers are compared

which keep the same kind of time, and both of which beat half seconds, it will mostly happen that the beats of the two instruments are not synchronous, but one will fall after the other by a certain fraction of a beat, which will be pretty nearly constant, and must be estimated by the ear. This estimate may be made within half a beat, or a quarter of a second, without difficulty, but it requires much practice to estimate the fraction closer, and with certainty.

Rate for Temperature.—Each chronometer should be accompanied with a record from a responsible chronometer maker, or, better still, from an Observatory, showing the *daily rates for mean temperatures* for each 10°, say from 40° to 100° Fahr.; then with a maximum and minimum thermometer in the chronometer-case, the actual temperature of the preceding day is recorded as soon as the case is opened for winding in the morning. Then, referring to the tabulated record of observed daily rates according to temperature, the rate for the preceding day is found by inspection, and, applying this according to its sign to the sum of the accumulated daily rates up to the previous day, there will be found the whole amount of the accumulated rate on the given day, to be applied as a correction to the primary error. Although the rates may differ with lapse of time, etc., it is more likely that the differences of rates for corresponding temperatures will remain the same, or nearly so.

These remarks have been taken from Chauvenet's "Manual of Spherical and Practical Astronomy"; *see* also Admiral Shadwell's excellent work, "Notes on the Management of Chronometers."

Application of the Original Error and Daily Rate of a Chronometer.—The chronometers carried on board British ships show Greenwich date *approximately.* When put on board, before the voyage commences, the optician who has had charge of the instruments furnishes the *error* by which each chronometer differs from Greenwich time on the noon of the day, and also the *rate* per day (of loss or gain on Greenwich time) that each instrument has been ascertained to keep during the period of supervision over them.

The given error—generally termed the *original error*—may indicate that the chronometer to which it is applicable is *fast* or *slow* on *Greenwich mean time*, and will be specified accordingly.

The given rate—called the *daily rate*—will be indicated as to whether the chronometer is *gaining* or *losing* on *Greenwich mean time.*

The application of the error and rate to the time shown by chronometer, to get the *correct* Greenwich mean time, is a purely *arithmetical* calculation, and may be shown here.

TO FIND GREENWICH MEAN TIME BY CHRONOMETER

I. *Given an Original Error and a Daily Rate*

1. Write down the chronometer time astronomically, with the month and day prefixed; under it write the *original error*, which subtract if the chronometer had been found *fast*, but *add* if found *slow*.

2. Find the number of days elapsed since the date the chronometer was rated; multiply this number by the daily rate, and the product will be the *accumulated rate* to be *subtracted* from the time if the chronometer had been found to *gain*, but to be *added* if it had been found to *lose*.

Example.—A chronometer indicating 5h. 40m. 42s. on November 11th had been found 19m. 58s. slow on Greenwich mean noon on July 2nd and was gaining 4·5s. daily; find the Greenwich mean time on Nov. 11th.

July 2nd		Elapsed days	132
	29	Daily rate	4·5
Aug.	31		660
Sept.	30		528
Oct.	31		
Nov.	11		60)594·0
Days	132		9 54
		for 6h.	+ 1
			Gain 9m. 55s.

	D.	H.	M.	S.
Time by chron., November	11	5	40	42
Slow, July 2nd		+	19	58
		6	0	40
Accumulated rate (gain)		−	9	55
Green. mean time, Nov. 11		5	50	45

Or the error of the chronometer at the Greenwich mean time may be found, and applied directly to the chronometer time.

Find the accumulated rate as in (2) and place it under the original error; then if the error was *fast* and the rate was *gaining*, or if the error was *slow* and the rate was *losing*, the error of the chronometer would now be greater than before, hence *add* the two together and keep the same name. But if the error was *fast* and the rate was *losing*, or if the error was *slow* and the rate was *gaining*, the error of the chronometer would now be found by subtraction, taking care to change the name of the error, if the accumulated rate was the greater.

The same Example as before—

	M.	S.	
Original error	19	58	slow
Acc. rate	9	55	gain
Diff.	10	3	slow

	D.	H.	M.	S.	
Time by chron., Nov.	11	5	40	42	
Error			10	3	slow
Green. mean time ,,	11	5	50	45	

If, during the voyage, when calling at a port where there is a good observatory, or by astronomical observations at a place the position of which is well known, an error and rate are determined anew, the process of finding the Greenwich mean time is exactly on the basis of that just described; and according to the *Rules* here given.

II. *Given Two Errors to find a Daily Rate*

When *two* chronometric *errors* have been found, and, by means of the elapsed time, you have to find a daily rate, proceed as follows:

1. Write down the time by chronometer, with the month and day before it; under this time write the *second error, adding* it if *slow, subtracting* it if *fast;* the result will be the *approximate* Greenwich date.

2. *For the Daily Rate.*—Write one error under the other, then—

Both errors fast, or both slow, take their difference;
One error fast and the other slow, take their sum;

the sum or difference must be converted into seconds, and divided by the number of days elapsed between the dates of the two errors. The result will be the daily rate, in seconds, or seconds and tenths, or perhaps tenths only.

3. *For the Accumulated Rate.*—Multiply the daily rate by the number of days elapsed between the date of the second error and the date by chronometer, allowing a proportion for the given hours. The product will be seconds, which, if above 60, reduce to minutes and seconds; the result will be the accumulated rate, and to know whether it has been a gaining or losing rate, *note* the following—

With 1st error *fast*, and 2nd error *faster* With 1st error *slow*, and 2nd error *not so slow* With 1st error *slow*, and 2nd error *fast*	Gaining rate.
With 1st error *slow*, and 2nd error *slower* With 1st error *fast*, and 2nd error *not so fast* With 1st error *fast*, and 2nd error *slow*	Losing rate.

4. *For the Greenwich Date, Mean Time.*—To the *approximate* Greenwich date apply the accumulated rate, *subtracting* it if *gaining*, but *adding* it if *losing*.

Example.—A chronometer showed 8h. 0m. 42s. on December 4th; on June 1st it had been found 0m. 12s. slow on mean noon at Greenwich, but on July 1st it was 4m. 27s. fast on mean noon at Greenwich. Find the Greenwich mean time on December 4th.

June	1st
	29
July	1
Days	30

	M.	S.
June 1st ..	0	12 slow
July 1st ..	4	27 fast
Sum	4	39
	60	
	30)27,9s.	
Daily rate	9·3s.	

July	1st
,,	30
Aug.	31
Sept.	30
Oct.	31
Nov.	30
Dec.	4
Days	156

Elapsed days	156	
Daily rate	9·3	
	468	
	1404	
	60)145,0·8s.	
	24	10·8
for 8h.	+	3·1
Gain	24m. 13·9s.	

	D.	H.	M.	S.
Time by chronometer, December	4	8	0	42
Fast, July 1st		—	4	27
		7	56	15
Accumulated rate (gain)		—	24	14
Green. date, mean time, Dec.	4	7	32	1

NOTE.—In these cases the dates have been considered as astronomical: their interpretation in reference to the longitude will always have to be taken into account.

In order that the longitude found shall be worthy of confidence, the greatest care must be bestowed upon the determination of the rate of the chronometer. As a single chronometer might deviate very greatly without being distrusted by the navigator, it is well to have at least three chronometers, and to take the mean of the longitudes which they severally give in every case.

But, whatever care may have been taken in determining the rate on shore,

the sea rate will generally be found to differ from it more or less, as the instrument is affected by the motion of the ship; and, since a cause which accelerates or retards one chronometer may produce the same effect upon the others, the agreement of even three chronometers is not an absolutely certain proof of their correctness. The sea rate may be found by determining the chronometer correction at two ports whose *difference* of longitude is well known, although the absolute longitudes of both parts may be somewhat uncertain. But for this purpose a small pamphlet—" List of Time Signals, established in various parts of the World "—is published by the Hydrographic Department of the Admiralty, compiled for the use of the navigator, as an aid for ascertaining the errors and rates of chronometers: this you should always have at hand with your epitome and charts.

An *unknown* error in the chronometer will have the following effect on the position of the ship as determined by an altitude of a heavenly body and the supposed known error and rate:

(1) If the chronometer is—

> Fast, and also faster than supposed, or
> Slow, but not so slow as supposed; then—

the *true* position will be *eastward* of that by *observation;* and as a consequence, when expecting to make land, sailing *eastward*, it will be made *earlier*, but sailing *westward*, *later*, than anticipated.

(2) Similarly, if the chronometer is—

> Slow, and also slower than supposed, or
> Fast, but not so fast as supposed; then—

the *true* position will be *westward* of that by *observation;* and as a consequence, when expecting to make land, sailing *westward*, it will be made *earlier*, but sailing *eastward*, *later*, than anticipated.

THE COMPASS

The invention of the compass has been placed at many dates; there are numerous indications that it was used in China over 2,000 years ago. It was in use in Europe in the twelfth century.

Almost the first historical compass is supposed to have been made by Flavio Gioja of Amalfi in the south of Italy. It was only marked to eight points.

A compass consists of four parts—the pivot, the bowl, the card, and the needles. There are many patterns, all of which aim at the same idea, that is, sensitiveness of action, which is obtained by two types, the dry compass and the wet or spirit compass.

The one most in favour is the compass patented by Sir William Thomson, afterwards Lord Kelvin, in 1877. The principal features of this most excellent compass are :—(1) reduction to the least possible weight; (2) short, light needles symmetrically arranged on each side of the central point; (3) mode of suspension; (4) its slow period of oscillation or swing.

The above features are attained by the application of well-known laws, and each will now be explained.

(1) *Reduction of Weight.*—This is attained by using short, light needles, and by cutting away the unnecessary central portion of the card, and, in order to give the necessary rigidity to the card, the rim is made of aluminium and suspended to the central cap by means of silk threads, which latter also absorb some of the vibration set up by the screw or other causes. The cap is of sapphire and the pivot iridium-pointed, and these two substances being amongst the hardest known, and the card being extremely light, friction is reduced to a minimum. The weight of a 10-inch Thomson compass card is about 180 grains, and will be found written on the under side.

(2) *Short, light Needles.*—The practical rules on which a compass adjuster works are based on the assumption that the compass-needle is a magnetic particle having no length. This assumption much facilitates the theory of calculation and compensation, and is most nearly attained by using a number of short needles—eight in the compass under consideration—the longest of which in a 10-inch card is only $3\frac{1}{4}$ inches.

(3) *Mode of Suspension.*—By keeping the point of suspension of the card and needles well above their centre of gravity the card has stable equilibrium, which has the effect of rendering the compass steadier and constrains it to move in a horizontal plane, which obviates the necessity of a counterpoise, in the form of a sliding weight, which was a feature of old pattern compasses.

(4) *Period of Oscillation or Vibration.*—One of the chief features of the Thomson compass is its slow period of swing which, in England, is about 40 30 seconds for the 10-inch card. The period of oscillation being slow conduces to greater steadiness, and practically eliminates the possibility of synchronising with the period or roll of the ship in a seaway, as a ship's period is much quicker.

The period of vibration of a compass can be lengthened by two different

methods; firstly, by increasing its moment of inertia; secondly, by decreasing the magnetic moment of the compass (the magnetic moment of a magnet is the product of the amount of magnetism imparted to one of its poles multiplied by the distance between its poles).

The moment of inertia of a compass can be increased by increasing its weight, but this would render the compass sluggish and also nullify the first point, viz., reduction of weight. It is, therefore, to the second point that we must look for the desired result, and this is attained by using *short, weakly-magnetised needles.* The loss of directive force in the needles is more than compensated for by the enormous reduction in the weight of the card as compared with old patterns. The period of oscillation is marked on the under side of the card.

A well-constructed compass will last for many years. Compasses are sometimes condemned as faulty when they are in good order and condition, the trouble being due to position on board, not construction.

THE LIQUID OR SPIRIT COMPASS

The card is made of mica, the degrees and points are painted on the outer edge; it is attached to a frame and central hollow float which is immersed in a mixture of distilled water and pure alcohol to prevent freezing; the liquid prevents any undue oscillation of the card. It is important that the point of suspension of the card should be in the horizontal plane passing through the gimbal ring, that the centre of flotation should be below the point of suspension, and that the centre of gravity of the card should be below the centre of flotation. The needles are sheathed in brass and are strongly magnetised, and are placed low on the card so as to prevent the card dipping with large changes of magnetic dip. The bowl should be kept full of the liquid. All air is removed by air pump before the cover is finally screwed down. The expansion and contraction of the liquid is met by having an elastic corrugated metal box attached to the bottom of the bowl. There should be no air bubbles present, and the card should not press upon the pivot with a weight of more than 100 grains. Both types of compass are fitted with a shadow pin and azimuth mirror with which to take bearings. Bearings, by the mirror, of objects more than 30 degrees high are not reliable.

GYROSCOPE AS A COMPASS

Originally constructed by M. Foucalt to make visible the rotation of the earth, the gyroscope is familiar to most people in the form of a spinning-top. Its adaptability as a compass depends upon the law that when a body symmetrical about an axis of figure is set rotating about that axis the tendency is for the axis to retain unchanged its directional position in space. The principle on which it proceeds is this—that unless gravity intervene, a rotating body will not alter the direction in which its permanent axis points.

In the gyroscope there is a rotating metal disc, the middle point of whose axis is also the centre of gravity of the machine. By this device the action of gravity is eliminated. It is so constructed that the axis of rotation can be made to point to some star. Then as the heavy disc revolves at great speed it is found that the axis continues to point to the moving star, though

in consequence of this apparently altering its direction relative to bodies on the earth. But if the axis be pointed to the celestial pole which is fixed no alteration in its position relative to bodies on the earth takes place. It has the dynamical property known as the moment of momentum (mass × velocity) into the distance from the axis. A gyroscope free to move in two planes, at any place on the earth other than the poles, tends to set itself with its axis of rotation parallel to the axis of the earth by reason of the relative rotations of the two bodies, meeting in this position the least resistance.

Used as a compass it has to possess a very large gyroscopic resistance strongly opposing any attempt to tilt its axis to any angle. It must therefore rotate at a very high speed, usually 20,000 revolutions per minute. The centrifugal force developed at the periphery is enormous, the stress amounting to 10 tons per square inch; the air friction consumes 95 per cent. of the energy. The disc and spindle are constructed from one solid piece of special nickel steel, so that nothing about it should work loose. The directive force developed is 15 times that of the best form of magnetic compass.

Owing to what is called the precession or swing to or from the meridian, a serious difficulty had to be overcome which was added to by the movements of the ship and the earth's rotation, causing the axis of the gyroscope to wobble; as the gyro is rigidly attached to the compass card, the axis and the north and south line being exactly parallel, this caused the north point of the card to swing steadily three or four degrees on each side of the meridian. This fault had to be overcome before the gyro could be used as a compass.

It was accomplished in a very ingenious manner. The gyro revolves inside a case and acts as a high speed centrifugal blower; a strong air blast is created which incidentally serves to keep the motor cool. A stream of this air was diverted through two small pipes led on opposite sides of the disc; the pipes were fitted with valves, which opened or closed with the inclination of the gyro; the jet of air opposed the inclination first one side, then the other, gradually reducing the amplitude of the swing, until in about two and a half to three hours the gyro ran with perfect steadiness; this process is called damping. The gyro must be free to move in two directions. The card, floats, and gyro are all attached and float in a bowl of mercury; the bowl is slung in gimbals in the same manner as an ordinary compass.

In order to keep the whole floating system central a steel stem is fixed in the centre of the glass cover and the lower end dips into a cup containing mercury carried on top of the float.

The gyro contains a motor by which it is driven. The best position for a gyro compass is as near the centre of gravity of the ship as possible.

If a "master compass" is thus carried transmitters worked electrically can be used in any place, in any position, either horizontally or vertically inclined. The gyro compass is of no service at the North or South Pole. It has to be corrected for course and speed. Tables for that purpose are supplied with the compass.

The Advantages are: Independence of magnetic disturbance and vibration (as the gyro points true north there is neither variation nor deviation to

be applied), much greater steadiness, no swing to and fro, more sensitiveness, and as a consequence the ability to steer a very straight course.

The Disadvantages are: Expensiveness, the need of electric installation, some knowledge required of electricity to be able to work the attachments in ordinary working or a breakdown.

A breakdown in the electric current would put the compass out of use.

THE MARINER'S COMPASS

When out of sight of land, the MARINER'S COMPASS is the only instrument that *shows* the DIRECTION in which the ship is moving; it is therefore necessary to understand its construction and use, together with the corrections which must be applied to its indications.

In the first place you must learn the divisions of the compass card, and know how to *box the compass*, that is, be able to repeat the points in their order, commencing at any given point, and going to right or left.

DIVISIONS OF THE COMPASS CARD.—The Compass Card is a circle divided into 32 equal parts, called *points of the compass;* and standing on the deck of a vessel at sea you must always suppose yourself to be in the centre of such a circle, the circumference of which is the *visible horizon*, and towards some point of which the vessel's head is directed. The *standard of direction* is the *meridian* passing through the place of the vessel, and a small part of this meridian is represented on the compass by a line drawn from one part of the circumference to another,—passing through the centre of the circle. One end of the meridional line is named North, because the line trends *in direction* towards the pole of that name, and the other end, directed towards the south pole, is named South. A second line, passing through the centre of the circle, but at right angles to the meridian, has one end named East, and the other West. When facing the North, the East (in which direction the sun and other heavenly bodies appear to rise) is on your right hand, and the West (towards which the same bodies set) is on your left. Hence the letters N., S., E., W., represent the four *cardinal* or chief points of the compass—corresponding to similar points of the horizon; and the two principal lines of which we have spoken (and which give these four points), by cutting each other at right angles, divide all the horizontal space around you into quarters, called *quarters of the compass.*

All the other points are named after the cardinal points as follow:—

By halving (bisecting) each of the four quarters of the compass, and drawing four lines in a new direction *from* the centre of the circle to the circumference, we get 4 more points; the N.E. (north-east) point, midway between N. and E., and so on with S.E. (south-east), S.W. (south-west), and N.W. (north-west), all midway between cardinal points; these new directions also give names to the four quarters of the compass, as when we say "the wind is in the N.E. quarter,"—meaning thereby not exactly N.E., but somewhere between N. and E.

Thus far, by halving the four quarters, or right angles, we have 8 *half*-right angles, thus making 8 points of the compass; and proceeding in a similar manner to halve these, we get 8 additional lines, which derive their names from the points to which they are contiguous, always placing first the nearest cardinal point, as N.N.E. (north-north-east), midway

between N. and N.E.; E.N.E. (east-north-east), midway between E. and N.E.; and so on with E.S.E. (east-south-east), S.S.E. (south-south-east), S.S.W. (south-south-west), W.S.W. (west-south-west), W.N.W. (west-north-west), and N.N.W. (north-north-west).

We have now 16 points of the compass, and the remaining 16 (to make up 32) are also derived by a similar process to that already explained, viz., halving the 16 angles last obtained, and the new points are named after the 8 principal ones by writing the word *by* before the next nearest cardinal point, as when we say N. by E. (north by east, *i.e.*, north in the direction towards east) for the point midway between N. and N.N.E.; N.E. by N. (north-east by north, *i.e.*, north-east in the direction towards north) for the point midway between N.N.E. and N.E.; and so forth, as best seen by looking at the compass points in their order on p. 73, "Mariner's Compass."

The space from point to point is usually divided into four equal parts, so that we say N. ¼ E. (*i.e.*, N. a quarter point towards E.), N. ½ E., N. ¾ E., N.E. ¼ N., N.E. ½ N., and so forth with any other point of the compass.

Note on expressing Half and Quarter Points.—There is no fixed system for expressing the divisions between two points, but the simplest (learnt by practice and attention) is the best; thus N.N.W. ½ W. is the same as N.W. by N. ½ N., but the first is preferable; you also hear E.N.E. ¼ E. as commonly as E. by N. ¾ N.; but you must beware of such absurdities as using E. by N. ½ E. instead of E. ½ N., or S. by E. ½ S. instead of S. ½ E.

The angular value of each point of the compass, or, more properly speaking, of the space from point to point, is 11¼° (obtained by dividing 360° by 32). The outer edge of the card is often also divided into degrees, and it is well to be able to convert points into degrees at sight, without direct reference to the compass card.

The points are frequently, in calculations, expressed in their numerical value, counting from the meridional line (N. or S.) in the direction of E. or W.; thus W.N.W. is N. 6 points W., and its angular value, in degrees, in the same direction is N. 67½° W. (*i.e.*, 11¼° multiplied by 6); similarly, N.E. ½ N. is N. 3½ points E., and its angular value N. 39° 22½′ E. (*see* Table of Angles, Plate III).

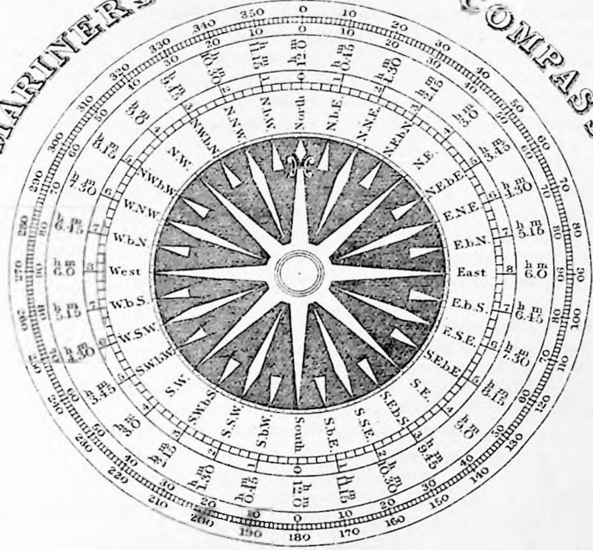
Finally, it is to be observed that the term *point* is commonly used as expressive of two different things; thus, when the question is put "How is her head?" and the reply given is S.E., the meaning is that the ship is sailing towards the S.E. point, *i.e.*, *direction;* but when the helmsman is told to "bring her up a point," in this case the meaning of point is a *change of direction through an angular space of* 11¼°; the mode of expression always removes any doubt as to what is intended.

The latest pattern compass is now graduated in degrees, and reckoned continuously from 0° at north through E., S., and W. to 360° at N.

Such is the principle on which the compass card is divided, and it is of the first importance that you make yourself master of its indications, both as to the points, and the rendering of these into degrees, as they are of constant application in Navigation and Nautical Astronomy.

The COMPASS CARD—HOW MADE.—For practical use the card, on which the points and degrees are figured, is made of stiff cardboard, or mica covered with paper, so as to be as light as possible; and before mounting it is not unfrequently termed the *fly*.

MARINER'S COMPASS.



A TABLE OF THE ANGLES

which every Point & Quarter Point of the Compass makes with the Meridian.

NORTH		POINTS	° ′ ″	POINTS	SOUTH	
		0 – ¼	2 . 48 . 45	0 – ¼		
		0 – ½	5 . 37 . 30	0 – ½		
		0 – ¾	8 . 26 . 15	0 – ¾		
N.b.E.	N.b.W.	1	11 . 15 . 0	1	S.b.E.	S.b.W.
		1 – ¼	14 . 3 . 45	1 – ¼		
		1 – ½	16 . 52 . 30	1 – ½		
		1 – ¾	19 . 41 . 15	1 – ¾		
N.N.E.	N.N.W.	2	22 . 30 . 0	2	S.S.E.	S.S.W.
		2 – ¼	25 . 18 . 45	2 – ¼		
		2 – ½	28 . 7 . 30	2 – ½		
		2 – ¾	30 . 56 . 15	2 – ¾		
N.E.b.N.	N.W.b.N.	3	33 . 45 . 0	3	S.E.b.S.	S.W.b.S.
		3 – ¼	36 . 33 . 45	3 – ¼		
		3 – ½	39 . 22 . 30	3 – ½		
		3 – ¾	42 . 11 . 15	3 – ¾		
N.E.	N.W.	4	45 . 0 . 0	4	S.E.	S.W.
		4 – ¼	47 . 48 . 45	4 – ¼		
		4 – ½	50 . 37 . 30	4 – ½		
		4 – ¾	53 . 26 . 15	4 – ¾		
N.E.b.E.	N.W.b.W.	5	56 . 15 . 0	5	S.E.b.E.	S.W.b.W.
		5 – ¼	59 . 3 . 45	5 – ¼		
		5 – ½	61 . 52 . 30	5 – ½		
		5 – ¾	64 . 41 . 15	5 – ¾		
E.N.E.	W.N.W.	6	67 . 30 . 0	6	E.S.E.	W.S.W.
		6 – ¼	70 . 18 . 45	6 – ¼		
		6 – ½	73 . 7 . 30	6 – ½		
		6 – ¾	75 . 56 . 15	6 – ¾		
E.b.N.	W.b.N.	7	78 . 45 . 0	7	E.b.S.	W.b.S.
		7 – ¼	81 . 33 . 45	7 – ¼		
		7 – ½	84 . 22 . 30	7 – ½		
		7 – ¾	87 . 11 . 15	7 – ¾		
East.	West.	8	90 . 0 . 0	8	East.	West.

J. W. Norie delin.

But to turn this card into the mariner's compass it is necessary to resort to *magnetism;* a small well-tempered steel bar is fully magnetised by drawing its opposite ends (from the middle in the direction of the ends) across the two opposite ends of a powerful magnet ; the bar thus acquires what is called *polarity*, and when suspended, but free to move only in a horizontal direction, its tendency is to rest, one end *towards* north, the other *towards* south ; and the same end will invariably turn (at any given place) towards the same point of the horizon, not indifferently, sometimes to one point, sometimes to its opposite. Two or more such bars, called *magnetic needles*, are fixed below the circular compass card, but parallel with its meridional (N. and S.) line, and in such manner that the North-*seeking* ends of the needles shall coincide (in direction) with the N. end of that line, and the South-*seeking* ends of the needles with the S. end of the same line. An inverted conical brass socket, called a *cap*, with a hard stone in its centre, is passed through a hole in the centre of the card ; and the whole, when *accurately balanced* on a pivot, will rest horizontally.

COMPASS BOWL.—The *compass bowl* is of non magnetic alloy ~~brass or copper~~, and sufficiently large to admit of the card moving freely within it when placed on a hard metal pivot rising from the middle of the bottom of the bowl ; the point of the pivot on which the cap of the card rests should always be sharp and smooth ; the cover of the bowl is glass, which, while protecting the card from wind and weather, admits of its indications being distinctly seen. There is also a vertical line drawn inside the bowl, which is called the *lubber's line ;* finally, the bowl, weighted at the bottom, is hung in gimbals, so that it shall always rest horizontally whether the ship rolls or pitches.

When long and powerful needles were used it was customary, and, indeed, necessary, to make the bowl of pure copper, and to leave as little clearance as possible for the needles ; then as the compass swung a magnetic current in the opposite direction was induced in the copper bowl, thereby bringing the compass rapidly to rest without any loss of directive force.

In a Thomson compass the bowl is made of brass and weighted underneath so as to keep it steady by means of an expansion chamber partly filled with castor oil.

The BINNACLE.—To the deck, in front of the helmsman's position, there is firmly fixed a stand called a *binnacle*, which may be of any shape—square, octagonal, or pillar-like—sometimes of wood, sometimes of brass (like the annexed figure) ; within it are supports (*bearings*), on which rest the gimbals of the compass bowl ; and its movable top or cover is furnished with a glass front, and a lamp, or perhaps two lamps, to cast a light on the compass card by night.

Note.—The compass will be perfect in its indications (or readings) conditionally : (1) that the divisions of the points are equidistant ; (2) that

the direction of the magnetism of the needle or needles is parallel to the longitudinal N. and S. line of the card; (3) that the pivot and centre of the cap working on it are in the centre of the graduated (or divided) circumference of the card; and (4) that the horizontal position of the card is preserved, for which purpose the needles must be furnished with sliding weights to counteract the tendency to *dip*. These subjects are further described in connection with "compass adjustment."

The COURSE.—The meaning of the term *course* is *direction;* and in reference to a ship it is the point of the compass upon which she sails,—or in other words, the direction in which the helmsman is ordered to keep the ship's head by compass. To *shape a course* is to determine the direction in which a ship is to be steered in order to prosecute a voyage; when the wind is foul, she cannot *lie her course*, if free, she *steers her course.* When the course is neither on a meridian (N. or S.) nor on a parallel (E. or W.), it is said to be *oblique.*

RUMBS OR RHUMBS.—The points of the compass were formerly called *Rumbs*, and later *Rhumbs*, after the points of the horizon of which, as already explained, the compass card is the representation.

The STEERING COMPASS.—When the compass card, with magnetised needles attached to it, rests on the pivot in the bowl, and the bowl is placed in the binnacle with the lubber's line directed forward, *i.e.*, towards the ship's head, and at the same time in the midship line, the instrument for *steering* purposes is complete. The point of the compass close to the lubber's line is said to be the *direction of the ship's head by compass;* and if the ship is under way, it is the COURSE *by compass.*

An AZIMUTH COMPASS is an instrument similar to the steering compass, but of superior make; the card is more accurately divided, and the outside of the compass bowl is fitted with a movable ring, to which are attached (exactly opposite to each other, and in a line with the centre of the compass card) two sight-vanes that can be turned down when not in use. *Sight-vanes* are merely oblong pieces of brass with a vertical slit in each; the slit of the eye-vane is very narrow, and the wider slit of the other (and opposite or object) vane is fitted with a single horse-hair for a vertical line; when you look through the vanes and turn them in the direction of an object, you take the object's *bearing*, which is indicated by the compass card. In the Prismatic Azimuth Compass a *prism* is attached to the sight-vane for the purpose of more accurate observation; and thus the divisions of the card below it are read by reflection, at the same time that the bearing is taken; but be careful not to read the card-indication the wrong way—for example 62° instead of 58°.

The azimuth compass is placed in a higher binnacle than is the steering compass, or sometimes on a tripod removable at pleasure, and in the latter case the compass is kept in a box (for safety) when not in use. The position of the azimuth compass should be such as to command a clear view in every direction around it. And it is of the first importance in taking bearings,—(1) that the compass card is perfectly horizontal; (2) that the surface of the prism is horizontal; (3) that the two vanes are vertical; and (4) that the line of sight passes directly over the pivot of the card.

NOTE.—The terms *azimuth* and *bearing* are synonymous. When *observing bearings*, if the compass is slightly vibrating take the *mean* of two or more bearings, read off as quickly as convenient. Never stop the card to read.

To reverse a bearing or course, by which you get the *opposite* point, simply reverse the letters which compose it; thus the opposite to N.W. ½ N. is S.E. ½ S.; similarly the opposite to N. 37° E. is S. 37° W.

The STANDARD COMPASS is that to which all the indications of the other compasses on board are referred, and by which the course is set. It should be fixed in a firm binnacle, or on a wooden pillar—the card not less than four, nor more than five, feet high, and well within direct vision—in a well selected part of the ship both as regards the ship's magnetic character and for the purpose of a clear unobstructed view around in every direction—to facilitate the taking of bearings rapidly. In this sense, and with the various appliances of a good compass, as already indicated, the standard compass is the azimuth compass, and should be fitted as such in every respect. The errors of this compass receive all the necessary attention. When you set a course, as for instance S.W. by S. by standard compass, you say to the man at the wheel—"How is her head?"—he may reply "S.W. by S. ½ S."; and that is the reading of the steering compass, corresponding to S.W. by S. by standard; you say, "Keep her so"; the officer of the watch refers from time to time to the standard compass to see that the proper course is kept; thus you know the course is being made good that you intend.

DEAD RECKONING.—The position of a ship when determined from the distance run by log, and the courses steered by compass, subsequently rectified, is the position by *dead reckoning;* it is generally expressed by the initials D.R.

THE SEXTANT

The first known instrument with which the early navigators tried to measure the altitude of celestial objects and from thence obtain the latitude was the Astrolabe.

It was a very crude instrument consisting of a graduated disc with a ruler (or alidade or a bable, as it was called) that moved on a centre and carried a sight.

About 1590 Captain John Davis invented the Back Staff. It consisted of two concentric arcs and three vanes; the longer radius had an arc of 30°, the shorter radius had an arc of 60°.

This instrument was in common use until 1730, when John Hadley invented a reflecting instrument called the Quadrant.

The Sextant of to-day is simply an extension and improvement of Hadley's Quadrant; the principle of construction remains the same.

The first idea of a *reflecting instrument*, with only a single mirror attached to a movable radius, is due to Hooke, but it was not adapted to the purposes of the navigator. It is not easy to understand why so much pains have been taken to deprive Hadley of the merit of his invention. Sir Isaac Newton, it is said, had previously invented a reflecting instrument with two mirrors; this, however, was not known to Hadley, whose discovery was published in 1731, that of Newton in 1742. Lalande mentions Godfrey, of Philadelphia, as having anticipated Hadley; but it was not known in England until after the publication of Hadley's paper in the *Philosophical Transactions*.

The SEXTANT derives its name from the extent of its limb, which is the sixth part of a circle, or 60°, but being an instrument of double-reflection it is divided into 120°. It is used for measuring *angles*—as the altitudes of, and distance between, heavenly bodies—and also altitudes of, and angles between, terrestrial objects.

BRIEF DESCRIPTION OF THE PARTS OF THE SEXTANT.—The form of the Sextant is that figured on Plate VIII.; but the student, in reading the description of the various parts, should place the instrument itself before him.

The flat upper surface of the Sextant is the *plane of the instrument*, and the following are the principal parts—

A A′ is the *arc* (or *limb*) which is graduated from right to left, from 0 (the *zero* point) to 120° or 150°; this is the *arc proper*, and the subdivisions of each degree are by 10′, 15′, or 20′, according to the size and perfection of the instrument. To the right of 0—*i.e.*, from left to right of the arc, there is also a small portion of graduated arc which is called the *Arc of Excess*.

I R V is the radius, or *index*-bar, movable along the arc and round a centre, and having a dividing scale V (called the *vernier*) close to the arc, by which the subdivisions of the arc are read off.

I is the *index-glass*, a reflector or mirror which moves with the index-bar, and is fixed on it in such a manner that its plane is over the centre of motion of the index, and perpendicular to the plane of the instrument.

THE SEXTANT.

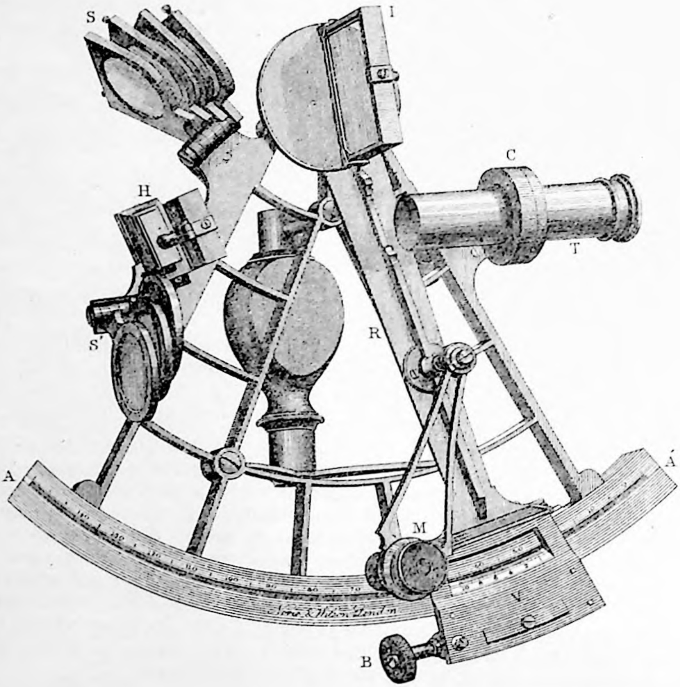


Fig. 1.

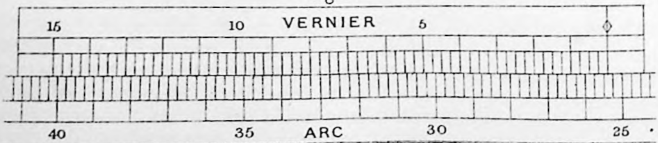
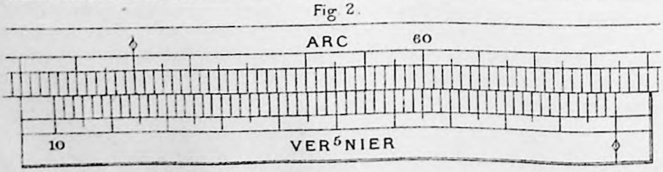


Fig. 2.



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H is the *horizon-glass,* one-half of which is a reflector or mirror, and the other half plain glass to admit of objects being seen through it; it is fixed perpendicularly to the plane of the instrument, and parallel with the index-glass when 0° on the vernier stands at 0° on the arc.

At S are the *index shades,* of coloured glass, to be turned down between the index-glass and horizon-glass, as the eyesight requires, in order to moderate the brightness of an object—as the sun or moon.

At S′ are the *horizon shades,* of coloured glass, to be turned up (as required) beyond the transparent part of the horizon-glass.

T is the *telescope,* to be screwed in the collar C, which is a double ring. When the telescope is absent, the collar is the place of the sight vane.

M is the *microscope* (movable) for reading accurately the divisions of the arc and vernier.

B is a *tangent screw* for giving a small motion to the index after it has been partially fixed by a *clamping screw,* which is at the back of the lower end of the index-bar and under the *vernier* (V).

The *handle* of the sextant, by which it can be held in any position between the vertical and horizontal, is partly shown beneath the index-bar.

The arc (or *limb*).—The limb is the circular part of the frame of the sextant, into which is let the arc (A A′) consisting of a thin piece of platinum, or silver.

The Index-Glass (I) is a plane reflector or mirror of quick-silvered glass, set in a brass frame, and so placed that the face of it is perpendicular to the plane of the instrument, and immediately over the centre of motion of the index-bar. This mirror, being fixed to the index-bar, moves with it, and changes its direction as the direction of the index-bar is changed.

This glass is designed to reflect the image of the sun, or any other object, upon the horizon-glass, whence it is reflected to the eye of the observer. The brass frame of the index-glass is fixed to the index-bar by two screws directly at the back; and a third (*outer*) screw serves to adjust it in a position perpendicular to the plane of the instrument.

Note.—The graduation of the arc may be perfect in itself, but if the index-glass and index-bar are not properly *centered,* to correspond to the arc of the instrument to which they are attached, then you have the *centre of another arc,* and consequently the vernier in passing along the arc of the instrument will give *inconsistent* readings as it is moved to different parts of the limb!

The Horizon-Glass (H) is parallel to the index-glass when 0 on the vernier stands at 0 on the arc; this mirror receives the rays of the object reflected from the index-glass, and transmits them to the observer. The horizon-glass is only silvered on its lower half, the upper half being transparent (or plain), in order that any object may be seen *directly* through it—as, for instance, the horizon or any object towards which the line of sight is directed. The glass is set in a brass frame, standing perpendicularly to the plane of the instrument, and fixed there; on the back of the frame are two screws—one towards the top, and the other near the bottom (or base), of the glass, towards one side, to effect any required adjustments of the horizon-glass.

The Coloured Glasses or Shades (S S′) are used to prevent the rays of the sun, or glare of the moon, from affecting the eye when taking observations. Each glass is set in a brass frame which turns on a centre. Four shades are placed between the index-glass and horizon-glass, to screen the

eye from the brightness of the reflected solar image, or the glare of the moon, and may be used separately, or together, as occasion requires. Three more such glasses are placed behind the horizon-glass to weaken the rays of the sun or moon when they are viewed directly through the horizon-glass. The pale glass is sometimes used in observing altitudes at sea, to take off the strong glare of the horizon.

These glasses, when examined singly, should exhibit no streakiness or flaws, and each glass should show a uniform shade of colour over its whole area ; also each glass should work easily and parallel to its fellows on either side.

The TANGENT SCREW (B) and *Clamping Screw*.—In order to observe with accuracy, and make the images of the observed objects come precisely in contact, an adjusting, or *tangent screw* (B), is attached to the index, by which it may be moved with greater regularity than can be done by hand ; but it must be observed this screw does not act until the index-bar is fixed by the *clamp* or finger *screw*, placed at the back of the lower part of the index-bar. Care should be taken not to force the tangent screw when it arrives at either extremity of its arc. When the index is to be moved any considerable quantity, the clamping screw must be loosened ; but when the index is brought nearly to the division required, this back screw should be slightly tightened, and then the index be moved *gradually* by the tangent screw.

The *vernier* will require a special description presently.

Accuracy of the instrument.—When the *joints* of the framework are close and tight, and the various *screws* fit closely and act well—when the *centering* of the instrument is perfect—when the *graduation* of the *limb* and the *vernier* is accurate in every part—when each of the *reflectors* or *mirrors* has its two faces parallel, and the glass perfectly clear—and when the *shades* have clear glasses, the two faces of each glass parallel, and the set work with all their faces parallel—the instrument may be considered perfect, as regards the optical and mechanical principles of its construction, and without any sensible error but what the adjusting screws can rectify.

Inside the box which holds the sextant, when not in use, are various telescopes and glasses.

TELESCOPES.—The sextant is generally furnished with a plain tube without any glasses—which is merely a sight vane. Also to render objects still more distinct, it has likewise two telescopes ; one to represent objects *erect*, or in their natural position ; the other, a longer one, which shows the objects *inverted ;* but the latter has a larger field of view, and a greater magnifying power, with other advantages : use and experience will soon accustom the observer to the inverted position, and the instrument can then be as readily managed by it as with the plain tube alone. By a telescope the contact of the images is more perfectly distinguished ; and by the place of the images in the field of the telescope it is easy to perceive whether the sextant is held in the proper plane for observation. By sliding the tube that contains the eye-glasses in the inside of the other tube, the object is suited to different eye-sights, and objects made to appear perfectly distinct and well defined.

The telescope is to be screwed into a circular ring or collar (C) : this ring rests on two points, against an exterior ring, and is held thereto by

two screws; by turning one and tightening the other, the axis of the telescope may be set parallel to the plane of the sextant. The exterior ring is fixed on a brass stem that slides into a socket; and by means of the screw at the back of the sextant, it may be raised or lowered so as to move the centre of the telescope to point to that part of the horizon-glass which shall be judged the most fit for observation.

A circular eye-piece, with coloured glasses, accompanies the sextant and is to be screwed on the eye-end of the tube, or on that of either telescope.

To these appendages are added a small screw-driver, to adjust the screws; a magnifying glass, to read off the observation with greater accuracy; and a microscope, for the same purpose, made to fit into a tube fixed at the lower end of the index-bar.

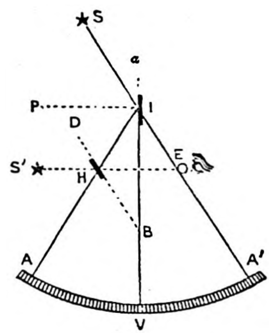
PRINCIPLE OF THE SEXTANT

The principle on which the sextant or quadrant and other reflecting instruments depend is connected with certain principles of *optics*, and the laws relating thereto—

(1) *The angle of incidence and angle of reflection are equal;* that is, if a ray of light is *incident* (or falls) upon a plane reflecting surface, and is thence received by the eye, or passes into space, the *incident* ray makes, with the perpendicular to the reflecting surface, an angle equal to the angle made with the same perpendicular and the *reflected* ray. Again—

(2) *If a ray of light suffers two successive reflections in the same plane by two plane mirrors, the angle between the first and last directions of the ray is equal to twice the inclination of, or angle between, the reflecting surfaces of the mirrors.*

In the annexed fig. which represents the outline of the sextant, the two mirrors are: I the index-glass, and H the horizon-glass; when the index-bar I V lies in one with I A′ the two mirrors are parallel. Move the index-bar I V to the middle of the arc: let a ray of light coming from the sun S fall on the index-glass I, at a certain angle, and thence be reflected at the same angle to the half-silvered part of the horizon-glass H, whence it is again reflected along the line H E to the telescope, or observer's eye at E. The telescope, or eye-piece, is always so placed that the lines E H and I H make equal angles with the direction of the mirror H; then, by the law of reflection, a ray falling on the mirror H, in the direction I H, is always reflected in the direction H E. The observer looking along the line E H will see, through the transparent part of



the horizon-glass, the sea-horizon, and also at the same time the image of the sun in the silvered part of the horizon-glass.

The angle S E H is the angular height of the sun, which is its *altitude;* and this angle is twice the angle of the mirrors, that is twice I B H or twice A′ I V. The arc A′ V which measures this angle is then the measure of one-half the altitude, or angular distance of S from the sea-horizon. The vernier at V indicates the *exact value* of the arc; but in order to avoid

the necessity of doubling this value after reading, a half degree of the arc is numbered as a whole degree: thus an arc of 60° (as on the sextant) is divided into 120 equal parts, each of which is reckoned as a degree.

PROOF OF THE PRINCIPLE OF THE SEXTANT

Let I A′ be the position of the index bar when the mirrors are parallel to each other, and I V the position when moved to V. It is required to prove that the angle S E S′ is double the angle V I A′ or B I E.

Draw P I perpendicular to the index glass I, bisecting the ∠ S I H, and draw B H parallel to A′ I. Now ∠ S I *a* = ∠ H I B because they are each the complement of the equal angles S I P and P I H, also ∠ S I *a* = ∠ B I E (Euc. I. 15), therefore ∠ H I E = twice the ∠ B I E or twice the angle of the inclination of the mirrors.

Now the ∠ S′ H I = ∠ H I E + ∠ H E I (Euc. I. 32); produce B H to D, bisecting the ∠ S′ H I. Now ∠ D H S′ = ∠ H E I, and the remaining ∠ D H I = ∠ H I E, therefore the ∠ H I E = ∠ H E I, and as ∠ H I E = 2 B I E, it is proved that ∠ S E S′, the altitude, is equal to twice the inclination of the mirrors to each other.

TO READ THE SEXTANT

The VERNIER (*see* Plate VIII. Figs. 1 and 2) is a small scale attached to the lower part of the index-bar of the instrument; it is slightly inclined to the face of the limb, and moves, with the index-bar, in close contact with the graduated arc A A′: by means of the *vernier* we are enabled to read off aliquot parts of the smallest spaces into which the arc of the instrument is divided.

To read off the sextant by means of the vernier; look at an instrument while you read the description: first, note that the starting-point of the vernier, on the right, is sometimes 0, and sometimes an arrow head or other device. The limb (arc) of the instrument is divided (graduated) to degrees, and parts of a degree; the degrees are indicated at intervals by numerals, the intermediate long strokes are also degrees, and the shortest divisional strokes are parts (*i.e.*, a certain number of minutes) of a degree. You will also see long and short strokes on the vernier—the long ones are minutes of a degree, and the shorter ones a certain number of seconds of a minute. Both are read from right towards left.

If the degrees on the arc are divided into three equal parts then each stroke equals 20′, and each stroke on the vernier equals 20″; when the degrees on the arc are divided into four equal parts, each stroke equals 15′ and each stroke on the vernier equals 15″, and when the degrees on the arc are divided into six equal parts each stroke equals 10′ and each stroke on the vernier equals 10″.

The principle on which the vernier is graduated is as follows: Suppose a vernier is desired for a foot rule; eleven inches would be the length of the vernier, which would then be divided into twelve parts, so with the sextant, 60 divisions on the vernier correspond to 59 on the arc. Thus—

$$60\,v = 59\,a$$

$$\therefore \quad 1\,v = \tfrac{59\,a}{60\,v} = 1 - \tfrac{1}{60}$$

that is, one division on the vernier is $\frac{1}{60}$ of 10′ = 10″ shorter than one on the arc.

Read the arc or limb to minutes of a degree, and when you understand

this you will soon be able to read to seconds. *First*, examine well the sequence of degrees and minutes.

Take a sextant divided to 10′; then, on any part of the arc, the first short stroke is 10′, the second 20′, the third 30′, the fourth 40′, and the fifth 50′; if the 0 of the vernier exactly coincides with a long stroke of the arc, the reading is degrees, and *no minutes*; it may be 10, 14, 20, etc.—any number. Now put 0 of the vernier to coincide exactly with the fifth short stroke to the left of 20° on the arc, and the reading will be 20° 50′, since each short stroke of the five beyond 20° represents 10′. Lastly, fix your eye on the space between 42° and 43°; now put 0 of the vernier to stand somewhere between the third and fourth short strokes to the left of 42°; in the first place the reading will be 42° 30′, but it must be something more because the vernier indicates minutes between 30′ and 40′; now look along the line of the vernier and see which minute stroke on it coincides with any stroke on the arc of the sextant; let us say that it is the seventh minute stroke; then the reading will be 42° 37′. In Plate VIII. Fig. 2, the 0 on the vernier is somewhat more than three divisions, or 30′, to the left of 56°; and the division on the vernier, coinciding with one on the arc, is 5′ 20″, therefore the angle pointed out by the index division is 56° 35′ 20″.

The arc of excess.—Thus far we have been reading *on* the arc; next learn to read *off* the arc, that is, on *the arc of excess* to the right of 0 on the arc: the subdivisions are the same as on the arc, but now you read from left to right; the vernier is also read in the same way; and on the sextant divided to 10′ the 10 of the vernier reads as 0, then 9 will be 1, and 8 will be 2, &c.; say the 0 of the vernier stands between the third and fourth strokes to the right of 0 on the arc, and the fourth minute stroke of the vernier coincides with a stroke on the arc, then the reading on the arc of excess, that is *off*, will be 36′, since the fourth stroke is 6 when reckoned from the left of the vernier.

ADJUSTMENTS OF THE SEXTANT

The theory of the sextant as a reflecting instrument requires the following conditions, viz.—

(*a*) The two surfaces of each mirror and the shade glasses must be parallel planes;

(*b*) The graduated arc, or limb, should be a plane, and with its vernier accurately divided;

(*c*) The axis should be at the centre of the limb, and perpendicular to its plane;

(*d*) The index-glass and horizon-glass should be perpendicular, and the line of sight parallel, to the plane of the limb.

(*a*), (*b*), and (*c*) are carefully attended to by the maker; they admit of being tested and any deviation found, but only by well-devised observations. The complete theory provides formulæ for ascertaining their defects and the corrections of such errors; (*d*) indicates the principal adjustments to be made by the observer.

To adjust a sextant is to set the index-glass and horizon-glass perpendicular to the plane of the instrument, and their planes parallel to each other when the index-division is at 0 on the arc; also, to set the axis of the telescope parallel to the plane of the instrument; each of these particulars must be examined before an observation is taken, and the adjustments, if requisite, made according to the following directions.

I. *To set the Index-Glass perpendicular to the Plane of the Instrument*

Place the index-bar near the middle of the arc, and holding the instrument in a horizontal position, with its plane upwards, the index-glass close to the eye, and the arc away from you, look obliquely into the glass in such a manner that you may see the arc by direct view, and by reflection, at the same time; if the arc seen by reflection forms an exact plane with the arc seen by direct view, the glass is perpendicular to the plane of the instrument, and it is in adjustment; but if the reflected part of the arc appears lower than the true arc, tighten the adjusting (outer) screw at the back of the frame; if higher, slacken it.

II. *To set the Horizon-Glass perpendicular to the Plane of the Instrument*

Screw the plain tube, or the common telescope, into the collar; set 0 on the vernier to 0 on the arc; and, holding the instrument horizontally, look through the telescope and the horizon glass at the sea-horizon, and observe if the reflected and true horizons appear in one line; if they are, the horizon-glass is perpendicular to the plane of the instrument, and in adjustment; otherwise, turn the uppermost screw at the back of the instrument till they perfectly coincide.

This adjustment may also be made by directing the view through the telescope to the sun, 0 on the vernier coinciding with 0 on the arc; hold the instrument perpendicularly and direct the telescope to the object; move the index-bar so that the reflected image shall pass over the direct object; if the reflected image be to the right or left of the direct object, turn the screw (as before) till they coincide with each other, when the glass will be perpendicular to the plane of the instrument. If the adjustment be made by a star, move the index backwards and forwards slowly, and observe if the reflected image, in passing the star, coincides with it.

When this or the following adjustment is made by observing the sun, the inverting telescope is always to be used, and one or more of the shades, both before and behind the horizon-glass, are to be turned up, in order to screen the eye from the bright solar rays proceeding from the direct and reflected suns, which are to be made, by means of the shades, to appear as nearly as possible of the same degree of brightness.

III. *To set the Horizon-Glass parallel to the Index-Glass, when the Index Division is at 0 on the Arc*

Make the index division of 0 on the vernier to coincide exactly with 0 on the arc; and, in order to make the coincidence as perfect as possible, examine them through the magnifying glass, or microscope, and fix the index by the clamp under it; screw on the telescope. Having done this, hold the sextant perpendicularly, and direct the sight through the telescope to the horizon; then, if the true horizon seen through the clear part of the horizon-glass appears in a line with the reflected horizon on the silvered part, the horizon-glass and index-glass are parallel. But if the reflected and true horizons do not coincide, turn the lower screw at the back of the horizon-glass till they are made to appear in the same straight line, then will the planes of the horizon-glass and index-glass be parallel.

IV. *To set the Axis of the Telescope, when screwed into the collar, parallel to the Plane of the Instrument (Error of Collimation)*

In measuring angular distances, the line of sight, or axis of the telescope, should be parallel to the plane of the instrument, as a deviation in that respect may occasion a considerable error in the observation, and this is most sensible in large angles; to determine the error use the telescope in which are placed two wires parallel to each other, and equidistant from the centre, to which are generally added two others at right angles to these, and parallel to each other. By means of these wires the adjustment may be made thus: screw on the telescope, and turn the tube containing the eye-glass till two of the wires are parallel to the plane of the instrument; then take two objects, as the sun and moon, or the moon and a star, or two stars, whose angular distance must not be less than 90 to 100 degrees, because the error is more easily discovered when the distance is great: bring them exactly into contact at the wire which is nearest to the plane of the sextant, and fix the index; then by altering a little the position of the instrument, make the objects appear on the other wire. If the contact still remains perfect, the axis of the telescope is in its right situation; but if the limbs of the two objects appear to separate at the wire that is farthest from the plane of the instrument, it shews that the object-end of the telescope inclines towards the plane of the instrument, which must be rectified by tightening the screw of that part of the collar nearest to the sextant, having previously slackened the screw farthest away. If the images overlap each other at the wire farthest from the sextant, the object end of the telescope is inclined from the plane of the instrument, and therefore adjust in a contrary manner. By repeating this observation a few times, the contact will be precisely the same at both wires, and consequently the axis of the telescope will be parallel to the plane of the instrument, or the line of collimation will be correct.

The inverting telescope is a lunar instrument. It is now of no particular use except for taking altitudes in connection with the artificial horizon. It is of little use for altitudes at sea, since we are never sure of the horizon to half a minute owing to abnormal refraction.

To find the Index Error

The *index error* is the number of minutes and seconds pointed out by the vernier, when the direct object and its reflected image coincide with each other, and may be found (1) by the sea-horizon, or (2) with greater accuracy by the sun.

By the horizon.—Set 0 on the vernier to 0 on the arc, hold the sextant vertically, and look at the horizon through the horizon-glass: if the horizon and its image are not in one line, move the tangent screw till they are so: the reading is the error.

The reading may be on the arc proper, or on the arc of excess: it is on the latter when 0 on the vernier is to the right of 0 on the arc, and the reading is then said to be *off* the arc; when 0 on the vernier is to the left of 0 on the arc, the reading is said to be *on* the arc. The error is really the difference between the position of 0 on the arc and 0 on the vernier.

? measure diameter of Sun

If o on the vernier falls to the right of o on the arc, *i.e.*, *off* the arc proper but on the arc of excess, every measurement will be too small, therefore index error is additive: if o on the vernier falls to the left of o on the arc, *i.e.*, *on* the arc proper, every measurement will be too great, therefore the index error is subtractive.

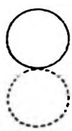
This method of finding the index error is not one of great precision.

By the sun.—Having screwed the inverting telescope into its place, adjusted the eye-tube to distinct vision, and turned up the proper shades, ? place o on the vernier about 30′ to the *right* of o on the arc, and tighten the clamp under the index of the sextant; then, holding the instrument perpendicularly, bring the direct and reflected suns in exact contact by the tangent screw, and read off the minutes and seconds pointed out by the vernier on the *arc of excess*, which note down, and call it *off*; next place o of the vernier about 30′ to the *left* of o on the arc, and make the contact of the two suns correct, as before; read the minutes and seconds indicated by the vernier on the arc proper, which call *on*, and set them under the first reading; then half the difference between the two readings will be the index error, which is *additive* to all angles taken with the sextant, when the greater reading is on the *arc of excess*, and *subtractive* when the greater reading is on the *arc proper*. The direct and reflected suns will appear through the inverting telescope thus—

Appearance on the *arc of excess*.

Direct (real) sun...

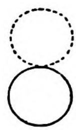
Reflected sun.... .



Appearance on the *arc proper*.

Reflected sun........

Direct (real) sun....



If the following observations had been taken to determine the index error, mark *off* with a *positive* sign, and *on* with a *negative* sign.

Example I.

Off	+	31′ 45″
On	—	33 0
		2)1 15
	—	0 37

Index error 37″ *subtractive*, because the arc to the left, or *on*, is greater than the arc to the right, or *off*.

Example II.

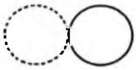
Off	+	32′ 10″
On	—	31 20
		2)0 50
	+	0 25

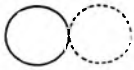
Index error 25″ *additive*, because the arc to the right, or *off*, is greater than the arc to the left, or on.

To prove that the contacts were made correctly, add the arcs together and divide their sum by 4; the quotient should then be equal to the sun's semi-diameter, as given in page II. of the given month in the Nautical Almanac. Thus, suppose the observations in Example I. were made on February 26th, 1888: here the sum of the arcs is 64′ 45″, the fourth part of which is 16′ 11″, agreeing nearly with the sun's semi-diameter (16′ 11″·1) as

given on that day in the Nautical Almanac; it may therefore be presumed that the contacts were correctly made.

NOTE.—If the altitude of the sun should be less than about 20° at the time of taking the above observations, the sun's horizontal, instead of the perpendicular, diameter should be measured; for as refraction affects the lower limb more than the upper, it occasions the perpendicular diameter to be less than the horizontal, which is that given in the Nautical Almanac: in this case the sextant is to be held horizontally, with the face upwards, and the reflected sun brought into contact alternately with the right and left limbs of the direct (or real) sun, as before explained; the contacts will then appear thus—

Reflected sun.  Direct sun.

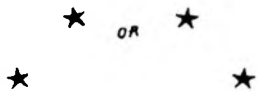
Direct sun.  Reflected sun.

By a star.—Set the sextant exactly at 0. Select a moderately bright star, look at it through the telescope and horizon-glass in the usual way, holding the sextant vertically. If the reflected and real stars coincide there is no index error and the sextant is in adjustment.

If the horizon-glass is not perpendicular to the plane of the sextant the reflected star will appear to one side thus—

If the horizon-glass and the index-glass are not parallel to one another the reflected star will appear thus—

If both adjustments are out they will appear thus—



While still holding the sextant vertically slip the small lever found in the box into one of the holes in the adjusting screws and turn the screws, first one and then the other, until the reflected and the real stars coincide.

USE OF THE SEXTANT

To observe Altitudes at Sea, by the Sea-Horizon

The altitude of any object is measured by the position of 0 on the vernier, when, by reflection, the object appears to be on the sea horizon: the face of the observer is directed towards that part of the horizon immediately under the object, and the instrument must be held perpendicular to the horizon so that its plane produced would pass through the object.

TO TAKE THE SUN'S ALTITUDE.—Set the index at 0; fix what shades you require; direct the eye through the sight vane and horizon-glass to that part of the horizon exactly under the sun; move the index from you, along the arc, and the image of the sun will appear to descend towards the horizon; when the sun's lower limb touches the horizon with accuracy

in the centre of the field, stop, and read off the altitude on the arc of the instrument.

Or having proceeded as just described, and having taken a rough or approximate altitude, you can clamp the index, screw in the telescope, and use the tangent screw to get a more perfect observation when the sun's lower limb just touches the horizon.

FOR AN ALTITUDE OF A STAR.—In order to be sure that you have the right star: when the index is at 0, direct the sight to it, and bring it down gradually to the horizon by moving the index from you.

Both for Sun and Star it may be better to give the instrument a slight vibratory motion to right and left, to be sure that you have measured a part of a *vertical* circle.

The *meridian altitude* will have to be watched for some minutes, and the use of the tangent screw is indispensable; the altitude rises until the object is on the meridian, then seems to be stationary for the moment, and subsequently descends (dips).

For the *moon's altitude* observe the *enlightened limb*; it may be the upper or lower limb according to her position.

The Artificial Horizon

When altitudes are to be taken on shore with a sextant, where the observer has not the advantage of the sea-horizon, he is obliged to have recourse to an *artificial horizon*, which is a horizontal plane with a smooth or polished surface, on which the rays of the sun or other object falling are reflected back to an eye placed in a proper position to receive them; the angle between the real and reflected objects being then measured with a sextant, will be double the altitude above the horizontal plane.

Such a horizontal plane may be obtained by pouring a quantity of oil, tar, treacle, or other fluid and viscous substance, into a shallow vessel; and to prevent the wind giving a tremulous motion to its surface, a piece of thin gauze, muslin, or plate-glass, whose surface is perfectly plane and parallel, may be placed over it when used for observation.

An artificial horizon sometimes consists of a plane mirror, fixed in a brass frame, standing upon three adjusting screws, by which its surface may be made horizontal with the assistance of a spirit-level placed on its surface in various positions; observing that the screws be turned until the air-bubble always rests in the middle of the tube. The under surface of the plate of glass is sometimes unpolished and blackened, so that the image of the sun can only be reflected from the upper surface, which would be carefully polished, and be a perfect plane; by this means the errors that might arise from a defect of parallelism in the two surfaces are avoided.

But the *best* and most approved kind of *artificial horizon* is that produced by quicksilver, which being poured into a small shallow trough will always preserve an exact horizontal plane at its surface: over this is placed a roof to protect the quicksilver from the action of the wind; in the roof are fixed two plates of glass, the two sides of each being ground perfectly plane and parallel. These are usually packed in a mahogany box, with a vessel containing a quantity of quicksilver, ready for use when wanted.

Observations by means of the artificial horizon should be made at a spot sheltered from the wind and as quiet as possible. The mercury should be clean and pure ; and for a clear, brilliant surface, before pouring the mercury from the bottle, " place the finger over the orifice, and give the bottle a shake in an inverted position, holding it over the trough previously cleaned: ease the finger, and allow the mercury to flow gently, keeping the bottle inverted, and taking care to stop the opening of the bottle before the last portion, with the dross, flows out."

When the mercury is pure, its surface clean, and the glass of the roof without flaw, trustworthy observations may be made. Errors arising from the glass roof may be partly eliminated by reversing the cover between each pair of observations.

When one of these instruments is used, the observer is to place himself at a convenient distance, in such direction that he may see the object reflected from the artificial horizon as well as the real object ; then, having screwed on the telescope of the sextant (using no shades, but only a dark glass at the eye-piece), the upper or lower limb of the sun's image, reflected from the index-glass, is to be brought into contact with the opposite limb of the image reflected from the artificial horizon, observing that when the inverted telescope is used the upper limb will appear as the lower, and *vice versa ;* the angle on the instrument being then read off, and the index error applied to it, will give double the altitude of the limb above the horizontal plane.

NOTE.—It will perhaps be more easy to the observer if he first brings the images of the sun nearly into contact by the naked eye, and afterwards screws on the telescope, and makes the contact perfect by the tangent-screw.

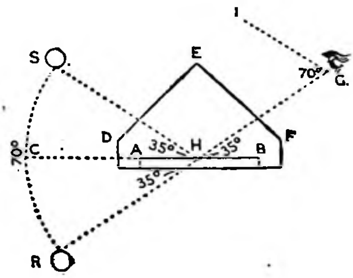
It is usual to observe three, five, or seven angles, and take the *mean* by dividing by 3, 5, or 7, as the case may be. Then, to the mean of the instrumental measurements apply the index error, if any, and the result will be double the apparent altitude : therefore dividing by 2 will give the correct apparent altitude of the limb observed.

When observing the moon, place a green shade in front of the horizon-glass ; you require no dark glasses.

It is always difficult to observe a star by the artificial horizon.

No altitudes less than 15°, or much beyond 60°, can be observed by the artificial horizon.

The following diagram will illustrate the method of observing altitudes with an artificial horizon.



Let A B represent the horizontal surface of the mercury contained in a shallow trough, whose plane is continued to C ; D E F is the roof, in which are fixed two plates of glass, D E and E F ; and O is the sun at S, whose altitude is required. Now the ray S H, proceeding from the sun's lower limb to the surface of the mercury, will be reflected thence to the eye, in the direction H G, and the upper limb of the sun's image, reflected from the mercury, will appear in the line H G, as if it proceeded from a point R, whose

angular depression A H R, below the horizontal plane, is equal to the altitude, A H S, of the object above that plane. If, then, I G is a direct ray from the object parallel to S H, an observer at G can measure with the sextant the angle I G R=S H R=twice S H C (or 2 S H A) by bringing the image of the object reflected by the index-glass into coincidence with the image R reflected by the mercury and seen through the horizon-glass. The instrumental measure corrected for index error will be double the apparent altitude. Thus, if we suppose the angle S H R, measured by a sextant, to be 70°, the altitude of the sun's lower limb will be 35°.

Example.—Suppose the observed angles between the lower limb of the sun, reflected from the index-glass of the sextant, and the upper limb reflected from the artificial horizon on shore, to be as follow; the index error of the sextant being 2′ 20″ to add; required the apparent altitude of the sun's lower limb.

Observed angles of sun's lower limb	99°	20′	15″
,, ,, ,, ...	,,	25	30
,, ,, ,, ...	,,	30	0
,, ,, ,, ...	,,	34	45
,, ,, ,, ...	,,	39	20
	5)	149	50
	99	29	58
Index error......................	+	2	20
	2)99	32	18
Apparent altitude of sun's lower limb	49	46	9

No correction is required for the height of the eye (dip), as in observations made by the sea horizon; but the other corrections, as usual (*see* "Correction of Altitudes").

Example.—Suppose the following angles of the sun's lower limb were observed by means of an artificial horizon; the index error of the sextant being 1′ 50″ to subtract; find the apparent altitude of the sun's lower limb.

Observed angles	101°	52′	40″
,, ,, ...	101	58	40
,, ,, ...	102	4	10

Ans. 50° 58′ 20″

CONCLUDING OBSERVATIONS AND NOTES

An artificial horizon fitted to the sextant, if it gave accurate results, would have inestimable value; it has been the hope of many an observer and inventor to produce such an instrument; but all the attempts hitherto made in that direction, if they have not been total failures, have yielded no satisfactory results, inasmuch as strict accuracy under all circumstances was not a certainty.

When the altitude of a body is more than 60°, it may be observed from the opposite point of the horizon as well as from the point of the horizon immediately beneath it.

See that the tangent screw has plenty of run onward or backward, as the case may be.

When two screws work against each other, be sure when tightening one to loosen the other, if necessary.

When reading off the sextant, whether at night or by day, do not hold the instrument *sideways* to the light, but take care that the light comes *directly* along the index-bar to the vernier; neglect of this may cause an error of one or two minutes of arc.

When using the telescope, close the eye not required for vision; with the tube it is sometimes preferable to keep both eyes open, because the image being seen by both eyes under the same magnitude one assists the other.

The shades, if they are of doubtful character, are not always necessary. For the sun, by using only a dark glass at the eye-piece of the telescope, correction for shade error is avoided.

What is called a *star telescope* is now very often fitted to a sextant, and is strongly recommended by those who take altitudes of stars to determine the ship's position; by its aid the horizon on a dark night can generally be discerned, though otherwise not to be distinguished.

In the tropics the great heat of the sun will certainly affect the instrument if it be kept too long exposed.

Before putting your sextant away in its box, wipe the glasses with a piece of soft chamois leather: do not use a pocket-handkerchief or rag for this purpose. Be careful not to use much pressure, otherwise the adjustments may be disturbed. Moisture allowed to remain on the mirrors will soon impair the silvering.

The *vernier* was invented by Pierre Vernier in 1631; but it differed slightly from that now in use: the term *nonius* for this invention is quite a mistake, as it is only applicable to another kind of subdivision.

If the navigator is desirous of possessing a good and reliable instrument he should send it to Kew Observatory, where all its defects will be ascertained, and errors given, for a small fee; when purchasing a new, first-class sextant, ask the optician for the Kew certificate.

Principle: A Column of Mercury about 30 inches in length is balanced by a column of air on the same sectional area

THE BAROMETER

The barometer is an instrument with which to measure the variations in the weight or pressure of the atmosphere. The principle of the mercury barometer was discovered by Torricelli, a pupil of Galileo, in 1643.

A mercury barometer consists of a glass tube (*see* Fig. 1) about 33 inches long, closed at one end and filled with pure mercury, which has been boiled to get rid of all air bubbles; the tube is then inverted and its open end immersed in a cistern of mercury (D). This cistern has a small hole (H) in it covered with a leather washer on the inside to prevent the mercury escaping from the cistern, and it is through this small hole that the atmosphere exerts its pressure on the mercury in the cistern and causes the mercury in the tube to rise and fall as the atmospheric pressure gets greater or less. The pressure of the atmosphere is about 14·7 pounds per square inch at sea level at the temperature of 32° F.

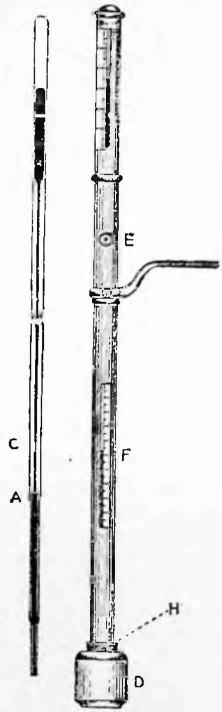


Fig. 1.

The latest pattern barometers are graduated in inches and decimals of an inch on one side and in centibars and decimals of a centibar on the other side. A vernier, the principle of which is thoroughly explained in the chapter on the sextant, is fitted to the scale to facilitate the reading of small variations of pressure. The vernier is moved up and down by a rack and pinion, which is operated by a milled screw as shown in Fig. 1 (E). It is only necessary to graduate a sufficient length of the tube at the top to measure all the variations in the atmospheric pressure, which varies from about 27·5 inches to a little over 31 inches.

In a marine barometer the tube is very much contracted for the greater part of its length, and at one portion (C) it is of capillary dimensions, in order to prevent what is technically known as "pumping," which is caused by the labouring of a vessel when in a heavy sea, and also to lessen the weight the tube has to support.

The tube is also fitted with an air trap (A) which consists of a small funnel, or pipette, whose function is to prevent air from getting into the tube above the mercury, which space should be a perfect vacuum. If any air should find its way into the tube above the mercury, the higher the reading the greater the error from this cause, because the pressure of a gas is inversely proportional to its volume.

A thermometer (Fig. 1, F) is also attached to the barometer and should

Specific gravity of Mercury = 13.7

always be noted when recording readings for the purpose of reducing them to standard temperature. This attached thermometer shows absolute temperature and the graduations are centigrade.

Fig. 5 is an enlarged drawing of the attached thermometer shown at F in Fig. 1. It is graduated on the left-hand side in Centigrade divisions, and reckoned from the absolute zero; on the right-hand side it is graduated in accordance with the Fahrenheit scale, whose freezing-point is + 32°. It will be observed that 273° A. are coincident with + 32° F., hence it follows that the freezing-point on the absolute scale is 273°. As 5° C. are equal to 9° F., it follows that every 5° on the absolute thermometer above and below the freezing-point must be coincident with every 9° on the Fahrenheit thermometer. Thus, it will be found that 278°, 283°, 288°, and 293° A. are coincident with + 41°, + 50°, + 59°, and + 68° F. respectively, and 268°, 263°, 258°, and 253° A. are coincident with + 23°, + 14°, + 5°, and — 4° F. The spaces between the divisions on any thermometer are equal, because equal increments of heat give equal increments in the length of the thread of mercury.

The marine barometer is so constructed as to obviate any correction for capillarity which tends to depress the mercury in the tube, or for the ever varying height of the mercury in the cistern caused by the mercury rising and falling in the tube as the pressure gets greater or less. There are, however, other corrections which cannot be eliminated in the construction, such as changes of temperature, changes of level, and change of latitude, each of which will now be explained.

Temperature correction.—As a column of mercury lengthens when heated and shortens when cooled it is necessary to apply a correction for temperature to show what the readings would have been at 32° F., which is the standard temperature to which all barometer readings are reduced for purposes of comparison.

CORRECTIONS FOR REDUCING READINGS BY MERCURY BAROMETER.

To Temperature of 273° Abs.		To Sea Level.		To Standard Gravity at Latitude 45° N. or S.					
Temp. by Att. Ther. Abs.	Correction.	Height in feet.	Correction.	Lat. N. or S.	Correction. At 27 ins.	Correction. At 30 ins.	Lat. N. or S.	Correction. At 27 ins.	Correction. At 30 ins.
°	in. —	ft.	in. +	°	in. —	in. —	°	in. +	in. +
272	·00	10	·01	0	·07	·08	90	·07	·08
275	·02	20	·02	10	·07	·07	80	·07	·07
277	·03	30	·03	20	·05	·06	70	·05	·06
283	·06	40	·04	25	·05	·05	65	·05	·05
289	·09	50	·05	30	·04	·04	60	·04	·04
294	·11	60	·07	35	·02	·03	55	·02	·03
300	·14	70	·08	40	·01	·01	50	·01	·01
305	·16	80	·09	45	·00	·00	45	·00	·00

The correction is to be added when the sign +, and subtracted when the sign — is at the head of the column.

Boiling Pt Mercury 675 F.
Freezing Pt " -40 C -40 F

Height correction.—As the pressure of the air is reduced as we rise above sea level it follows that there will always be a correction to be applied whenever the cistern is above sea level. The correction from this cause amounts to about ·001 inch for each foot above sea level and is always additive.

Gravity correction.—The earth being a spheroid flattened at the poles, the polar radius is less than the equatorial radius, and as the force of gravitation varies inversely as the square of the distance from the centre of mass and also as centrifugal force is directly opposed to gravity at the equator and vanishes at the poles, it follows that a column of mercury is shorter at the poles than it would be at the equator; and as it is necessary for the sake of precision in comparison that some latitude shall be agreed upon as a standard for the measurement of weight, the latitude chosen is 45°.

From what has been said it is clear that a column of mercury between the equator and 45° N. or S. is longer than it would be at the standard latitude, and in order to reduce it to "standard" a minus correction would have to be applied, whereas a column of mercury between 45° N. or S. and the pole is shorter than at the standard latitude, and the correction to reduce it to "standard" would be plus.

SETTING AND READING A BAROMETER

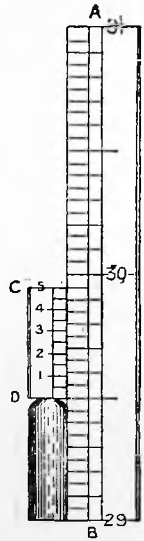


Fig. 2.

When setting the vernier and reading the barometer great care should be taken to have the eye on the same level as the top of the mercury, and the tube must be allowed to hang vertically in the gimbals and not held in the hand while setting the vernier, because any inclination of the tube causes the mercury to rise. If the barometer is pumping at the time of observation the vernier should be set for reading when the mercury has completed its downward movement. It should be borne in mind that it is the mercury in the tube that rises and falls with the motion of the ship, and not the mercury in the cistern.

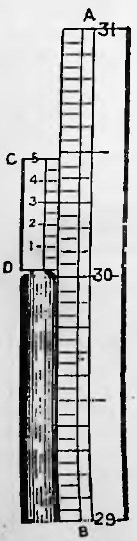


Fig. 3.

In Fig. 2, the bottom of the vernier (D) having been brought into coincidence with the top of the mercury, the scale line 29·50 exactly coincides with the zero of the vernier, and whenever this is the case the uppermost graduation (C) on the vernier should always coincide with a scale graduation as shown.

In this example the reading is 29·5, or 29·50, or 29·500 inches.

In Fig. 3 the bottom of the vernier (D) is above 30·00

and below 30·05; it is therefore obvious that the reading lies between 30·00 inches and 30·05 inches. Looking carefully at the graduations on the vernier it is seen that the figure 3 nearly coincides with a scale graduation; the figure 3 indicates ·03; on a closer examination it is seen that the graduation on the vernier half-way between 3 and 4 is in exact agreement with a scale reading; this graduation represents ·005, so that D is ·035, or thirty-five thousandths of an inch above the scale reading next below D.

The reading can be put down in the following form—

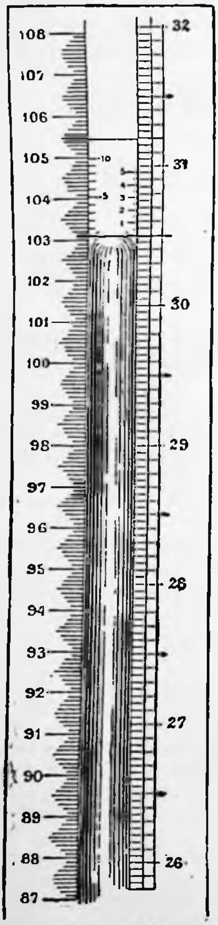
Reading on scale in inches	30·000
Reading on vernier in hundredths of an inch....	·030
Reading on vernier in thousandths of an inch..	·005
Actual reading of barometer	30 035

or 30 inches point thirty-five thousandths of an inch.

note

Fig. 4 shows the graduations of a Kew pattern barometer; it is graduated on one side in centibars with millibar divisions, and on the other in inches with twentieths of an inch divisions. This drawing is an exact reproduction of a barometer of the latest pattern, and shows the graduations exactly as they appear on the instrument; the equivalent number of millibars corresponding to any reading of inches and tenths being the same as shown by the instrument, but it should be observed that this does not agree with the Table of Equivalents of inches and millibars as shown on page 9 of the Seaman's Handbook of Meteorology. The barometer in this figure is reading at 30·5 inches, which corresponds to 1,030·8 millibars. The Standard Temperature stamped on the instrument is the temperature at which a reading of 1,000 millibars of pressure would be correct at sea level in lat. 45° N. or S. It is, therefore, the "Fiducial Temperature" for that instrument in lat. 45°, and the cistern of the barometer at sea level. Should the barometer not be at sea level a new "Fiducial Temperature" will have to be found by adding 1° for every five feet, or 1·5 meters, that the cistern is above sea level, to the temperature stamped on the instrument, the result will be the Fiducial Temperature for that height in lat. 45° N. or S., when the pressure is 1,000 millibars. The Fiducial Temperature is different in different latitudes, and it is recommended to compile, by means of Table I., a table of "Fiducial Temperatures" from the Equator to 75° N. or S., making due allowance for the height of the cistern above sea level. To correct a reading the observer has then only to consider the difference between the "Actual Temperature," as shown by the attached thermometer (Fig. 5), and the "Fiducial Temperature" shown by the table for that particular latitude The correction for this difference of temperature amounts to ·1 millibar for every ·6 of a degree of temperature that the Actual differs from the Fiducial, or 1 millibar for every 6° of difference; when the attached thermometer reads higher than the Fiducial Temperature the correction is subtractive, and when the attached thermometer reads lower than the Fiducial Temperature, the correction is additive.

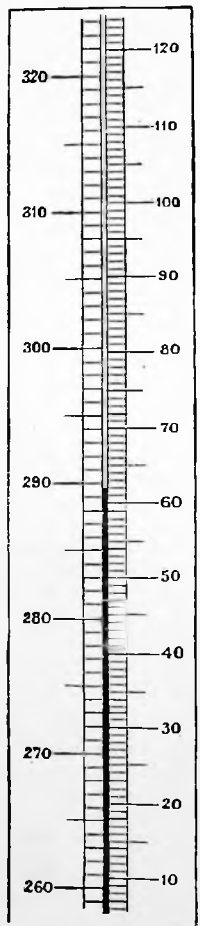
GRADUATIONS OF A KEW PATTERN BAROMETER AND THERMOMETER



Centibars, with millibar divisions.

Inches, with $\frac{1}{20}$-inch divisions.

Fig. 4.



Centigrade from the Absolute Zero.

Fahrenheit.

Fig. 5.

?

SUPPLEMENTARY CORRECTIONS FOR SPECIAL ACCURACY

The correction in Table II. is in reality a fractional part of the pressure, and ought to be adjusted proportionally for different points in the range of atmospheric pressure. The adjustment is made by adding 1 per cent. to the quantity taken from Table II. for each centibar above 100, and subtracting 1 per cent. for each centibar the barometer reading is below 100 centibars.

One per cent. only begins to be appreciable when the correction from Table II. is about 10 millibars and, except on rare occasions, may be neglected.

Correction for Scale Error

This can be provided for by the table of Kew corrections, which gives the Standard Temperature at different points of the scale. A properly graduated scale should have the same Standard Temperature throughout its range. If correction for Standard Temperature in different parts of the scale be necessary, it can be found from Table II.

Example of scale correction.

Barometer M. has Standard Temperature 286° A. at 1,000 millibars, and 280° A. at 900 millibars. Find the scale correction when the reading is 920 millibars.

The Standard Temperature at 920 millibars would be 281° A. or 5° below standard conditions, and is equivalent to reducing the " Fiducial Temperature " by 5°, which, in Table II. A., gives ·8 millibar to be subtracted from the reading. ·6

TABLE I.

Latitude	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
Subtract Degrees A.	16	16	15	14	12	10	8	6	3	0
Latitude	90°	85°	80°	75°	70°	65°	60°	55°	50°	45°
Add Degrees A.	16	16	15	14	12	10	8	6	3	0

TABLE II.

(A.) Actual Temperature above the Fiducial Temperature.

Actual Temperature — Fiducial Temperature.	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
Subtract	·2	·3	·5	·7	·8	1·0	1·2	1·3	1·5	1·7 mb.

(B.) Actual Temperature below the Fiducial Temperature.

Fiducial Temperature — Actual Temperature.	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
Add	·2	·3	·5	·7	·8	1·0	1·2	1·3	1·5	1·7 mb.

Relation of millibars to inches. (*See* Fig. 4).—The units on the absolute scale are related to one another as follows:—

10 millibars = 1 centibar.
10 centibars = 1 decibar.
10 decibars = 1 bar.

The millibar is adopted as the working unit in the Daily Weather Service. The scale of millibars is related to the conventional scale of mercury inches as follow:—

Normal pressure for British Isles,
29·92 mercury inches = 1013·2 millibars.
Highest recorded pressure for the British Isles,
31·11 mercury inches = 1053·5 millibars.
Lowest recorded pressure for the British Isles,
27·33 mercury inches = 925·5 millibars.
1 millibar = ·029 mercury inch.

Thus one-tenth of a millibar corresponds with ·003 mercury inch, which may be taken as the limit of accuracy to which it is possible to read a barometer under favourable conditions.

A few examples will now be given of the method of correcting barometer readings recommended by the Meteorological Committee.

Barometer M. 12 metres (40 feet) above sea level in lat. 52° N. reads 1013·1 millibars; attached thermometer 285° A. Find the Fiducial Temperature in lat. 52° and correct the reading, the Fiducial Temperature for that instrument in lat. 45° at sea level being 286° A.

Fiducial Temperature in lat. 45° at sea level		286° A.
Correction for 12 metres	+	8°
Fiducial Temperature in lat. 45° at a height of 12 metres		294° A.
Latitude correction, Table I.	+	4°
Fiducial Temperature in lat. 52° at a height of 12 metres		298° A.
Actual Temperature Attached Thermometer		285° A.
Diff.		13°

Uncorrected reading		1013·1 millibars
Correction for 13° diff. between Actual and Fiducial Temperatures from Table II. B.	+	2·2 millibars
		1015·3
Proportional adjustment $1\frac{1}{3}$ per cent. of 2·2 negligible		·0
Scale Error—nil		·0
Corrected reading		1015·3 millibars

Barometer MA. 50 feet above sea level in lat. 60° S. reads 1,028 millibars; attached thermometer 278° A. Find the Fiducial Temperature for that latitude, and correct the reading, the Fiducial Temperature for that instrument in lat. 45° at sea level being 285° A.

33 millibars = 1 inch (approx.

Fiducial Temperature in lat. 45° at sea level	285° A.
Correction for 50 feet +	10°
Fiducial Temperature in lat. 45° at a height of 50 feet	295° A.
Latitude correction, Table I.	8°
Fiducial Temperature in lat. 60° at a height of 50 feet	303° A.
Actual Temperature Attached Thermometer	278° A.
Diff.	25°

Uncorrected reading	1028 millibars
Correction for 25° diff. between Actual and Fiducial Temperatures from Table II. B. +	4·1 millibars
	1032·1
Scale Error —nil	·0
Proportional adjustment 3 per cent. of 4·1	·12
Corrected reading	1032·22 millibars

From the above it is obvious that a table of Fiducial Temperatures for the height of the "Barometer Cistern" above sea level in different latitudes much facilitates the correcting of barometer readings, as, for practical purposes, there is only the correction given in Table II. for the difference of temperature between the Actual Temperature and the Fiducial Temperature, in the particular latitude the observer is in, to be applied to the reading as shown by the barometer in order to reduce the reading to Standard Conditions.

BAROMETER (ANEROID)

The aneroid barometer is another instrument for measuring changes in pressure. It consists of a circular metallic chamber partially exhausted of air and hermetically sealed. By an arrangement of levers and springs a hand is worked which indicates the pressure. / it is exaggerated 650 times

This instrument is particularly useful in ships, as it can be placed in a position immediately under the eye of the officer on deck, which, generally speaking, is not a practicable or advantageous position for a mercury barometer. The aneroid should be frequently compared with the mercury barometer, and corrected, when necessary, by means of the adjusting screw at the back. Whenever such an alteration of the index error is made, the fact should be clearly stated in the log book, or on any records of observations, as a guide to persons consulting the data for use in the future.

Readings of aneroids do not require correction for temperature, but only for height above sea level and index error. The figure given for the correction of the aneroid barometer of ships in communication with the Meteorological Office is frequently a combined result, and makes allowance for both height and index error.

Principle. Unequal expansion of liquids + solids

THE THERMOMETER

This instrument shows increase or decrease of temperature but is not sensibly affected by changes of the pressure of the air. It consists of a glass tube of very small bore, closed at one end, and united at the other to a bulb, which is commonly filled with mercury. Thermometers intended for use in very cold climates are filled with spirit instead of mercury, which would freeze and solidify at the low temperatures of the Arctic regions, whereas spirit would not freeze. Mercury freezes at a temperature of about — 38·2° F. = 39° C.; spirit (pure alcohol) becomes a thick liquid at — 130° F., and solidifies into a white mass at — 202° F. Almost all substances expand when they are heated, and contract when they are cooled, but they do not all expand equally. Mercury expands more than glass, and so when the thermometer is heated the mercury in the bulb expands, and that portion of it which can no longer be contained in the bulb rises in the tube, in the form of a thin thread. The tube being very minute, a small expansion of the mercury in the bulb, which it would be difficult to measure directly, becomes readily perceived as a thread of considerable length in the tube. When the instrument is cooled the mercury shrinks, and the thin thread becomes shorter as the mercury subsides towards the bulb. By observing the length of the thread of mercury in the tube, as measured by the graduation on the scale at its side, or marked on the tube, the thermometer shows the temperature of the bulb at the time, which thus indicates the temperature of the surrounding air, or of any liquid in which the bulb is immersed.

The indications of a thermometer are recorded in degrees, the scale for which is obtained as follows. There are two fixed points on the scale according to which thermometers are graduated, viz., that at which ice melts, and that at which water boils. In the thermometers in ordinary use in England, the distance between these two points is divided into 180 parts, or degrees. When surrounded by melting ice an accurate thermometer on this scale indicates 32°, and if placed in boiling water, when the barometer reading is 30 inches, the reading is 212°. This graduation was adopted by Fahrenheit, a native of Dantzig, in the year 1721. Other graduations were devised about twenty years later; one by Celsius, a professor at Upsala, in 1742, and another by Réaumur, a French physicist, at about the same period. Celsius suggested that the boiling-point be called zero, and the freezing-point 100°. The modern Centigrade scale, which is an adaptation of the Celsius, is in general use at the present time in most Continental countries, the freezing-point is taken as zero, and the boiling-point as 100°. Réaumur framed a scale similar to the Centigrade but divided the interval between the freezing and boiling-points into eighty divisions. This scale, which at one time was commonly employed on the Continent, is now almost obsolete.

The Absolute scale is yet another measure of temperature that has been introduced, based on the researches of the late Lord Kelvin, Dr. J. P. Joule, and others, who found the absolute zero of temperature to be 273° Centigrade below the freezing-point of water, or 459° on the Fahrenheit scale. This zero of temperature is based on the doctrine of the dissipation of energy, heat having for a long time previously been recognised as a form of energy. It represents, so far as our present knowledge goes, the temperature at which the whole of the heat of any substance whatever would have been converted into some other form of energy. The principal advantage of the Absolute scale for meteorological work is that all negative values are avoided.

Formula--

$$F^\circ = C^\circ \times \frac{9}{5} + 32^\circ, \text{ and } C^\circ = (F^\circ - 32^\circ) \times \frac{5}{9}$$

where F° = degrees Fahrenheit and C° = degrees Centigrade.

Also see Norie's Tables, Table for conversion of temperature readings of Fahrenheit and Centigrade scales to the Absolute scale.

Max Diff ashore 15°

THE HYGROMETER

Max diff - sea 10°

This instrument measures the humidity of the air. There are several kinds of hygrometers, but the easiest to make and to manage consists of a pair of thermometers placed near each other. If one of these be fitted with a single thickness of fine muslin or cambric fastened tightly round the bulb, and this coating be kept damp by means of a few strands of cotton wick, which are passed round the glass stem close to the bulb so as to touch the muslin, and have their lower ends dipping into a cup of water placed close to the thermometer, it will usually show a temperature lower than that shown by the other thermometer which is near it, the amount of the difference, commonly called the depression of the wet bulb, being dependent on the degree of dryness of the air.

To ensure correct records of the temperature and humidity of the air, the dry- and wet-bulb thermometers should be placed in a screen, the sides of which are protected from the sun and rain by narrow sloping boards overlapping each other, but with spaces between, so as to let in the air freely.

Principle

THE HYDROMETER

Archimedes

This instrument is employed for determining the specific gravity of liquids. The hydrometer used at sea is constructed of glass. If made of brass the corrosive action of salt water soon renders the instrument erroneous in its indications. The form of the instrument in common use is shown in Fig. 1. It consists of a glass tube ending in a globular bulb partly filled with mercury or small shot, to act as ballast and to make the instrument float steadily in a vertical position. From the neck of the bulb the glass is expanded into an oval or cylindrical shape, to give the instrument sufficient volume for flotation; above this it is tapered off to a narrow upright stem closed at the top, attached to which is an ivory scale. The divisions on the scale read downwards, so as to measure the length of the stem which stands above the surface of any fluid in which the hydrometer is floated. The denser the fluid, or the greater its specific gravity, the higher will the instrument rise; the rarer the fluid, or the smaller its specific gravity, the lower it will sink.

The indications depend upon the well-known principle that any floating body displaces a quantity of the fluid which sustains it, equal in weight to the weight of the floating body itself. According, therefore, as the specific gravities of fluids differ from each other, so will the quantities of the fluids displaced by any floating body, or the depth of its immersion, vary, when it is floated successively in each.

The specific gravity of distilled water, or its relative weight, compared at the temperature of 62° F., to an equal volume of other substances, being taken as unity, the depth at which the instrument remains at rest when floating in distilled water is the zero of the scale on which its indications are recorded. If the specific gravity or the density of the water be increased, as it is by the presence of salt in solution, the hydrometer will rise, and the scale is so prepared as to indicate successive increases of density up to 4 per cent., or 40 in the thousand parts. The graduations thus extend from 0 to 40, the latter corresponding to the mark on the scale which will be level with the surface when the instrument is placed in water, the specific gravity of which is 1·040. In recording observations, the last two figures only—being the figures on the scale—are written down. There has recently been introduced an hydrometer of more open scale, which has a range of from 15 to 35 (Fig. 2), instead of from 0 to 40, as in Fig. 1. This change will facilitate reading, and serve nearly every purpose for observations on board ship.

The instrument is used to show the relative density of different parts of the ocean. It may float at 40 or even higher in some parts of the Suez Canal, where the water is exceedingly salt. On the western side of the North Atlantic, in the Tropics, Bay of Bengal, and Black Sea, and in the vicinity of the mouth of a large river, the hydrometer will sink much deeper, owing to the comparative freshness of the water. The water employed for taking the specific gravity of the sea should be drawn in a bucket from over the ship's side, forward of all ejection pipes, and its temperature immediately observed and recorded, so that by its aid the specific gravity may be reduced to what it would have been at the temperature of 62° F. as explained below. The hydrometer should be slightly spun in the centre of the bucket; it soon loses any up-and-down motion; and the scale can be read before the turning motion has entirely ceased.

Fig. 1.

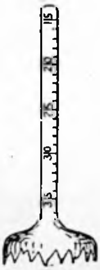


Fig. 2.

Whenever the temperature of the water tested differs from 62°, a correction to the reading is necessary, for the expansion or contraction of the glass, as well as for the temperature of the water itself, in order to reduce all observations to one generally adopted standard.

When using the hydrometer, it should be scrupulously clean, all dust, smears, or greasiness being got rid of by wiping the instrument with a clean soft cloth, before and after use.

THE TIDES

AND ON THE CORRECTION FOR SOUNDINGS

Tidal phenomena present themselves under two aspects: as alternate elevations and depressions of the sea, and as recurrent inflows and outflows of streams. Careful writers, however, use the word *tide* in strict reference to the *changes of elevation* in the water, while they distinguish the recurrent streams as *tidal currents.* Hence, also, *rise* and *fall* appertain to the tide, while *flood* and *ebb* refer to the tidal current. *Stand* should be used specifically for the period of time, at high or low water, when no vertical change can be detected; and *slack* for the period of time when no horizontal motion can be detected. *Set* and *drift* are applicable only to tidal currents, as indicating *direction* and *velocity.* The *range* of the tide is the height from low water to high water.

The *cause of the tides* is the combined action of the sun and moon. The relative effects of these two bodies on the oceanic waters are directly as their mass, and inversely as the square of their distance; but the moon, though small in comparison with the sun, is so much nearer to the earth that she exerts the greater influence in the production of the great *tide-wave:* thus the mean force of the moon, as compared with that of the sun, is as $2\frac{1}{2}$ to 1.

The attractive force of the moon is most strongly felt by those parts of the ocean over which she is vertical, and they are, consequently, drawn towards her; in the same manner the influence being less powerfully exerted on the waters furthest from her than on the earth itself, they must remain behind. By these means, at the two opposite sides of the earth, in the direction of the straight line between the centres of the earth and moon, the waters are simultaneously raised above their mean level; and the moon, in her progressive westerly motion, as she comes to each meridian in succession, causes two uprisings of the water—two high tides—the one when she passes the meridian above, the other when she crosses it below; and this is done, not by drawing after her the water first raised, but by raising continually that under her at the time; this is the *tide-wave.* In a similar manner (from causes already referred to) the sun produces two tides of much smaller dimensions, and the joint effect of the action of the two bodies is, that instead of four separate tides resulting from their separate influence, the *sun merely alters the form of the wave raised by the moon;* or, in other words, the greater of the two waves (which is due to the moon) is modified in its height by the smaller (sun's) wave. When the summit of the two happens to coincide, the summit of the combined wave will be at the highest; when the hollow of the smaller wave coincides with the summit of the larger, the summit of the combined wave will be at the lowest.

If the earth presented a uniform globe, with a belt of sea of great and uniform depth encircling it round the equator, the tide-wave would be perfectly regular and uniform. Its velocity, where the water was deep and free to follow the two luminaries, would be 1,000 miles an hour, and

the height of tide inconsiderable. But even the Atlantic is not broad enough for the formation of a powerful tide-wave. The continents, the variation in the direction of the coastline, the different depths of the ocean, the narrowness of channels, all interfere to modify it. At first it is affected with only a slight current motion towards the west—a motion which only acquires strength when the wave is heaped up, as it were, by obstacles to its progress, as happens to it over the shallow parts of the sea, on the coasts, in gulfs, and in the mouths of rivers. Thus the first wave advancing meets in its course with resistance on the two sides of a narrow channel, it is forced to rise by the pressure of the following waves, whose motion is not at all retarded, or certainly less so than that of the first wave; thus an actual current of water is produced in straits and narrow channels; and it is always important to distinguish between the tide-wave, as bringing high water, and the tidal stream—between the rise and fall of the tide, and the flow and ebb.

In the open ocean, and at a distance from the land, the tide-wave is imperceptible, and the rise and fall of the water is small: among the islands of the Pacific 4 to 6 feet is the usual spring rise. But the range is considerably affected by local causes, as by the shoaling of the water and the narrowing of the channel, or by the channel opening to the free entrance of the tide-wave. In such cases the range of tide is 35 to 45 feet or more, as in the Bay of Fundy, in the river Severn, and at St. Malo, and where the tidal stream is one of great velocity. It may under such circumstances even present the peculiar phenomenon called the *bore*, as in the Hooghly and the Amazon Rivers, where a wave comes rolling in with the first of the flood, and, with a foaming crest, rushes onwards, threatening destruction to shipping, and sweeping away all impediments lying in its course.

It is certain that in the open ocean the *great tide-wave* could not be recognised as a wave, since it is merely a temporary alteration of the sea-level.

The progress of the tide-wave as it circulates round the globe is shown on a physical chart by a series of irregular curves called *co-tidal lines*; and thus these curves pass through all such places as have, at full and change, a contemporaneous tidal hour.

In enclosed seas, as the Baltic and the Black Sea, there are no tides: in the Mediterranean and Red Sea, open at their entrance, the range is small.

Shortly after the time of *conjunction*, when the moon is *new*—and at *opposition*, when the moon is *full*—the moon and sun are in such positions that their attractions produce the greatest effect on the waters, and the result is the highest tides, called the spring tides.

When the action of the moon and sun are contrary, as when the moon is in quadrature, the tides have the least range and are called neap tides—the moon's action being then the least possible, and the sun's the greatest possible.

High water occurs on the average of 28 days comprising the lunar month, at about the same interval after the time of the moon's crossing the meridian. This nearly constant interval, expressed in hours and minutes, is known as the *lunitidal interval*. The observed interval at the time of full and change at any port is the *establishment of the port*, which is an element necessary for the determination of the tidal hour to be derived from

standard tables of reference. The *corrected establishment*, used in the United States Coast Survey, is the mean of all the intervals of the tides and transits of half the month. The time elapsed between the original formation of the tide and its appearance at any place is called the *age* of the tide, and sometimes the *retard*. The difference between the lunitidal interval and the correct establishment is the *semi-monthly inequality*.

The height of the tide is subject to considerable perturbation from the weather; and the effect of winds from different directions in raising or lowering the mean level of the water is well known. The water also stands higher with low, and lower with high, barometer; to what exact extent is uncertain; estimates vary from 7 to 20 inches rise of water for an inch fall of the mercury. Again, the times of high and low water must not be considered to always coincide with the times of slack and change of current, the two phenomena being frequently quite distinct. In estuaries and rivers the water often still runs up-stream for long after the tide has turned, and when the water-level is falling; the converse is true of ebb and low water: the current in the offing compared with that near the shore often presents these peculiarities.

In many estuaries and rivers the water rises much more rapidly than it falls. In some places there is a double high water, called *tide and half-tide*; the second high water occurring within an hour or two of the first, making four high tides in the day, generally caused by some peculiarity in the coastline. Southampton has a double tide caused by the tide flowing in first at the Needles then again round St. Catherine's.

The possibility of computing an accurate *tide-table* depends on the knowledge of certain tidal constants appropriate to the port, and most of these constants must be obtained from a series of observations at the port. The basis of the computation, whether for time or height of high water, chiefly depends on the time of the moon's transit, the semi-monthly inequality, and the corrections for the moon's parallax, the moon's declination, the sun's declination, and the sun's parallax. And in not a few places the tides are affected by a diurnal, and even by a semi-diurnal, inequality, the effect of the latter being that one high and one low water may be succeeded by a second high and low water of considerably diminished range; and again at certain of the moon's quadratures, in years of large lunar declination, the *inferior* tide may disappear altogether, and there will be only one high and one low water during the day.

It is the custom of the Admiralty to publish, in advance, annual "TIDE TABLES," which are indispensable to the navigator. These Tables give, for every day in the year, the predicted times and heights of the tides at all the principal ports of Great Britain and Ireland, and the times of low water at several prominent ports; also the times and heights of the tides at the principal ports of the world; and then by a system of "Tidal Constants" or tidal differences the times and heights at any port can be readily calculated.

At the end of the Tide Tables is also given a description of the general set of the tides in the neighbourhood of several parts of the coast, including tides on the west coast of Scotland; a full account of the streams among the Orkneys, and through the Pentland Firth; the development of the movement of the great tide-wave up the English and Irish Channels, and into the North Sea; to which has been added a description of the set of the

tides in the vicinity of Rathlin Island on the north coast of Ireland, and remarks on the tidal streams among the Channel Islands.

The time of high water at Dover for every other day of the year is given upon a detachable leaf to be used with a pocket atlas of tidal streams round the British Isles.

Lastly, there is appended for various places on the globe, arranged according to the apparent progress of the tide-wave, as well as alphabetically, the time of high water on the days of full and change; with the rise of the tide at springs and neaps.

The United States Coast Survey publishes similar Tide Tables, with the necessary differences, for the Atlantic and Pacific coasts of the United States; and the Indian Government also issue, annually, very elaborate Tide Tables for the Indian ports and Hong Kong, giving the times and heights of both high and low water.

By the aid of these special Tide Tables and the given differences, or through the *difference of the establishments* of two ports, the time of high water (H.W.) and the height of high water on any given day may be found, as will be seen in the sequel.

Those who wish to know more about the tides should consult any of the following works:—Professor Haughton's "Manual of Tides and Tidal Currents"; "An Elementary Treatise on the Tides," by James Pearson, M.A., F.R.A.S.; or the Article on Tides by Professor G. H. Darwin, in the new edition of the Admiralty Manual of Scientific Enquiry; Captain Ruthven, and others.

Time of High Water by Admiralty Tide Tables

A port other than the standard one being given, to find the a.m. and p.m. times of high water on a certain day, *proceed as follows:*

Turn to the Admiralty Tide Tables; seek for the given port, take out the time difference, and note its sign (+ or —) as well as the standard *port of reference*; turn to the given *month*, seek out the *standard port*, and for the given day will be seen the morning and afternoon times of high water at that port. If a blank or (—) occurs in either column it indicates *no high water*, and consequently there is but one high water on that day.

At all ports there is always one high water every day, and since the greatest tidal difference never exceeds 8 hours, it follows that if a tidal difference be added to the morning time of high water for any day at the *port of reference*, the resulting time must remain in that day. Or again, if the tidal difference be subtracted from the afternoon time of high water for any day at the *port of reference*, the resulting time must still be in that day. And if the *port of reference* has only one time of high water on any day, this must be within *one* hour of noon, hence in this case also, whether the tidal difference be added or subtracted, the resulting time must be in the day used. Therefore *one time of high water* on a certain day can always be found by the *following rule.*

When the time difference is +, that is, *additive* to the tidal hours at the *port of reference*, write down the *morning* time at the *port of reference* on the given day, and *add* the difference. If the sum is less than 12 hours the resulting time is a.m., but if the sum exceeds 12 hours, reject 12 hours and call the remainder p.m.

When the difference is —, that is, *subtractive* from the tidal hours at the *port of reference*, write down the *afternoon* time at the *port of reference* on the given day and *subtract* the difference, borrowing 12 hours if necessary. If 12 hours are borrowed, the resulting time is a.m., if not, then it is p.m.

When there is only *one* time in the tables at the *port of reference* on the given day, write that time down and *add* or *subtract* the difference according to its sign, rejecting or borrowing 12 hours as necessary. If the operation is performed without rejecting or borrowing 12 hours the time keeps its name; but if 12 hours are rejected or borrowed the time changes its name.

To find the Time of the other Tide (if any)

When the time found as above is a.m. take from the tables the time following the one already used and apply the difference, rejecting or borrowing 12 hours as necessary. If the operation is performed without rejecting or borrowing 12 hours the time keeps its name; but if 12 hours are rejected or borrowed the time changes its name.

When the time *first* found is p.m., take from the tables the time preceding the one already used and apply the difference, rejecting or borrowing 12 hours as necessary. If the operation is performed without rejecting or borrowing 12 hours, the time keeps its name; but if 12 hours are rejected or borrowed, the time changes its name.

If the time now found has a different name to the first found time, it is the time required. But if it has the same name as the first found time, reject it, for it shows that there is no second time of high water on the given day.

The following examples illustrate the use of the "Admiralty Tide Tables"—

Jan. 8th, 1913. Find the time of high water at St. Malo, a.m. and p.m.

		H. M.			H. M.	
St. Helier, Jersey, Jan. 8th		6 53	a.m.		7 10	p.m.
Difference for St. Malo	—	0 25		—	0 25	
H.W. at St. Malo		6 28	a.m.		6 45	p.m.

Jan. 29th, 1913. Find the time of high water at Dartmouth, a.m. and p.m.

		H. M.			H. M.	
Devonport, Jan. 29th		10 23	a.m.		10 45	p.m.
Difference for Dartmouth	+	0 33		+	0 33	
H.W. at Dartmouth		10 56	a.m.		11 18	p.m.

Jan. 20th, 1913. Find the time of high water at Gravesend, a.m. and p.m.

		H. M.			H. M.	
London Bridge, Jan. 20th		11 44	a.m.		0 19	a.m. 21st
Difference for Gravesend	—	0 53		—	0 53	
H.W. at Gravesend............		10 51	a.m.		11 26	p.m. 20th

Jan. 25th, 1913. Find the time of high water at Picton, a.m. and p.m.

			H. M.
Father Point, Jan. 25th	— —		4 46 p.m.
Difference for Picton		—	4 29
	no a.m.		0 17 p.m.

Jan. 18th, 1913. Find the time of low water at Liverpool, a.m. and p.m.

	H. M.	H. M.
Jan. 18th	0 46 a.m.	1 26 p.m.

Jan. 7th, 1913. Find the time of high water at Annan Fort, a.m. and p.m.

		H. M.	
Liverpool, Jan. 7th		11 14 a.m.	
Difference	+	0 42	
		11 56 a.m.	no p.m.

Find the a.m. and p.m. tides at the following places. Use 1915 Tide Tables.

Jan. 1st St. Malo.	Jan. 6th Chatham.	Jan. 22nd Heligoland.
,, 2nd Fowey.	,, 7th Margate.	,, 4th Antwerp.
,, 3rd Selsea Bill.	,, 8th Harwich.	,, 9th Brisbane.
,, 4th Hastings.	,, 9th Bridlington.	

Answers to Tides

	H.	M.		H.	M.	
St. Malo	5	50	a.m.	6	10	p.m.
Fowey	5	5	,,	6	9	,,
Selsea Bill	0	33	,,	0	53	,,
Hastings	0	15	,,	0	37	,,
Chatham	3	38	,,	9	50	,,
Margate	3	3	,,	3	28	,,
Harwich	3	47	,,	4	15	,,
Bridlington	9	35	,,	10	0	,,
Heligoland	4	18	,,	4	44	,,
Antwerp	8	33	,,	8	50	,,
Brisbane	2	52	,,	2	54	,,

Reduction to Soundings

The soundings marked in small figures on the chart only agree with the lead line when a cast is taken at low water ordinary spring tides. At any other time the lead line will measure more or less water than is shown on the chart—mostly more. Less water can be shown only by a cast taken at dead low water extraordinary spring tides. It follows that at most times a correction is necessary, especially where the range of the tide is large.

The correction is called the Reduction to Soundings.

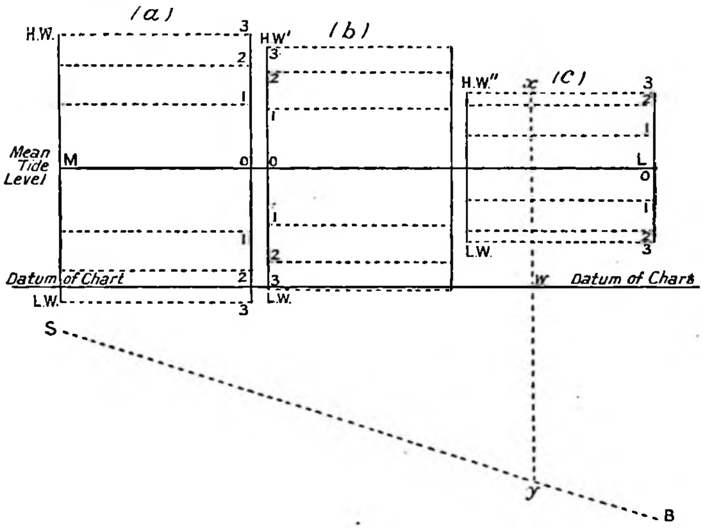
The following figure will give an idea of what is required.

M.T.L. is the mean level of the sea, and the line worked from; the tide (generally speaking) rises three hours from this line and falls three hours below it.

Taking (*b*), suppose the range is 40 feet, that will be 20 feet above M.T.L. and 20 feet below M.T.L. The Table B grades this 20 feet proportionally to the rush of tide; (*b*) represents an ordinary spring tide, therefore dead low water and the chart soundings represented by the datum line agree. This datum line is generally the level of the sea at mean low water springs (M.L.W.S.)

(*a*) represents an extraordinarily high tide which rises above and falls below the ordinary, therefore at dead low water a vessel would be at L.W. some inches below the chart datum line, and to bring the data to chart level that quantity should be added to the depth found by lead line.

(*c*) represents a neap tide which neither rises as high nor falls as low as an ordinary tide.



If S B is the sea bottom then the vertical distance between S B and datum line of the chart will be the soundings as recorded on the chart.

A vessel tide borne at H.W." (*c*) would have to subtract the value of a vertical line equal to the distance between H.W." and chart datum to get the soundings as per chart. That is, a line is dropped from x to y; the corrected soundings would be $xy - xw = wy$, the soundings on the chart.

Excellent diagrams showing the height of the tide at any time for any place, constructed by Captain T. H. Tizard, R.N., C.B., F.R.S., are now inserted in the Admiralty Tide Tables and take the place of Table B because they can be used at sight for 5h. 6h. or 7h. tides.

Table B is now obsolete, and the reduction to soundings is shown graphically by means of a diagram which is constructed from data obtained as follows—

(N.B.—" Tidal Constants " are now called " Tidal Differences.")

To find the Time of High Water

At standard ports the times of high water are tabulated for every day in the Admiralty Tide Tables. In some cases the times of both high and low water are given.

At ports which are not standard ports the time of high water is found by applying the time difference according to its sign to the time of high water at the standard port; the result will be the time of high water at the port required.

If the time differences for both high and low water be given, these differences must be applied to the times of high and low water at the standard port according to their signs, and the result will be the times of high and low water at the port required. It is necessary to notice whether a spring or neap difference is required.

To find the Duration of the Tide

If the times of high and low water are given, take the difference between them and the result is the duration of the tide.

At standard ports where only high waters are given, the time of rise and fall is tabulated at the foot of the table. If only high-water time differences are given, the times of rise and fall at the required port must, for the present, be assumed to be the same as at the standard port.

If the cast were taken on a rising tide, take the difference between the time of preceding low water and the time of high water, the result will be the duration required; but if the cast were taken on a falling tide, the difference between the time of preceding high water and the time of the following low water will be the duration required.

If mean time at ship be given, it must be turned into standard time by applying the longitude in time to it in order to get standard time at ship.

To find the Interval between Time of Cast and High Water.

If the cast were taken before high water, subtract the time of cast from the time of high water, and the result will be the required interval before high water; but if the cast were taken after high water, subtract the time of high water from the time of cast, and the result will be the required interval or time after high water.

To find the Angle corresponding to the Time from High Water

Multiply 180° by the time from high water and divide the result by the duration of rise or fall according as it is a rising or falling tide on which the cast was taken, and the result will be the required angle.

To find the Mean Tide Level (M.T.L.) and Half Range of Tide when the Tide does not fall below the Datum Line

If both high and low waters are given, take their difference, which will be the range of that tide; divide the range by two and the result will be the half range, to which add the height at low water above the datum line and the result will be the mean tide level. Where both high and low waters are given the mean tide level must be found for every tide.

At ports for which high and low water height differences are given, apply the high and low water differences to the height of high and low water at the standard port; then find the range and half range as above, and to

the half range add the height of low water above the datum line, and the result is the M.T.L.

At standard ports where only high water is given the M.T.L. is given at the foot of the page.

At ports for which only high water height differences are given, apply "Half the Spring Difference" to the M.T.L. at the standard port, and the result is the M.T.L. at the required port; and in order to find the half range for the radius of the circle in the diagram, subtract the M.T.L. thus found from the height of high water, and the result is the half range required.

When the Tide falls below the Datum Line

Add together the height of high water and the distance the tide falls below the datum line; this will give the range of the tide; divide by 2 and the result is half range; from the half range subtract the distance the tide falls below the datum line and the result is mean tide level.

In other cases the quantity by which the half range exceeds the mean tide level is the distance the tide falls below the datum line.

N.B.—When a tide falls below the datum line and the cast is taken at low water the distance the tide falls below the datum line is the correction to be *added* to the cast.

To lay off the Angle corresponding to the Interval between the Cast and High Water

Draw a line in a vertical direction as in the following diagrams, and by means of a protractor or Field's parallel rulers lay off the angle above the M.T.L. if the time from high water is less than half the duration of the tide, but below M.T.L. if the time from high water is greater than half the duration of the tide.

If the angle be 90° the M.T.L. is the correction to be applied to the cast. The angle must always be laid off from the vertical line, never from the M.T.L. line; if the angle exceed 90°, subtract it from 180° and lay the result off from the vertical line below the M.T.L. line.

If the candidate will study the following examples he will find no difficulty in solving any Reduction to Soundings.

The diagrams can be drawn to any scale, and if ruled paper be used the distance between the lines can be used as feet, or half feet, as desired.

Example I

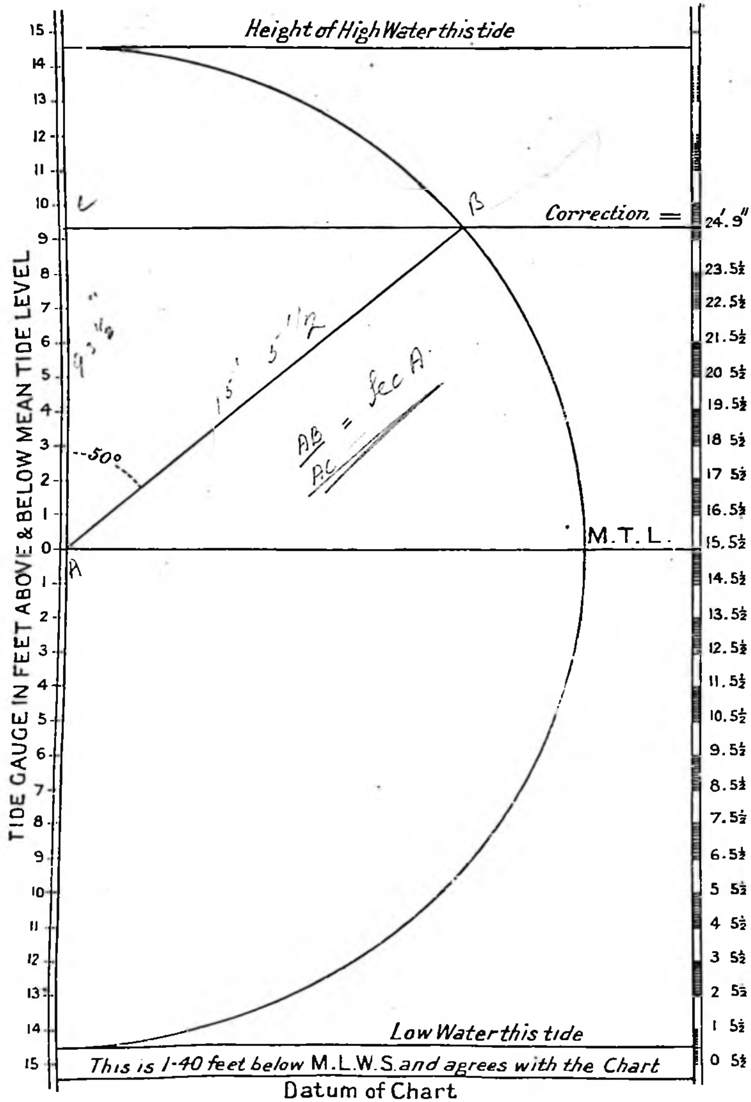
REDUCTION TO SOUNDINGS

On Aug. 26th, 1915, at 1h. 45m. a.m. standard time at ship, being off Barrow Docks. Required the correction to be applied to the depth obtained by the lead line before comparing it with the chart.

	H.	M.	
H.W. L'pool	11	49	p.m. 25th
Constant	—	4	
H.W. Barrow Dock	11	45	
L.W. ,, ,,	6	56	
Duration of fall	7	11	

	H.	M.	
L.W. L'pool	6	46	a.m. 26th
Constant	+	10	
L.W. Barrow Dock	6	56	a.m.
Time of H.W. Barrow Dock	11	45	p.m. 25th
Time of cast	1	45	a.m. 26th
Time after H.W.	2	0	

Example I.


Height of High Water this tide
Correction = 24′. 9″
B
A
50°
15′ 5″ 1/2
9″ 1/8
AB/AC = Sec A.
M.T.L.
Low Water this tide
This is 1·40 feet below M.L.W.S. and agrees with the Chart
Datum of Chart
TIDE GAUGE IN FEET ABOVE & BELOW MEAN TIDE LEVEL
15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
23.5½ 22.5½ 21.5½ 20 5½ 19.5½ 18 5½ 17 5½ 16.5½ 15.5½ 14.5½ 13.5½ 12.5½ 11.5½ 10.5½ 9.5½ 8.5½ 7.5½ 6.5½ 5 5½ 4 5½ 3 5½ 2 5½ 1 5½ 0 5½

	ft.	in.
Ht. of H.W. at L'pool	29	10
H.W. spring constant	+	2
Ht. of H.W. Barrow Dock	30	0
Ht. of L.W. ,, ,,	0	11
Range at Barrow Dock	29	1
Half-range ,, ,,	14	6½
Ht. of L.W. ,, ,,	+	11
Mean tide level	15	5½

	ft.	in.
Ht. of L.W. at L'pool	0	6
L.W. spring constant	+	5
Ht. of L.W. Barrow Dock	0	11

To find the Angle.

180°
·2
7·2)360(50°

Data for diagram.
Angle 50°

	ft.	in.
Half range	14	6½
M.T.L.	15	5½

To calculate the Correction

Multiply the half range by the cosine of the angle from high or low water and the result is to be added to M.T.L. if time from high water is less than half duration of tide, and to be subtracted from M.T.L. if greater than half duration.

	ft.	in.	
M.T.L.	15	5½	
+	9	4	
Correction by calculation	24	9½	to subtract
,, ,, diagram	24	9	,, ,,

Half range in inches 174·5 log. 2·241795
50° cos. 9·808067

ft.	in.		in.		
9	4	or	112	log.	2·049862

EXAMPLE II

On 4th March, 1915, at 1h. 21m. a.m. standard time, being off Holyhead, took a cast of the lead. Required the correction to be applied to the cast before comparing it with the chart.

	H.	M.	
Time of H.W. at Holyhead	11	39	p.m. 3rd
Time of cast	1	21	a.m. 4th
Interval or time after H.W.	1	42	

	ft.	in.
Ht. of H.W. above datum	16	11
L.W. below ,,	+	8
Range of tide	17	7
Half range	8	9½
Distance of L.W. below datum	—	8
Mean tide level	8	1½

To find the Angle

180°
1·7
6·1)306·0(50°
305
10

To find Duration of Fall

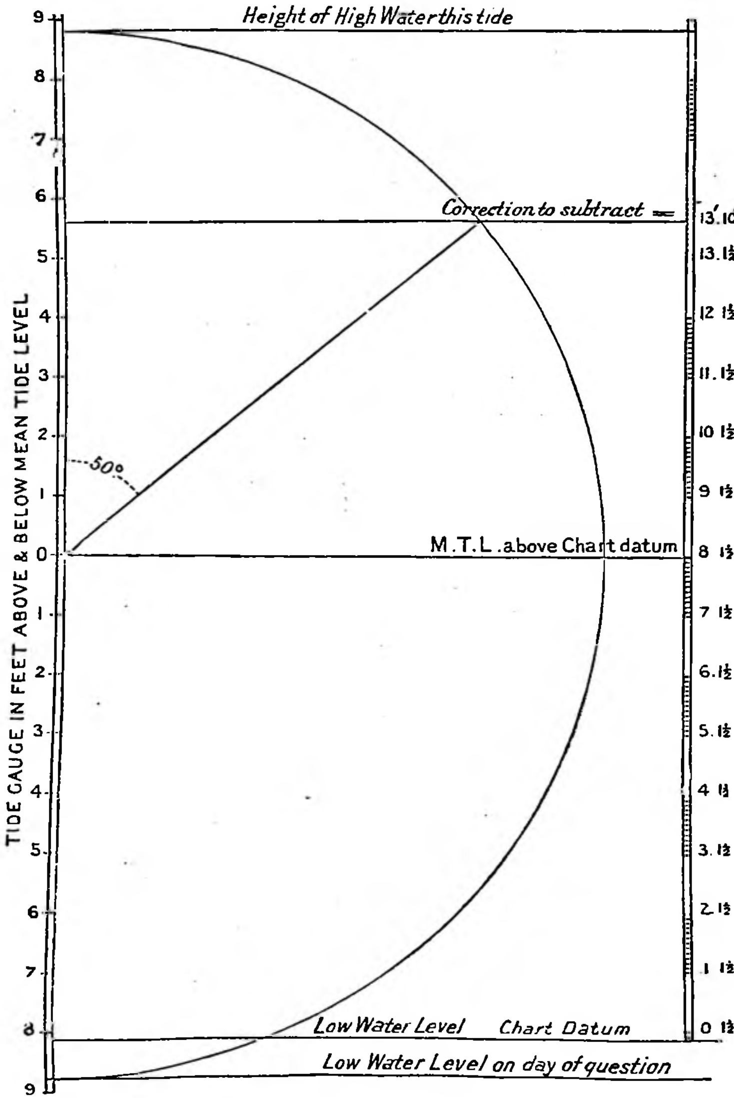
	H.	M.	
Time of H.W.	11	39	p.m. 3rd
Time of following L.W.	5	44	a.m. 4th
Duration of fall	6	5	

Data for constructing Diagram—

Angle = 50°

	ft.	in.
Half range = Radius reqd. =	8	9½
M.T.L.	8	1½

EXAMPLE II.


Height of High Water this tide
Correction to subtract = 13′.10″
M.T.L. above Chart datum
50°
Low Water Level
Chart Datum
Low Water Level on day of question
TIDE GAUGE IN FEET ABOVE & BELOW MEAN TIDE LEVEL
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
13.1½
12 1½
11.1½
10 1½
9 1½
8 1½
7 1½
6.1½
5.1½
4 1½
3.1½
2.1½
1 1½
0 1½

To find the Correction by Calculation

Half range in inches	105·5	log. 2·023252
	50°	cos. 9·808067
	in. 67·81	log. 1·831319

	ft	in.
M.T.L.	8	1½
+	5	8
Correction =	13	9½

The diagram shows that 67·87 in. are to be added to the M.T.L. and it also shows that the correction is to be subtracted from the cast.

	ft.	in.	
Correction by calculation	13	9½	to subtract
,, ,, diagram	13	10	,, ,,

Example III

On Sept. 11th, 1915, at 1h. 4m. a.m. mean time at ship, being off Maryport, took a cast of the lead. Required the correction to be applied before comparing it with the chart.

	H.	M.	
H.W. at Liverpool	11	59	p.m. 10th
Constant	+	5	
H.W. Maryport	0	4	a.m. 11th
Time of cast	1	18	a.m. 11th
Interval or time after H.W.	1	14	

	H.	M.	
Mean time at ship, 11th	1	4	a.m.
Longitude in time	+	14	
Standard time, M'port	1	18	

	ft.	in.
Height of H.W. Liverpool	30	0
Spring difference	— 3	11
Height of H.W. Maryport	26	1
M.T.L. ,,	13	3
Half range Maryport	12	10

	ft.	in.
Height of H.W. at Liverpool	30	0
Height of L.W. ,,	—	5
Range at ,,	29	7
Half range at ,,	14	9½
Height of L.W. ,,	+	5
M.T.L. ,,	15	2½
Half spring diff. Maryport	— 1	11½
M.T.L. Maryport	13	3

To find the Angle

```
      180°
       1·2
6·97)216·0(31°
      209·1
        690
        697
```

To find Duration of Fall

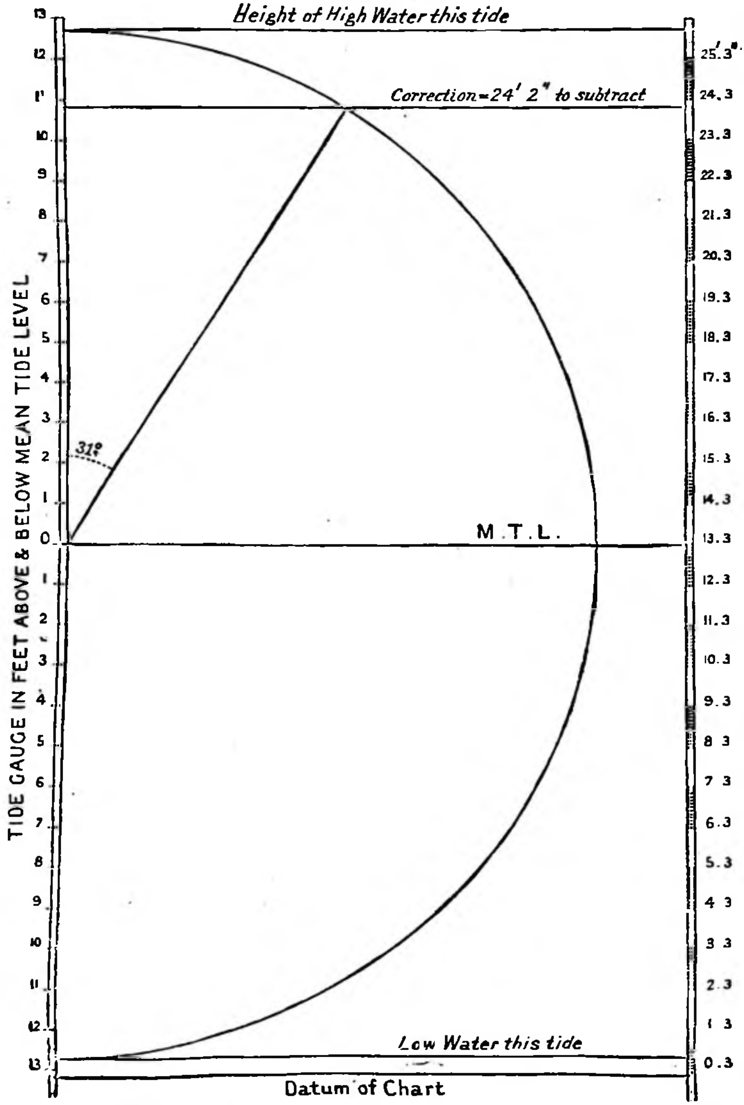
	H.	M.	
Time of H.W. Liverpool	11	59	p.m. 10th
Time of L.W. ,,	6	57	a.m. 11th
Duration of fall	6	58	

Data for constructing Diagram—

Angle 31°

	ft.	in.
Half range = Radius	12	10
M.T.L.	13	3

EXAMPLE III.


Height of High Water this tide
Correction = 24' 2" to subtract
31°
M.T.L.
TIDE GAUGE IN FEET ABOVE & BELOW MEAN TIDE LEVEL
13
12
11
10
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
25'.3"
24.3
23.3
22.3
21.3
20.3
19.3
18.3
17.3
16.3
15.3
14.3
13.3
12.3
11.3
10.3
9.3
8.3
7.3
6.3
5.3
4.3
3.3
2.3
1.3
0.3
Low Water this tide
Datum of Chart

As there is no low water time difference given for Maryport, the duration of tide at Liverpool viz., 6h. 58m., is to be used.

Correction by Calculation

				ft. in.
Half range in inches	154	log. 2·187521	M.T.L.	13 3
	31°	cos. 9·931691	+	10 11·6
	in. 131·6	log. 2·119212	Correction	24 2·6 to subtract

Correction by diagram the same.

The diagram shows that 131·6 inches are to be added to M.T.L. to find the correction, and it also shows that the correction is to be subtracted from the cast.

Example IV

On 31st Oct., 1915, at 5h. 14m. p.m., mean time at ship, being off Portree, find the correction to be applied to the lead line before comparing it with the depth on the chart.

	H. M.			ft. in.
M.T. ship	5 14 p.m.		M.T.L. Thurso	6 7
Long. in time	+ 25	to nearest minute	Half spring diff.	+ 10½
Standard time at ship	5 39 p.m.		M.T.L. Portree	7 5½

	H. M.		ft. in.
Time of H.W. Thurso	2 13 p.m.	Height of H.W. Thurso	8 8
Time difference —	1 56	Neap difference +	1 3
Time of H.W. Portree	0 17 p.m.	Height of H.W. Portree	9 11
Time of cast	5 39 p.m.	M.T.L. Portree	7 5½
Interval after H.W.	5 22	Half range, Portree	2 5½

As there is no low water time difference, the duration of the tide is to be taken as being the same as at the standard port, in this case 6 hours. When the duration of the tide is 6 hours, divide the interval from H.W. by 2 and the result will be the required angle.

	H. M.
Interval from H.W.	5 22
	60
	2)322
Required angle	161°
	180
Supplement	19°

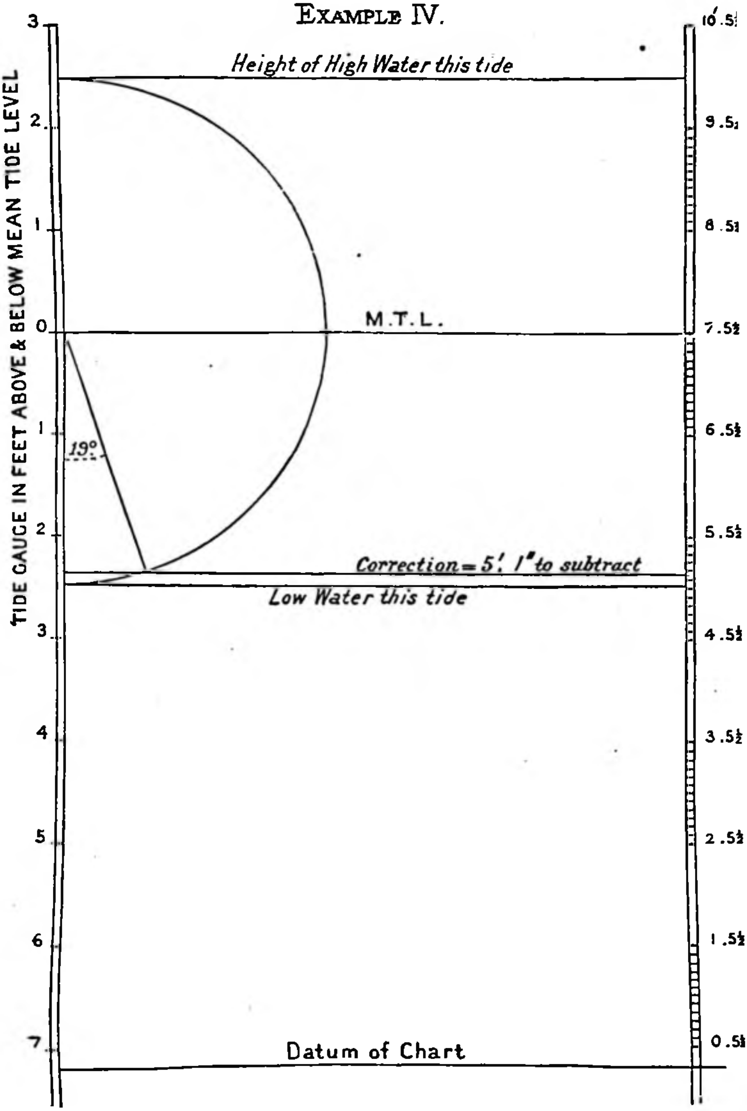
Data for Diagram—

	ft. in.
M.T.L.	7 5½
Half range	2 5½

Angle 19°

(to be laid off from the vertical line below M.T.L.)

Example IV.



TIDE GAUGE IN FEET ABOVE & BELOW MEAN TIDE LEVEL
Height of High Water this tide
M.T.L.
19°
Correction = 5' 1" to subtract
Low Water this tide
Datum of Chart
3
2
1
0
1
2
3
4
5
6
7
10'.5½
9.5½
8.5½
7.5½
6.5½
5.5½
4.5½
3.5½
2.5½
1.5½
0.5½

To find the Correction by Calculation

	in.	
Half range	29·5	Log. 1·469822
	19°	Cos. 9·975670
	27·89 inches	Log. 1·445492

As the interval between H.W. and time of cast is greater than half the duration of the tide, the quantity found by calculation is to be subtracted from M.T.L. to find the required correction.

	ft.	in.	
M.T.L.	7	5	
Amount tide has fallen below M.T.L.	2	4	
Correction	5	1	to subtract.

Examples for Practice

1. On Aug. 26th, 1915, at 5h. 32m. a.m., mean time at ship, being off Barrow Docks. Required the correction to be applied to the depth obtained by lead line before comparing it with the chart.

Ans.—

	H.	M.
Duration of fall	7	11
Time of cast after H.W.	6	0
Angle	150°	
	ft.	in.
Half range	14	6½
M.T.L.	15	5½
Correction to subtract	2	10½

2. On April 14th, 1915, at 11h. 36m. p.m., standard time at ship, being off Thurso. Required the correction to be applied to the depth obtained by lead line before comparing it with the chart.

Ans.—

	H.	M.
Duration of fall	6	0
Time of cast after H.W.	3	0
Angle	90°	
	ft.	in.
Half range	6	5
M.T.L.	6	7
Correction	6	7 to subtract.

3. On March 3rd, 1915, at 1h. 14m. p.m., M.T.G., being off Devonport. Required the correction to be applied to the depth obtained by lead line before comparing it with the chart.

Ans.—

	H.	M.
Duration of fall	6	14
Time of cast	Low water	
Angle	180°	
	ft.	in.
Half range	8	9½
M.T.L.	7	8½
Correction to add	1	1

A Line of Soundings

The following method is much used at sea—

Draw in pencil upon the chart a line from A to B upon which it is desired to keep the ship; this line will cover a line of soundings on the chart. Procure a long strip of paper an inch wide upon which the soundings are recorded to scale, using the graduated meridian for that purpose.

The distance the ship has run between the casts with the reduced depth and nature of the bottom all being placed upon the strip of paper, a comparison is made with the line upon the chart. If the vessel is on the line the soundings upon the paper will agree with the soundings upon the chart. If they do not the slip of paper should be moved up or down, backward or forward, until they do agree. For if the soundings are carefully taken and correctly reduced the line must be found on the chart.

If it is desired to alter the course, place the paper upon a compass on the chart and give the slip of paper a half-turn equal to the angle of the new course. Done with care, soundings can be recorded continuously for long distances.

The Lead

There are two kinds of leads, the hand and the deep-sea lead.

The hand lead is used in shallow waters, chiefly going in or out of port.

The depth is obtained by actual measurement.

The weight of the hand lead is from 10 to 14 pounds; it is bent on to a water-laid line $1\frac{1}{8}$ inch in circumference and is 25 fathoms in length.

The lead line is marked as follows—

At	2	fathoms	Two strips of leather	At	13	fathoms	Blue bunting
,,	3	,,	Three ,, ,,	,,	15	,,	White ,,
,,	5	,,	White bunting	,,	17	,,	Red ,,
,,	7	,,	Red ,,	,,	20	,,	Two knots
,,	10	,,	Leather with hole in it				

There are nine marks and eleven deeps.

The deep-sea lead used with a hemp line weighs from 28 to 30 pounds. The line is 100 fathoms long; it is marked the same as the hand lead up to 20 fathoms and has an additional knot for each 10 fathoms, that is, three knots at 30 fathoms, four knots at 40 fathoms, etc., and every five fathoms is marked by a single knot.

SOUNDING MACHINES

The Sounding Machine invented by Sir William Thomson has been the type of other designs. Three hundred fathoms of thin pianoforte wire are wound round a drum, which has a V-shaped groove around its rim. When running out the drum runs freely round a spindle and is stopped by a frictional brake consisting of wooden chocks on each side. To the end of the wire is secured, by a hemp line, a specially constructed lead with a long iron shank weighing about 24 pounds.

On the hemp line is lashed a brass guard tube in which is placed a glass tube 24 inches long; the tube is placed open end down inside the guard tube.

The glass tube is coloured inside with chloride of silver, which makes the glass appear red. The salt water penetrating turns the colouring matter white.

The tube is withdrawn after a cast, placed upon a graduated scale, and read off.

The principle is an application of Boyle's law of the compressibility of gases.

The temperature remaining the same, the volume of a given quantity of gas is inversely as the pressure to which it is subjected.

Thus, in a tube 24 inches long when it has descended—

10	fathoms	the	compression =	8·34	inches
20	,,	,,	,,	5·00	,,
30	,,	,,	,,	3·71	,,
40	,,	,,	,,	2·95	,,
70	,,	,,	,,	1·28	,,

These soundings are affected by the atmospheric pressure. The following corrections must be applied if the barometer is above 29·50—

Barometer	29·75	add 1	fathom	in	40
,,	30·00	,,	,,	,,	30
,,	30·50	,,	,,	,,	20
,,	31·00	,,	,,	,,	15

LOG-SHIP AND LOG-LINE,

WITH THE

LOG-GLASS AND PATENT LOG.

THE LOG AND LOG-GLASS

The common LOG consists of the LOG-SHIP and LOG-LINE.

The Log-ship.—The log-ship is a quadrantal-shaped piece of hard wood, about 5 or 6 inches radius, and a quarter of an inch thick; the circular part is loaded with lead to make it float perpendicularly, and just sufficient to immerse it.

The outer extremity of the log-line terminates in two or three ends,—for as many holes as are in the flat of the log-ship,—and which, when fitted, forms a sling or bridle; at the end of one part of the sling is a wooden peg, which is fitted into a socket seized on to the log-line at X, and draws upon being checked, after the operation of *heaving the log* has been effected, and thus the log-ship is more easily hauled in: the other (or inner) end of the log-line is attached to a REEL, around which is wound 120 fathoms of line.

The Log-line.—The log-ship end of the line is marked off, to the length of 10 fathoms or more, according to the size of the ship, by a bit of white rag or bunting; and this length is called *stray*-line,—its use being to carry the log-ship out of the eddies of the ship's wake before counting commences. The line from the stray part inwards is divided into equal lengths—called *knots*—by pieces of cord let into the strands of the log-line,—each piece of cord carrying the requisite number of knots to distinguish it. The sub-divisions of a knot are called fathoms. Each knot is the representative of a nautical mile, and its length is proportionate to the seconds of the log-glass that is used when heaving the log. The log-line is subject to variation in length, due to wetting and strain.

The hand Log-line is usually marked as follows; first half knot, one end of leather; first knot, 2 ends of leather; 1½ knots, 3 ends of leather; 2 knots a piece of cord with two knots; 2½ knots a piece of cord with one knot; 3 knots a piece of cord with 3 knots, and so on, an additional knot for each knot, and a single knot at each half-knot.

Heaving the Log.—When the log is hove, a seaman holds the reel by the two ends, another seaman takes the log-glass, and an officer of the watch asks, "All clear?" On receiving a reply, "All clear," he throws the log-ship well out to leeward from the lee quarter; as soon as the whole of the stray line has gone, the officer calls "turn"; the seaman then turns the sand end of the glass uppermost; the log-ship, being perpendicular in the water, and presenting a face toward the ship, by its resistance to the ship's progress draws the marked part of the line off the reel; when the sand has run out, the seaman calls "stop"; the officer at that instant clutches tight the line, and the number of knots which have passed out indicates how many

nautical miles per hour the ship is moving through the water. In a heavy sea the line requires to be paid out rapidly at the time the stern is rising, and slightly retarded as the stern is falling. In heaving the log, great care should be taken to veer out the line as fast as the log takes it ; for if the log be left to turn the reel itself, it will come home, and give an erroneous distance.

The Log-glass.—The log-glasses, of the same shape as an hour-glass, are glasses filled with sand or metal filings, and run to seconds ; the *long glass* runs out in 30 seconds, or in 28 seconds ; and the *short glass* in half the time, viz., in 15 seconds, or in 14 seconds. When the ship's rate is more than 5 knots, the short glass is used, and the number of knots shown by the log-line is doubled.

As the glasses may be affected by variation in the temperature, and the sand certainly by damp weather, it is necessary to examine their accuracy, from time to time, by comparing them with a seconds' watch ; they can be made true, by drying the sand, or changing its quantity, to do which it will be necessary to remove the cork at the stoppered end of the glass.

The Nautical or Sea Mile.—In Navigation, distance is invariably measured in Nautical Miles ; and one such mile is considered to be the 21600 part of the earth's circumference ($360° \times 60 = 21600$).

The *geographical mile* of the trigonometrical survey is 6087·23 feet, which corresponds to the admeasurement obtained from the *data* of Sir G. B. Airy, Bessel, and Col. A. R. Clarke, using the earth's *equatorial* radius as the basis of computation ; whence we get respectively 6086·5 ft., 6086·5 ft., and 6087·1 ft. ; and the geographical mile, as 1′ (minute) of the Equator = 2029·1 yards = 1·1529 Eng. statute miles.

The geographical mile is generally defined to be the length of a minute of arc of the earth's equator ; but the *nautical or sea mile* as defined by hydrographers is the length of a minute of arc of the meridian, and is different for every latitude. It is equal to a minute of arc of a circle whose radius is the radius of the curvature of the meridian at the latitude of the place. Thus—

Length of a nautical mile in lat.	0°	=	6045·93	feet.
,, ,, ,,	45°	=	6076·82	,,
,, ,, ,,	90°	=	6107·98	,,

And the length of a mean nautical or sea mile = 6076·91 feet, or 2025·63 yards, or 1·1509 statute miles. This admeasurement agrees with those obtained from the *data* of Sir G. B. Airy, Bessel, and Col. A. R. Clarke, using the mean of the equatorial and polar radii of the earth, and whence are obtained respectively 6076·3 ft., 6076·3 ft., and 6076·7 ft., as the length of a mile.

The Admiralty knot, however, is = 6080 feet, or $2026\frac{2}{3}$ yards ; and this is the admeasurement usually adopted for the nautical or sea mile.

Length of a Knot.—Coming to the length of a knot on the log-line, you can at once understand that the *length between two adjacent knots should be the same part of a nautical mile, that the seconds of the glass are of an hour.*

Before, however, commencing to determine this length, you must, in the first place, know that it is *safest to have the reckoning ahead of the ship* (to apprise the seaman the sooner to look out when approaching the land), an

allowance in the length of the knot is made such that it shall obviate any causes of error arising when heaving the log; hence, for practical purposes at sea, the length of the nautical mile is often taken to be 6,000 feet. Then, on this rough basis, for the 30-second glass we readily get the length of the knot; 30 seconds, or half a minute, is the 120th part of an hour, and 6,000 divided by 120 gives 50 feet, which is the length generally taken; and it must be clear to you that if the 120th part of a mile (as measured on the log-line) runs out in the 120th part of an hour, the ship would be dragging along at the rate of 1 mile per hour; on the same basis, if eight 120ths passed out, she would be sailing at the rate of 8 miles an hour. But you must not assume that this is more than an approximation.

RULE.—For accuracy a method of computation must be adopted equally applicable and simple for all glasses. Now there are 3,600 seconds in an hour, and 6,080 feet in a nautical mile, hence—

$$\begin{array}{rlcrcrcrl} & \text{s.} & & \text{s.} & & \text{ft.} & & \text{ft.} & \\ \textit{Example}\text{—} & 3600 & : & 30 & :: & 6080 & : & 50{\cdot}666 & = 50 \text{ ft. } 8 \text{ in. nearly.} \\ \text{and} & 3600 & : & 28 & :: & 6080 & : & 47{\cdot}29 & = 47 \text{ ft. } 3\tfrac{1}{2} \text{ in. } ,, \end{array}$$

These are the *correct lengths* for the 30 sec., and 28 sec., glasses; and the statement is the same for any other glass, whatever it runs to.

ROUGH RULE.—Throwing away the odd 80 feet in the nautical mile, we have 6,000 feet; and since the ratio of 3,600 to 6,000 is as 6 to 10, or as 3 to 5, hence the rule; affix a cipher to the seconds run by the glass, and divide by 6, for the length of a knot in feet. Or, multiply the seconds run by the glass by 5 and divide the product by 3.

Example.—Suppose, a 28-sec. glass, find the length of the knot.

$$\begin{array}{l} 6)\underline{280} \\ \quad 46{\cdot}66 = 46\tfrac{2}{3} \text{ feet.} \end{array} \qquad \text{Or,} \qquad \begin{array}{r} 28 \\ \underline{5} \\ 3)\underline{140} \\ 46{\cdot}66 \text{ feet.} \end{array}$$

It is well, however, always to be accurate in your work: therefore, the rate of the glasses should often be examined by the aid of the chronometer, and the length of the knots measured. It is no uncommon thing to mark the length of the knot by two copper nails driven into the deck, at the proper distance apart; so this measure is always at hand.

The subdivisions of the knot, already referred to, and erroneously called fathoms (8 to a knot), are really nautical furlongs, after the same manner as 8 statute furlongs make a statute mile. The division into tenths is preferable.

Always bear in mind that the value of the operation of heaving the log depends conjointly on the accuracy of the instruments and the care bestowed in using them.

NOTE.—The log-line no less than the glass varies in its indications. You may consider that—one being correct, and the other faulty—

For error of seconds' glass—Glass too short gives distance too short;
Glass too long gives distance too long.
For error on log-line—Knot too short gives distance too long;
Knot too long gives distance too short.

If BOTH are *faulty* :—Multiply faulty length of knot by erroneous distance, and divide the product by faulty time that the glass runs; three-fifths of the result gives the *true* distance.

The following rules for correcting the ship's run, on account of the errors in the log-line or half-minute glass, are given on a supposition that the knot ought to measure 50 feet, and the glass to run 30 seconds.

It would, of course, be much simpler to find the length of the knot for the actual number of seconds run by the glass.

I. *When the Log-line is truly divided, and the Glass faulty*

RULE.—Multiply the distance given by the log by 30; divide the product by the seconds run by the glass, and the quotient will be the true distance.

Example 1.—If a ship sails 8 knots by the log, while the glass is running out, which when measured is found to run 34 seconds, what is her true rate of sailing?

Distance by log 8 knots.
30
34)240(7 knots.
238
2

Example 2.—Suppose the distance sailed by the log be 75 miles, and the glass runs out in 27 seconds, what is the true distance run?

Distance by log 75 miles.
30
27)2250(83·3 true distance sailed.
216
90
81
90
81
9, etc.

II. *When the Glass is true, and the Log-line faulty*

RULE.—Multiply the distance sailed by twice the measured length of a knot; then point off two figures to the right, and the remainder will be the true distance.

Example 1.—A ship sails 9 knots in half a minute, by a log measuring 52 feet; required the true rate of sailing.

Distance by log........ 9 knots.
Twice the length of a knot 104
True rate .... 9·36
or 9 knots 4 tenths nearly.

Example 2.—If a ship sails 195 miles by a log which measures 48 feet, what is her true distance run?

Distance by log.......... 195 miles.
Twice the length of a knot 96
1170
1755
True distance ....187·20 miles.

III. *When the Glass and Log-line are both faulty*

RULE.—Multiply the distance sailed by the log, by six times the measured length of a knot, and divide the product by the seconds run by the glass; the quotient, pointing off one figure to the right, will be the true distance.

Example 1.—If a ship runs 5 knots of a log-line of 45 feet to a knot, while a glass of 25 seconds is running out, what is her true rate of sailing?

Distance run by log ..	5
6 times the length of a knot	45 × 6 = 270
Seconds run by glass	25)135·0
True rate of sailing or 5 knots 4 tenths.	5·4

Example 2.—Suppose the distance sailed by the log be 150 miles, the measured length of a knot being 51 feet, and the glass running 28 seconds; required the true distance run.

Distance by log....	150 miles.
6 times length of a knot	306
	28)4590·0
True distance run	163·9 miles.

IV. *To find the Length of a Knot corresponding to a Glass running any given Number of Seconds*

RULE.—Add a cipher to the number of seconds run by the glass, and divide this by 6; the quotient will be the proportional length of a knot in feet.

Example 1.—What ought to be the length of a knot when the glass runs 33 seconds?

6)330
55 feet.

Example 2.—Required the length of a knot corresponding to a glass that runs 28 seconds.

6)280
46·67 or 46 feet 8 in.

The Ground Log.—An adaptation of the common log is used in shoal water when the ship is drifting in a tideway, or amidst currents, with no land visible, or no distant object is seen whereby to fix the position. A lead, of 4 or 5 lbs., is made fast to a log-line and then cast overboard; thus the lead rests on the bottom, and the rate and drift are indicated, irrespective of current, and can be noted as usual; especially will this log show the *drift* of the current as it is hauled in.

The Dutchman's Log is a very old contrivance, and perhaps not the least accurate. On a ship's rail mark off a given distance. When about to take the rate, an observer must be stationed at each extremity of the distance. A bottle or log of wood is thrown overboard, from the forward station, and forward of the direction in which the ship is progressing; as

the log passes the forward station the time is noted, and similarly noted by the aft observer as it passes the aft station. Thus there is a distance, and an interval of time; divide the distance by the number of seconds of interval, and multiply the quotient by ·6; or if you wish to be more accurate multiply by ·59.

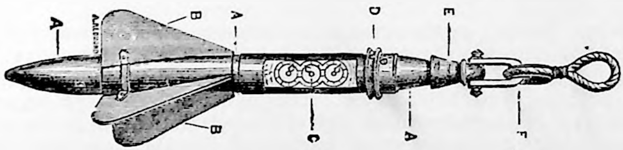
Note.— $\frac{3600}{6080} = \cdot 592$.

Example.—Suppose, distance 190 feet, and interval of passage 15 seconds, then—

$\frac{190}{15} \times \cdot 6 = 7\cdot 6$ Or, $\frac{190}{15} \times \cdot 59 = 7\cdot 5$ miles, rate per hour.

Various logs have been devised at different times—for instance, screw logs, pressure logs, and electric logs. Few successful. The log best known is that originally proposed by Massey.

Patent Log.—In days gone by there were no other mode of finding the ship's rate of progress through the water except by the use of the log and seconds' glass, but the PATENT LOG has now, to a certain extent, superseded the old log-ship, especially in the case of steamers. This instrument, consisting of a rotator and register is kept towing astern with sufficient length of tow-line to carry it out of the immediate wake of the ship, and then the revolutions of the rotator indicate on the register the distance run, which can be ascertained from time to time by hauling it in. The mechanism of the patent log is the same in principle as that of the screw propeller; the rotator, of three or four flanges (B), revolves more or less rapidly according to



the rate at which it is drawn through the water, and, by revolving, sets in motion a system of wheel-work which turns the hands of three indices on the register (C). The distance run, according to the number of revolutions of the rotator, is registered first in ¼ miles, then up to 10 miles, and lastly up to 100 miles.

It is needless to dwell on the patent log, as there are many different kinds by different makers, and "*descriptions for use*" accompany all of them. Up to a recent date it has been necessary to haul in the log to ascertain by the register the distance run; but the latest improvement in the mechanism is the arrangement by which the rotator alone tows, and the register is connected to it *on board*, so that without any hauling in the distance run can be known at any moment. The makers are numerous, and the logs of each have specialities of their own—some shipmasters preferring one maker's instrument, and others another maker's.

ordinary
1" on barometer = $\frac{23}{24}$ in.
Fortin Barometer inch is true

METEOROLOGY

Meteorology is now understood to be the science which deals with the conditions and changes in the atmosphere. That is weather science.

To the sailor the weather is an all-important subject. Strong winds make high seas, and flat calms are often associated with dense fogs. The weather is contained in the atmosphere which, resting upon the earth, extends upwards to about 200 miles, becoming more and more rarefied with distance from the earth.

The average weight of a column of atmosphere is about 15 pounds to the square inch at the sea level. The human body supports a pressure of about 14 tons. The atmosphere is sometimes compared to the earth as the skin to an orange, but it is more true to scale if we say that it is comparable to the tissue paper in which an orange is wrapped.

The atmosphere is composed in volume of 77 parts of nitrogen, 21 parts of oxygen. There are small quantities of carbonic acid gas, argon, and other gases. There is also much water vapour in suspension; the density is measured by the barometer (*see* "Barometer").

The average height at the sea level is 29·92 inches, or 760 centimetres.

At the height of		18,000	feet the pressure is		$\frac{1}{2}$
,,	,,	36,000	,,	,,	$\frac{1}{4}$
,,	,,	60,000	,,	,,	$\frac{1}{10}$

Balloons having meteorographs (a combination of barometer and thermometer) attached have recorded heights of 79,200 feet (15 miles). Modern meteorologists divided the atmosphere into troposphere and stratosphere, an inner and outer layer. The inner, the troposphere, extends from sea level to about five miles high at the poles and seven at the equator, varying also with the height of the barometer. In the inner layer of atmosphere, the troposphere, the temperature of the air varies, uniformly decreasing one degree for every 300 feet of altitude.

The peculiarity of the outer layer, the stratosphere, is that the temperature does not vary uniformly, but is more or less constant over the British Isles at a height of from five to seven miles, the temperature being from — 20° F. to — 80° F. The temperature is highest at the poles, lowest in the neighbourhood of the equator. In Central Africa (Victoria Nyanza) the temperature of the stratosphere is about — 119° F.

The study of the upper air is still in its earliest stages. It is not yet possible to speak definitely of what takes place there. There is, however, a peculiar co-relation in the troposphere between geographical positions widely apart. One of the first to attract attention was the atmospheric oscillation between Iceland and the Azores. It has been found that in a given month, if the pressure at Iceland was above the average, at the Azores it would be below the average, and *vice versâ*. Again, there is a high pressure system over Siberia and the low pressure over the North Pacific; also there seems to be a barometric see-saw between India and

the Argentine. We look forward to the time when, with sufficient observation, a law can be established.

An instance is here given as to how these oppositions and variations are of interest to the navigator.. In a paper in the *Meteorologisch Zeitschrift*, W. Mienardus suggests tentatively that the following phenomena are closely related—

A.—(1) Weak Atlantic circulation during the period from August to February, when the pressure difference between Iceland and the Azores is less than the normal, the consequence of which would mean less strong and less persistent south-westerly winds over that region than is normally the case.

(2) Low water temperatures on the coasts of Europe from November to April.

(3) Low air temperature over Central Europe from February to April.

(4) Little ice off Newfoundland in spring.

(5) Much ice off Iceland in the spring.

B.—(1) Strong Atlantic circulation from August to February. Pressure difference pronounced, gradients steep; southerly winds strong and persistent.

(2) High water temperature on the coast of Europe from November to April.

(3) High air temperature in Central Europe from February to April.

(4) Much ice off Newfoundland in spring.

(5) Little ice off Iceland in spring.

The atmosphere maintains life upon the earth. It diffuses the light and heat of the sun and prevents too rapid radiation. It must be dense enough to store heat, but if it were too dense the sun could not penetrate it and reach the earth. If there is too little, as on mountain tops, there is little heat. We are accustomed to consider the cold of the poles and the heat of the equator as extreme, but the range of temperature is very limited compared with the difference between the heat of the sun, 10,000° F. and the absolute Zero — 459° 4 F. Life is only possible within a limited range of temperature.

Air currents (winds) are caused by the disturbed atmosphere endeavouring to reach a common level, that is, obeying the universal law of gravitation. The disturbance is brought about chiefly by variation in temperature causing warm air to ascend and cold air to descend. There is a notable exception to this rule, when a downward rush of mountain air becomes compressed by increased pressure and gains heat in consequence, just as a bicycle pump gains heat as the air becomes compressed in the tyre. The Föhn wind of Alpine valleys is the most familiar case of descent. This is a strong wind which rushes down from the mountain-tops into the valleys. The wind is very warm and dry. Both are the effect of compression due to the greater pressure at the lower level. It has been found that the temperature increased 1° for every 180 feet of descent. A more notable case is due to the Chinook winds on the eastern slopes of the Rocky Mountains. The wind has been known to sweep down the mountain-side and in twelve minutes dissolve two and a half feet of snow, the temperature rising from — 13° F. to + 36° F. in seven minutes.

The Puna winds are an interesting variant of the mountain winds. To the eastward of Arequipa in Peru there is a barren tableland between two great ranges of the Andes, called the Punos, which for four months of the

year is swept by dry, cold winds. These winds cross the lofty range of the Cordilleras, where they are intensely cooled; they then descend and become a little warmer but intensely dry. Prescott states in his "Conquest of Peru" that it was in this district the ancient inhabitants buried their dead. Bodies of animals exposed to the air soon turn into mummies.

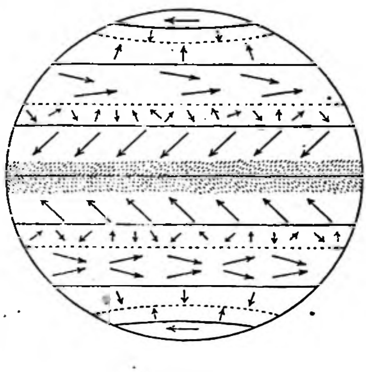
The upward movement of air is not exceptional. A gradually inclined rise will cause condensation, that is, rain; a rapid upward rush, a cloud-burst; an explosive uprush causes hail. The amount of water vapour in the atmosphere may vary from time to time, but for every temperature there is an amount which cannot be exceeded.

At a temperature of 60° there may be as much as 5·8 grains of water to a cubic foot of air, at 40° there will be half that, 2·9 grains. If air at a temperature of 60° at the earth's surface ascends until the temperature is 40°, 2·9 grains of water is condensed.

It has been found that drops ·2 inch in diameter fall at the rate of 25 feet per second; larger drops would fall faster but they cannot hold together, they break into smaller drops. If the upward rush of air is more than 25 feet per second the moisture is held suspended in the air, to fall ultimately as rain when the conditions are favourable. If the rain-drops increase in size in saturated air until there is a great accumulation and the ascending motion is checked, the suspended vapour falls through the force of gravitation in such quantities as to be called a cloud-burst.

If the uprush is very rapid the consequent cooling may be so great that the rain-drops are frozen, thus hailstones are formed. Some hailstones show concentric markings, caused, it is supposed, by the hailstones falling because of gravitation, then being carried upward again by an explosive uprush of air, that is, tossed up and down. Large hailstones are seldom seen in the British Isles, but on the continent of Europe and the other continents great hailstones do much damage, and a large business is done in hailstone insurance.

The ordinary heating and cooling of the air is responsible for two main movements, an upward movement called convection and a horizontal movement, translation, and wind, as we feel it, is a resultant of these components. As a consequence great masses of atmosphere move from place to place in great ovals, such as air expanding and rising about the equator and passing towards the pole, and to compensate for the loss of balance the air at a lower level passes from the poles to the equator, forming in a remarkable manner such air currents as are called Trade Winds. The laws that govern the air currents of our planet are not yet thoroughly understood; at present they appear very confused and complex. In a general way they are supposed to follow something of the following order.

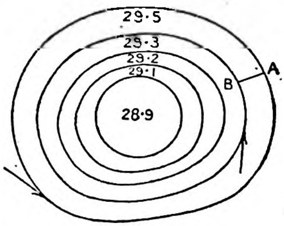


A neutral rainy calm belt about the Equator, then N.E. and S.E. trade winds moving to the N. and S. with the sun within the tropics, then a

variable zone extending between the trade winds, and the strong westerly winds peculiar to the forties in both hemispheres, but because of the absence of land more pronounced and constant in the Southern Hemisphere.

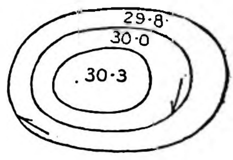
To the north in the Northern Hemisphere and to the south in the Southern Hemisphere there is an area of low pressure towards which the air currents flow from both north and south, causing variable winds. Within the Arctic and Antarctic circles easterly winds are said to prevail.

Because of hot and cold currents of water, presence or absence of land, mountain, and desert plain, conditions will alter as and where the different conditions exist, but in a general way the foregoing gives a fair outline of the great air movement. In detail, it is found that outside the great polar-equator current the winds are inclined to flow round centres of low and high pressure, forming what is called cyclonic and anticyclonic disturbances. The circular movement of the air is best studied from what are called Synoptic Charts (synoptic meaning seen at the same time). A number of barometric readings covering as large an area as possible are corrected to one level and one temperature, and then plotted on a chart. Places having the same readings are connected up by a traced line and are called isobars. Very frequently these lines so form that they make circles or ovals; the lowest reading will have the smallest circle, the highest reading the greatest circle. The strength of the wind may be estimated by the distance the isobars are apart—that is, the closer the isobars are the steeper the gradient. By gradient is meant the perpendicular distance, as A B. The difference between A B is divided by the difference between the readings; the result is the gradient, which will be inversely proportional to the perpendicular distance between the isobars; it follows that if the distance between the isobars on one chart is half what it is on another the wind velocity of the closer lines should be twice that of the other. Theoretically, the wind should follow the circles or isobars and probably does at from half a mile to a mile above the surface of the earth, but at the surface the wind is always inclined towards the centre of low pressure, as indicated by the arrows.



The statement of the relation between the direction of the wind and the barometric pressure is known as Buys Ballot's law.

In the Northern Hemisphere stand with your back to the wind and the region of lowest pressure will be on your left-hand side and slightly in front of you; in the Southern Hemisphere the reverse. Sailors have generally discussed this rule as facing the wind, reversing the rule. Anti-cyclones are the reverse of cyclones and are typical of fine weather. The highest reading is to be found at the centre; the wind revolves in the direction of the hands of a clock in the Northern Hemisphere and in the other direction in the Southern Hemisphere. It will be seen that the isobars are closer at the circumference than at the centre, and the winds increase in force proportionately. The paths followed by the centres of cyclones in the temperate zones are



not constant, but the general movement is toward the east. Motion from east to west is not unknown, but is very rarely observed. They move from 20 to 30 miles per hour, but are sometimes almost stationary. The erratic path of these storms has rendered the approach of storms cabled from America, or reported by wireless from ships, of little practical use. More knowledge is required before such reports can be used effectually.

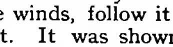
TROPICAL CYCLONES.—The cyclones above described are those of the temperate regions experienced chiefly between the parallels of 40° and 50°. The tropical cyclone is both more definite in form and constant in its progress, and to such a degree that certain laws called the "Law of Storms" have been compiled for the use of seamen, and which, reduced to a minimum, are published in all the Notices to Mariners published by the Hydrographic Office.

The cyclones discussed above are the usual type of disturbed weather where the barometer fluctuates irregularly and considerably.

Within the tropics the barometer fluctuates regularly and slightly day after day, the highest reading being obtained between 9 and 10 a.m. and p.m., and the lowest reading between 3 and 4 a.m. and p.m. Total fluctuation 0·413. The barograph shows a very constant zig-zag after this manner

the amount of the rise and fall being about ·07 of an inch. If this "diurnal range," as it is called, is a regularly recurring phenomenon, it follows that a suspension or cessation of its regularity must indicate an interference with natural law, and trouble must follow; therefore it is the first indication of a "change" for the worst and is preceded by the barometer *not* rising between 4 and 10, followed by a steady and rapid fall.

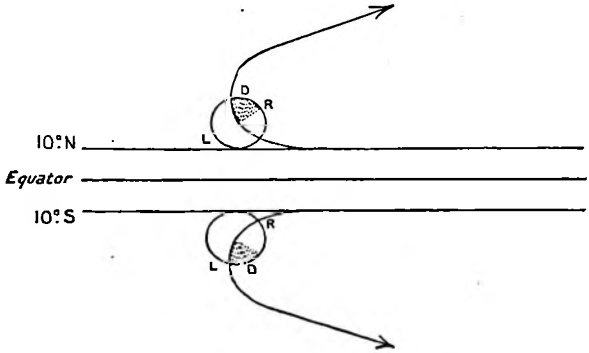
Plotted on a chart the isobars would be nearly circular and very close together, showing exceedingly steep gradients. The causes that generate these tropical storms are yet under discussion. It is found that they, like other cyclones, revolve against the hands of a clock in the Northern Hemisphere and with the hands of a clock in the Southern Hemisphere. This is supposed to be caused by the earth's rotation, a body of air travelling more rapidly on the equatorial side of the storm than on the polar side, which must give the air a twist and compel it to move as described. The storm has also the movement of translation from east towards west; in this particular it is different from the temperate zone cyclone. This is brought about by the intensity of the circular movement giving the storm a gyroscopic action, thus forcing it to remain in position while the earth turns on; it then follows the line of least resistance, trends polarwise, then expanding itself, quickly loses its gyroscopic action and gradually becomes incorporated with the temperate zone system or is lost. The cyclones generally form on the equatorial border line of the trade winds, follow it along, and break through where the trades are weakest. It was shown by Buys Ballot's law that the wind does not follow the isobar circle, but inclines towards the lower pressure (the centre) thus



; therefore the circle is really a spiral trending inwards. If a vessel were to run with square yards she would not make circles, but would (following the arrow)

gradually work her way towards the centre. It is always the object of the seaman to avoid the centre.

The line followed by the centre of the storm is called the line of progression, and it therefore follows that one-half of the storm-field must lie upon the right of this line and the other upon the left. This progression causes the wind on the right-hand side to shift right-handedly in both



hemispheres and on the left-hand side to shift left-handedly in both hemispheres. In the Northern Hemisphere the right side of the line of progression is the most dangerous semicircle and in the Southern Hemisphere the left-hand semicircle; this is for two reasons: first because the winds blow towards the line of progression (therefore the centre) and also the course of the storm moving W and N or W and S according to hemisphere compresses the air in the forward quadrant marked D, thus intensifying its action.

The opposite side is not so dangerous, since the wind does not blow towards the line of progression but away from it, and the air has room for expansion. If the storm remained stationary—that is, if it had no movement of translation—the wind would continue to blow from the same direction, no matter what part of the storm-field the vessel was in; this also occurs when the vessel is on the line of progression in front of the storm. A reference to the diagram will show clearly what should be done in the three possible cases—

(*a*) When in the most dangerous semicircle.
(*b*) When in the least dangerous semicircle.
(*c*) When on the line of progression.

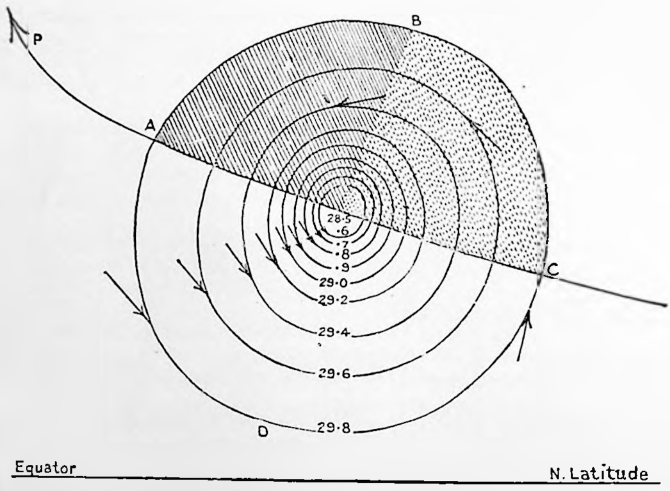
The object is to get away from the centre. It has been shown that if a vessel took the wind aft she would pass from one isobar to the other until she reached the centre or lowest depression.

(*a*) It follows (in the Northern Hemisphere) that if the wind was brought on to the starboard quarter the vessel would sail on an isobar. This would be of no advantage, therefore the vessel must be brought as close to the wind as possible; that is, in a sailing vessel, on the starboard tack, in a

steam vessel with the wind a little on the starboard bow. By so doing, if the vessel makes any headway she increases her distance from the centre; she also follows the wind movement to the right and heads the sea, and the storm moving along the line of progression increases the distance between the vessel and the centre of the storm.

(*b*) In the least dangerous semicircle there is liberty of action. The wind can be brought on to the starboard quarter or more abeam if possible, thus rapidly increasing the distance between the vessel and the storm-centre.

(*c*) When on the line of progression the least dangerous side should be entered with all possible speed (*see* "Concise Rules for Revolving Storms").



AB the most dangerous quadrant; ABC the most dangerous semicircle; ADC the least dangerous semicircle; PAC the line of progression.

Diagram showing the rotary motion of a circular storm in the Northern Hemisphere.

If the storm remained stationary, as they sometimes do, there would be "no shift of wind," only an alteration in direction as the vessel sailed towards or from the centre of the storm. In such a case in the Northern Hemisphere the ship should carry as much sail as she could with the wind as far as possible forward on the starboard side. In the Southern Hemisphere take the wind on the port side and make as much distance as possible. Stationary cyclones are most common in the Indian Ocean at the beginning and end of the season.

The problem seems complicated, because in most cases the storm has a progressive motion, and this causes the wind to shift as the storm passes over the vessel.

It should be understood that the rotary motion of the storm causes the wind to fly from the circumference in a constant direction, therefore the wind is *always* to be found from the same direction in the same part of the storm-field.

Because of the progressive motion the seaman has no liberty of action when on the most dangerous side; he can only hang off to the best of his ability, whereas when on the least dangerous side he may run away from the storm-centre, that is, if there is sea room.

GENERAL NOTICES

Concise Rules for Revolving Storms

1. Revolving storms are so named because the wind in these storms revolves round an area of low pressure situated in the centre. They have also local names, and are termed hurricanes in the West Indies and South Pacific Ocean, cyclones in the Indian Ocean, Bay of Bengal, and Arabian Sea, and typhoons in the China Sea.

2. In these storms the wind always revolves in the same way in the same part of the world; that is, against the movement of the hands of a watch in the Northern Hemisphere, and with the hands of a watch in the Southern Hemisphere. The wind does not revolve in circles, but has a spiral movement, inwards towards the centre.

3. Revolving storms have also, as a general rule, a progressive movement. Within the tropics they usually move from east to west at first, and then curve towards the pole of the hemisphere in which the storm is generated, and afterwards move from east to west.

4. The track which the centre of the storm takes is called the path of the storm, and the portion of the storm-field on the right of the path is known as the right-hand semicircle, and that on the left as the left-hand semicircle of the storm.

5. In the right-hand semicircle, if the observer be stationary, the wind will always shift to the right, and in the left-hand semicircle to the left. This law holds good in both hemispheres.

6. If a vessel be so situated in a storm that by running before the wind the path of the advancing storm will be crossed, this is considered to be the dangerous semicircle. This will always be the right-hand semicircle in the Northern Hemisphere, and the left-hand in the Southern.

7. These storms are most frequent in the Northern Hemisphere from July to November, and in the Southern Hemisphere from December to May. In the Bay of Bengal and Arabian Sea they, however, occur most frequently about the time of the change of the monsoon.

8. The area over which revolving storms have been known to extend varies in diameter from 20 miles to some hundreds of miles, and their rate of movement in the West Indies averages about 300 miles a day; in the China Sea, Bay of Bengal, and Arabian Sea about 200 miles a day, the more stationary storms occurring at the beginning and end of the hurricane season.

9. The indications of the approach of a revolving storm are: (i) an unsteady barometer, or even a cessation in the diurnal range which is

constant in settled weather ; (ii) a heavy swell not caused by the wind then blowing ; (iii) an ugly, threatening appearance of the sky.

10. In order to judge what is the best way to act if there is reason to believe a storm is approaching, the seaman requires to know (*a*) in which direction the centre of the storm is situated, (*b*) in which semicircle the ship is situated.

11. As these points cannot be determined if a vessel is moving with any speed through the water, the first proceeding should be to stop " heave to " and, as it is always best to assume, at first, that the vessel may be in a dangerous semicircle, she should be hove to on the starboard tack in the Northern Hemisphere and on the port tack in the Southern.

12. If an observer faces the wind the centre of the storm will be from 12 to 8 points on his right hand in the Northern Hemisphere and on his left in the Southern Hemisphere, 12 points when the storm begins, about 10 when the barometer has fallen $\frac{3}{10}$ of an inch, and about 8 points when it has fallen $\frac{6}{10}$ of an inch or upwards.

13. If the wind shifts to the right the vessel is in the right-hand semicircle ; if to the left in the left-hand semicircle ; and if the wind is steady in direction but increasing in force she is in the direct path of the storm.

14. If the seaman has reason to think that his vessel is in the direct path of the storm he should run with the wind on the starboard quarter in the Northern Hemisphere and on the port quarter in the Southern Hemisphere, until the barometer has ceased falling ; this would take the vessel into the less dangerous semicircle in both hemispheres. If she is in the right-hand semicircle in the Northern Hemisphere she should remain hove to on the starboard tack, but if in the Southern Hemisphere she should run with the wind on the port quarter ; if she is in the left-hand semicircle in the Northern Hemisphere she should run with the wind on the starboard quarter, but if in the Southern Hemisphere remain hove to on the port tack.

15. Should a vessel not have sufficient room to run when in the least dangerous semicircle, she should heave to on the port tack in the Northern and on the starboard tack in the Southern Hemisphere.

16. If in a harbour or at anchor, the seaman should be just as careful in watching the shifting of the wind and ascertaining the direction of the centre, as by doing so he will be able to tell on which side of the path of the storm he is situated, and be able to act according to circumstances.

17. Should the centre of a storm pass over a vessel the wind, after blowing furiously in one direction, ceases for a time, and then blows with equal fury from the opposite direction. This makes a pyramidal sea, which is especially dangerous.

18. Should the wind be west the vessel should remain hove to and the storm will pass away from her.

GENERAL NOTICES.

Sea Disturbance Scale.

Scale.	Description.	Height of waves in ft. from crest to trough.	Condition of surface.	Scale.	Description.	Height of waves in ft. from crest to trough.	Condition of surface.
0	Calm	—	Glassy	5, 6	Rough to very	5 to 10 ft.	Much disturbed; deeply furrowed
1, 2	Smooth	—	Rippled	7, 8	High to very	11 to 15 ft. 16 to 35 ft.	Rollers with steep fronts.
3	Slight to moderate	Under 5 ft.	Rocks buoys or small boat Furrowed	9, 10	Phenomenal	36 ft. and above	Precipitous: towering.

WIND FORCE.

Figures to denote force of wind.	Description of wind.	Mode of estimating on board sailing ships.	Equivalent velocity of wind in miles per hour.
0	Calm		0
1	Light air	Sufficient wind for working ship	2
2	Light breeze		5
3	Gentle breeze		10
4	Moderate	Forces most advantageous for sailing with leading wind and all sail drawing.	15
5	Fresh breeze		21
6	Strong breeze	Reduction of sail necessary with leading wind.	27
7	Moderate gale		35
8	Fresh gale	Considerable reduction of sail necessary even with wind quartering.	42
9	Strong gale		50
10	Whole gale	Close reefed sail running or hove to under storm sail.	59
11	Storm		68
12	Hurricane	No sail can stand even running.	above 75

OCEAN CURRENTS

The ocean currents are caused by the turning of the earth on its axis, by the differences of temperature in different parts of the ocean, and the prevailing winds; all of which cause a displacement of the water, while gravitation compels the restoration of equilibrium. There is therefore a close parallel between the air currents (wind) and the ocean currents, the more definite action of the water currents being due to its greater specific gravity.

If the water were to flow continuously from one point, as the Gulf Stream, for instance, the gulf would run dry, but the outward flow is balanced by an inward flow from the equatorial current, which in turn is supplied by other currents from north and south, thus making a complete chain of currents the seas over. There is, in fact, two well defined systems of currents in the Atlantics. That to the north of the equator circulates with the hands of a watch, the centre of which is the Sargasso Sea. To the south of the equator the currents circulate against the hands of a watch; this circulation occurs in a lesser degree in the Pacific and Indian Oceans, thus forming the great ocean currents.

Atlantic Ocean

The equatorial current commences on the coast of Africa in the vicinity of Anno Bom Island between long. 2° and 8° E. and runs to the westward between 2° N. latitude and 10° S. latitude with an average velocity of 30 miles, but occasionally attains a rate of 60 to 70 miles a day.

Off San Roque it splits, one part running to the southward forming the Brazil Current, which flows down the South American coast as far as Rio de la Plata at a distance of about 150 miles from the main; it then recurves to the eastward, forming the South Atlantic connecting current. The other part of the equatorial current flows along the north coast of South America with a velocity of nearly 100 miles a day and flows through the Caribbean Sea round the Gulf of Mexico and through the Straits of Florida, forming the commencement of the Gulf Stream.

The Gulf Stream

After leaving the Strait of Florida, where it is 50 miles wide, it gradually expands, flows to the northward for about 300 miles, then turns towards the north-east, then more east across the Atlantic right on to Norway. Off Cape Hatteras the width is about 120 miles, and beyond Bermuda 250 miles wide. It attains its greatest velocity, 120 miles a day, in the Strait of Florida.

The Arctic Current

This flows out of Davis Strait over and around the Bank of Newfoundland along the American coast inside the Gulf Stream. A part enters the Strait of Bellisle and flows down the western side of the island. Where the

Arctic current and the Gulf Stream join is well marked; the warm stream is blue and the cold stream green. The current has little strength.

The north-east trade drift is due to a wind current and flows very slowly to the westward and southward before the wind and joins the equatorial current in the neighbourhood of the West Indies.

The Rennel Current

This is an easterly current from the Atlantic, perhaps a portion of the Gulf Stream arrested by the westerly winds. It strikes the land on the N.W. corner of Spain and divides, one part going south along the land and towards the Straits of Gibraltar; the other part sweeps round the Bay of Biscay, passing about 15 to 20 miles off Ushant, and flows to the N.W. across the mouth of the English Channel.

The Mediterranean

The current sets in from the Atlantic and is caused chiefly by the great evaporation which goes on in the Mediterranean; generally the current sets east along the African coast, and to the west from the eastern end along the coast of Europe.

The Guinea Current

This current flows along the African coast to the eastward between Cape Roxo and the Bight of Biafra, extending southwards to about 3° of N. latitude. It extends to the westward as far as 23° W. longitude, extending more to the westward during the season, July to November. It attains a velocity of about 3 miles an hour off Cape Palmas and is warmer than the equatorial current.

The Cape Horn Current

This is an easterly current caused chiefly by prevailing winds. After passing Cape Horn it gravitates towards the north and in about 40° south latitude joins the South Atlantic connecting current.

The circulation is completed by a small portion of the Agulhas current, which passes over the banks of that name, and branching off to the N.W. joins up with the South Atlantic Connecting Current.

Indian Ocean

The equatorial current is broken into by the prevailing monsoon and does not run so true as in the Atlantic.

The S.E. trade drift is a current caused by the wind and runs to the westward at the rate of 20 to 25 miles a day between the parallels of 8° S. and 27° S. latitude. It separates on the east side of Rodriguez Island into two parts, one towards the north end of Madagascar at the rate of 40 to 60 miles a day, and the other part sweeps to the south end at the rate of 50 miles a day.

The northern branch again splits near Cape Delgado in latitude 11° S. on the African coast, one part running southward through the Mozambique Channel, in some places at the rate of 4 to 5 miles an hour.

In the latitude of Natal it again joins the stream that went south of

Madagascar and forms the Agulhas Current which flows south-west and west along the African coast at a distance varying from 5 to 120 miles, attaining a great velocity between Port Natal and the twenty-third meridian, making from 4 to 4½ miles per hour, its greatest strength being near the bank. It then in the main follows the edge of the bank, is deflected to the southward and then to the eastward, flowing back into the Indian Ocean with diminished speed and a lower temperature as far as 40° S. latitude. As before stated, a small portion flows round the coast to the N.W. The part that turned north off Cape Delgado runs north and north-east at the rate of 2 to 4 miles per hour and during the S.W. monsoon to the eastward across the Arabian Sea. The current runs out of the Red Sea during the S.W. monsoon and into it during the N.E. monsoon. In the Gulf of Suez and the Red Sea generally the currents are caused by the prevailing winds.

Pacific Ocean

The equatorial current, as in the other oceans, runs to the westward from near the American coast towards the east coast of Australia between the parallels of 5° N. and 20° S. latitude at a rate of half a mile to 3 miles per hour. It turns to the eastward when 40 to 60 miles from the Australian coast and helps to form the equatorial counter current which flows between the parallels of 5° and 8° north latitude.

The N.E. trade drift runs to the westward between 9° and 20° north latitude and is deflected by the Philippine Islands to the northward, forming the commencement of the Kino Sirvo which flows along the east coast of Formosa and Japan, sweeps along the south-eastern coast of Japan until it reaches the parallel of 50° N., where it is known as the Kamschatka current. The Oya Sirvo is a cold water current from the Kamschatka and Kuril Islands running southward along the east coast of Yezo Island, the N.E. coast of Nipon, and inside the Kino Sirvo.

China and Java Seas

In the China Seas the currents set with the prevailing monsoon at a varying rate of a half to 2 miles per hour, but on the east coast of China and to the eastward of the Pescadores Islands the northerly current sometimes runs 3 to 4 miles per hour.

The currents in the Java Seas are influenced in the same manner by the prevailing monsoon, and are generally stronger in the N.W. than in the S.E. monsoon, in the former, running at the rate of a mile an hour, in the latter, at half a mile an hour. Where the waters are confined in narrow channels the currents run much more rapidly and are complicated by the tides and are therefore uncertain and irregular.

The student will notice that, throughout, one current flowing away induces another, forming complete chains wherever the seas run.

For more detailed information on "Ocean Currents" see the Sailing Directions for the various Oceans.

INTRODUCTION TO NAUTICAL ASTRONOMY

To understand that branch of navigation which is based upon nautical astronomy it is necessary that the student should have a clear conception of the earth's position in the solar system and in the stellar universe. The following chapter is written for that purpose, and is purely elementary.

For further information consult "Elementary Lessons in Astronomy" (Lockyer), "The Story of the Stars" (Chambers), "Introduction to Astronomy" (Moulton), etc.

Imagine the earth reduced to a globe a few feet in diameter, and upon which we are so situated that we can see the sky in all directions. The sun's light would pervade everywhere, we should see nothing but, perhaps, a faint reflection from the moon.

To enable us to see the stars the sun's light must be blotted out; the planets, their satellites, and the moon would also disappear, because they shine with light reflected from the sun. We should be in complete darkness except for the faint light received from the stars. With normal sight we should see scattered about the sky about seven thousand stars, probably less—not the countless numbers generally supposed. All the stars would be in view, but nothing new, only the familiar stars in lonely splendour or clustered in beautiful constellations. They would be spread before us apparently in endless confusion, but with a little acquaintance they soon take order and are easily recognised.

If we look up into the northern sky we shall see Polaris, the North Star in the constellation of Ursæ Minor, the North Pole of the heavens, round which all the stars in the Northern Hemisphere swing. Far down in the southern sky faintly gleaming close to the South Pole can be seen Sigma Octantis; midway between lies a broad band of stars sweeping right round the heavens, the signs of the Zodiac, that thousands of years ago represented the months and seasons; a starry calendar, recording the sun's annual progress, and still familiar to us as Aries, representing Spring, Cancer, the Summer, Libra, the Autumn, and Capricornus, Winter.

An ancient rhyme gives the signs thus:

> The Ram, the Bull, the heavenly Twins,
> And next the Crab the Lion shines,
> The Virgin and the Scales;
> The Scorpion, Archer, and He-goat,
> The man that holds the water-pot,
> And Fish with glittering tails.

Put into their latinised names they are: Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, and Pisces. Between this equatorial band and the poles are many other constellations, stars of various brightness and colour, all suns much like our own sun, shining with their own light, but at such an immense distance that they are reduced

to a point of light. No ordinary measurement can convey any idea of their distances. The measurement commonly used is the distance light travels in a year, and since light travels 186,333 miles per second, the distance in a year will be equal to $186,333 \times 60 \times 60 \times 24 \times 365$—an inconceivable figure. Yet the nearest star to the Solar System, α Centauri, is distant four and a half years, and the most remote are estimated to be from 2,000 to 3,000 years distant. We can form a fair idea of the distance in another way. If we had a Morse flash-lamp powerful enough to reach the stars we could call up the moon in one and a half seconds, the sun in a little more than eight minutes, the nearest star in four and a half years. In the centre of this tremendous space the sun, "simply one star in the universe" is situated, surrounded by his family of eight planets. From this great multitude of stars the navigator selects about twenty of the brightest to assist him to find his way across the seas, but he must know them with precision. How is he to do this?

At this stage the astronomer comes to his aid. He draws horizontal and vertical lines (really portions of immense circles) upon the surface of the sky on the same principle that geographers draw parallels of latitude and meridians of longitude upon the earth's surface. The earth's poles are extended indefinitely until they reach the sky, the North Pole almost touching Polaris, and the South Pole Sigma Octantis. The earth's equator is also extended till it meets the stars, but this greater circle is called the equinoctial. Lines are drawn parallel to the equinoctial both above and below until the poles are reached. They are really circles which diminish in circumference, parallels of declination. Then great circles are drawn from pole to pole, cutting all these parallel circles at right angles, and are called meridians or hour circles. Imagine the sky now ruled by parallel and vertical lines, which, at the equinoctial, form squares, and at the poles triangles. It is only necessary to mark the intersection of parallel and hour circle to be able to fix the position of any object in the heavens. But two starting-points are necessary from which to measure, one upwards and downwards, the other to the right or left.

The equinoctial supplies the first from which declination is measured in degrees from 0° to 90° north or south. The second point requires a little explanation.

The sun in the course of a year traces a great circle among the stars, called the ecliptic; the equinoctial is also a great circle. If the ecliptic is horizontal the equinoctial is tilted about $23\frac{1}{2}°$, and as the one circle fits the other they must cut in two exactly opposite points—one where the sun ascends into northern declination, the other where it descends into southern declination.

Astronomers closely observed where the sun crossed the equinoctial going from south to north declination, and when its exact centre was on the intersection of the two circles a mark was made through them on to the sky beyond. As it happened, this mark did not fall on any star, it was simply an imaginary mark in a group of stars called Aries; it was called a point, a first point, the first point of Aries, and thus the second place from which to measure to the right or left was established.

If the mark was made to-day it would be 30° more to the westward, in the constellation of Pisces. This is due to a backward movement of about 50″ annually, called the precession of the equinoxes. It is caused

by the action of the sun and moon upon the earth's equatorial protuberance. They pull the equator downwards, causing the crossing point to constantly recede. Having established a point in the sky it is used as Greenwich is used, the first meridian is drawn through it, but with this difference —longitude is measured in degrees east and west from Greenwich, but from the first point of Aries right ascension is measured in hours, minutes, and seconds right round eastward from 0 hours to 24 hours. We now have a complete set of vertical and horizontal lines. Suppose the beginner knows one bright star, Sirius, for instance, and wished to find Rigel. He simply compares the right ascension and declination—

	H.	M.		°	′	
Sirius R.A.	6	40	Dec.	16	34 S.	Dep. 22°·25 and d. lat. 8°·25 in the Traverse Table give N. 70° W. dist. 24°.
Rigel ,,	5	11	,,	8	20 S.	
Difference	1	29	d. lat.	8°	14′	N. B.—The degrees are used as minutes in using the Table.
dep.	22°	15′				

Therefore Rigel will be found about N. 70° W. 24° distant from Sirius. Any other celestial object can be found in the same way. The right ascension and declination of 56 bright stars is given in Norie's Tables.

It will be seen that the 7,000 visible stars are not scattered without order through space, but that each star has its name or number, its right ascension and declination recorded. The stars visible to the naked eye have been gathered into groups called constellations. In very ancient times they were given the names now familiar to us, of men, birds, beasts, and fishes. In the constellations, generally speaking, the brightest stars have proper names, and in addition take the first letter of the Greek alphabet and the name of the group. Sirius is also α Canis Major, the Great Dog, Rigel, is β Orionis, and so on. When the Greek letters are exhausted the remaining stars are numbered in Flamsteed's catalogue, and when these are used up Lalande's and Lacaille's, etc., are referred to. Altogether, ancient and modern, there are 84 constellations. The most casual observer will notice that stars differ considerably in brightness, or, as it is called, magnitude. The term magnitude means their apparent brightness, not their size, which in most cases is not known. The visible stars are divided into six magnitudes. On reference to the Nautical Almanac and other stellar tables, it will be seen that the marking is peculiar. Sirius, the brightest star in the sky, is marked—1·4, and Canopus, the brightest southern star, —1·0; Capella is 0·2, Aldebaran 1·1, Markab 2·6, Orionis 4·6, etc. If stars like Vega 0·1 are used as a standard then Sirius and Canopus are brighter by 1·3 and ·9 respectively, and the others less bright according to the figures attached. As the magnitudes decrease so do the numbers of the stars increase, especially as the limit of vision is approached. A little attention will also discover that the stars are of different colours. Some are white with a bluish tinge like Sirius or Vega, some yellow like Capella, some have an orange colour like Arcturus, and some are red like Aldebaran, Antares, and Betelguese. The stars have what is called an apparent and a proper motion. This is very small and can only be detected by a telescopic observation The apparent motion is the movement as seen from the earth, the proper is the actual movement deduced by calculation in reference to the sun. These motions are of no consequence to the navigator beyond the

small correction they entail in looking out the stars' right ascension and declination, which are practically constant. The spectroscope shows that stars differ considerably in physical constitution. The camera and the spectroscope have in recent years added immensely to our knowledge of the stars.

Hitherto we have spoken of the earth as if it were the actual centre in space. This is not so, the sun is the centre of *our* system; it is simply another star, which, if placed at the same distance from us as α Centauri, would not shine so brightly.

The sun has a family of eight planets, which swing round it in circles varying in size and in the following order: Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune.

They are said to revolve round the sun in orbits of varying diameter in the plane of the ecliptic. The ecliptic may be compared to the horizon extended indefinitely, and the plane of the ecliptic to a boundless sea, wherein the sun and all the planets float half immersed. The sun, planets, and many other minor bodies, such as the moon, the planetoids, satellites, the asteroids, the meteors, and about 30 comets, make up the Solar System. It will assist us if we could imagine a model of it in proportion somewhat as follows:

First, to represent the ecliptic imagine an immense sheet of glass, circular in shape, with a radius of one and a quarter miles. The sun and all the minor planets should be half immersed in the ecliptic. At the centre should be placed the

SUN, a globe 2 feet in diameter,
MERCURY, a small pin-head distant 82 feet,
VENUS, a small pea distant 142 feet,
EARTH, a moderate-sized pea distant 215 feet,
MARS, a large pin-head distant 327 feet,
ASTEROIDS, grains of sand distant 500 to 600 feet,
JUPITER, a moderate-sized orange distant $\frac{1}{4}$ mile,
SATURN, a small orange distant $\frac{2}{5}$ mile,
URANUS, a good-sized cherry distant $\frac{3}{4}$ mile,
NEPTUNE, a good-sized plum distant $1\frac{1}{4}$ miles,

that is on the margin of the sheet of glass. Although the diameter of the circle through which the outside planet (Neptune) sweeps is several thousands of millions of miles of which the sun is the centre, it is a small thing compared with the distance of the nearest star, and compared with the more distant stars a negligible quantity.

Thus the sun and its planets float in space quite alone, the starry heavens nothing more than a far distant background.

For the rest of this chapter we must consider the earth in its full size and proper place among the planets.

It was desired in describing the foregoing model of the solar system to convey the impression that all the planets were floating round the sun in the same plane. This is not strictly correct, because each planet has its own plane, tilted a little more or less from the plane of the ecliptic, but for our purpose no error will be introduced if we continue to consider them

all in one plane. If all our models were furnished with visible poles it would be seen that none of them floated upright; each one would have a tilt or list. The earth's (the only one of interest to the navigator) has an inclination towards the ecliptic of $23\frac{1}{2}°$; this is the cause of the change of declination, for if the earth floated upright there would be no change in declination and no seasons. The declination and its changes are wholly due to the inclination of the earth's axis.

The so-called movement of the sun and the tilting of the ecliptic seem somewhat upside down, but they follow the common explanation which supposes the sun to go round the earth instead of the earth round the sun.

The planets Mercury and Venus revolve in orbits close to the sun, within the earth's orbit, consequently they can be seen only a short time before sunrise or a short time after sunset, always near the sun, Mercury very close. They are called inferior planets because their orbits are inferior in size to the earth's. The five outer planets are called superior planets for the reason that their orbits are greatly superior to the earth's; and because they revolve in orbits outside the earth's they may be seen in all parts of the sky, within the limits of their declination. If their path were traced in the sky they would not describe a steady forward course, like the sun and moon, but would go forward for a time, then make a loop, then forward again. They appear to wander about, hence their name—planet, or wanderer. The wandering is brought about by each planet, the earth included, having different sized orbits around which they travel at different speeds, and although each planet goes steadily onward, the earth appears to overtake a planet or is overtaken in turn and occasionally goes along for a time at apparently the same rate. This movement, with some change in the declination, seems to give a planet a very erratic motion. You will observe that the planets move among the stars like one of them, but they are infinitely smaller bodies close to us, and they shine with reflected light, and are humble companions of the sun, having no part with the glorious orbs of fire that lie ages of miles beyond them.

The earth and each of the superior planets have moons or satellites. The earth, one (the Moon); Mars, two; Jupiter, seven; Saturn, ten; Uranus, four; Neptune, one.

In ordinary navigation, the only satellite used is the Moon.

Tables of the movements of Jupiter's satellites are given in the Nautical Almanac; they are chiefly used by astronomers for finding the mean time at Greenwich.

It was the difference of time between the entrance and exit of the satellites across Jupiter's disc at its nearest and farthest distances that the speed with which light travels was confirmed.

The moon floats in the ecliptic almost upright, its plane is tilted from the ecliptic about 5°. It revolves on its axis in the same time that it takes to go round the earth, about 28 days, consequently we see but one side of the moon. It moves across the stars at a mean rate of about 51 minutes in the 24 hours from west to east, a very quick motion. This quick motion was early taken advantage of to determine the mean time at Greenwich and thence the longitude. Flamsteed, the first Astronomer-Royal, was established at Greenwich Observatory in 1675, chiefly for the purpose of calculating Lunar distances. The moon's distance from nine bright stars in her path, four planets and the sun, is calculated for

each three hours of Greenwich mean time and tabulated; then at any time, day or night, the moon being visible and the weather suitable, the distance is measured with the sextant between the moon and one of the tabulated bodies, and at the instant of observation the time is noted. If the corrected measurement agrees with the tabulated distance the time under which it occurs is the Greenwich mean time recorded; if the exact distance is not obtained then the time is found by proportion. This is the only independent method the navigator has of finding the longitude at sea. The mean distance of the moon from the earth is about 240,000 miles; its close proximity causes the right ascension and declination to alter very rapidly, and for this reason it is not used in ordinary navigation with the frequency of slower moving bodies, so many corrections being required.

In the foregoing explanation the earth's track round the sun is described as circular. Very few orbits are circular, they are mostly elliptical. The earth's orbit is an ellipse, the sun is not at the centre but nearer one end than the other. This brings the earth nearer the sun in winter by 3,000,000 miles than in summer. If the earth's orbit were circular the apparent and mean day would be alike, the clock and the sundial would always agree, but the orbit being an ellipse causes the clock and the sundial to disagree, necessitating what is called the equation of time. A simple figure will show how it occurs. Draw an ellipse; on the major axis place the sun a little nearer one end than the exact centre. Using the centre of the sun as a point, describe inside as large a circle as the ellipse will permit. Divide the circle into eight or more equal parts, produce the radii to the ellipse. It will be found that the circle will be divided into equal segments and the ellipse into unequal segments; the difference represents the equation of time and is used to make the time shown by the circle, which is mean time, agree with the time shown by the ellipse, which is apparent time, or vice versâ.

The earth turning upon its axis causes the heavenly bodies to rise and set. They move in great arcs across the sky with a precision and regularity that man vainly attempts to emulate. At every instant of time some celestial body is recording the hour angle or is indicating a true point on the horizon, or when reaching the meridian is recording the latitude. The navigator with his sextant, his chronometer, and his compass presses the sun, moon, and stars into his service and thereby guides his vessel over the trackless seas. How this is done, it is the province of another chapter to describe.

NAUTICAL ASTRONOMY

DEFINITIONS AND PRINCIPLES

In the previous pages the position of a ship at sea has been determined by that branch of Navigation which is mainly connected with the use of the Log and Mariner's Compass, the Lead, and the Chart; the methods, based on plane trigonometry, are sufficiently accurate; but the results obtained from the instruments are to a certain extent imperfect; hence the place of a ship is only *approximately* determined, and it is usually called the position by *Dead Reckoning* (D.R.), and sometimes *by Account.*

That branch of Navigation which determines the place of a ship by observations of the heavenly bodies is called NAUTICAL ASTRONOMY. The instruments for obtaining the *data* are the QUADRANT or SEXTANT, the ARTIFICIAL HORIZON, a Time-keeper called a CHRONOMETER, and the NAUTICAL ALMANAC; the use of these will be described in the sequel. For special purposes the Circle of Reflexion and a portable Transit Instrument are valuable.

In *Nautical Astronomy,* which is the application of the principles of ASTRONOMY, heights and distances come only incidentally into the calculations. The position of the observer depends upon the *angular* position of the heavenly bodies with respect to each other, and the horizon and meridian.

Through the methods of *spherical trigonometry* the necessary *data* determine the *place of the zenith on the celestial concave,* which, referred to the Earth, is the *Position of the Ship by Observation,* as distinguished from the position by *Dead Reckoning.*

I.—THE SOLAR SYSTEM AND FIXED STARS

The SOLAR SYSTEM is that assemblage of *planets, comets, meteors,* and *cosmical bodies* that have the Sun as their central luminary, revolving around that body at different distances, and with different periodic times.

The PLANETS of the System, in the order of their proximity to the Sun, are, Mercury, Venus, the Earth, Mars, the Asteroids, Jupiter, Saturn, Uranus, and Neptune. The planets shine by the reflected light of the Sun, as do all the bodies of the system; and it is characteristic of planetary motion that a *line drawn from the Sun's centre to a planet would sweep over equal areas in equal times.*

The *inferior planets* are the two whose paths are between the Earth and Sun: the *superior* planets move in paths at a greater distance from the Sun than the Earth's path.

SATELLITE is another name for MOON, and most of the planets in the solar system are accompanied by one or more of such bodies, which revolve around their primary, as a secondary planet. The Earth has one Moon; Mars two; Jupiter four; Saturn eight and also meteoric rings; Uranus

four ; and Neptune one or more. These moons or satellites revolve around their special primary planet in definite periodic times, and where there are more than one, at definite distances.

COMETS are luminous bodies whose very eccentric paths lie around the Sun.

METEOROIDS are small luminous bodies, or groups of small bodies, whose paths are around the Sun, and intermingled with the paths of the planets.

The following statistics are a compendium of the Solar System—

Symbol.	Name of Body.	Mean distance from the Sun.		Mean diameter in Miles.	Density.	Time of Rotation on Axis.	Periodic Time.	Orbital velocity in miles per Second.
		Astronomical Units	Millions of Miles.		Water = 1			
☉	Sun			860,000	1·444	25½ days		
						H. M. S.		
☿	Mercury	·3871	35¾	2,992	6·85	24 0 50	87·97 dys.	29·55
♀	Venus ..	·7233	66¾	7,660	4·81	23 21 22	224·70 dys.	21·61
⊕ or ♁	Earth ..	1·0000	93	7,926	5·66	23 56 4	365·26 dys.	18·38
♂	Mars	1·5237	141	4,211	4·17	24 37 23	686·98 dys.	14·99
ASTEROIDS.								
♃	Jupiter	5·2028	480	86,000	1·378	9 55 20	11·86 yrs.	8·06
♄	Saturn	9·5389	881	70,500	·75	10 14 0	29·46 yrs.	5·95
♅	Uranus	19·1834	1771	31,700	1·28	unknown	84·02 yrs.	4·20
♆	Neptune	30·0544	2775	34,500	1·15	unknown	164·78 yrs.	3·36

NOTE.—The Asteroids are a group of minor (telescopic) planets revolving around the Sun, in orbits between those of Mars and Jupiter. Only four, Ceres, Pallas, Juno, and Vesta, were known in the first years of the present century ; now they are known to number 458 (Jan. 1898), varying in diameter from 20 to 400 miles.

The Moon (☾) that accompanies the Earth has a diameter of about 2,160 miles and revolves around the Earth, as her primary, at a mean distance of about 237,700 miles, in 27d. 7h. 43m. 11·5s. ; but the revolution on which the Moon's *phases* depend extends to 29d. 12h. 44m. 2·7s.

THE FIXED STARS are distinguished from the planets, inasmuch as their relative positions are the same from year to year. Some of them are single ; others, attended by a companion, are double ; some appear grouped in clusters. They are divided into classes or magnitudes according to their apparent splendour, the lowest class visible to the average human eye being that of the sixth magnitude. Those used in Nautical Astronomy are of the first or second magnitude.

The Fixed Stars shine by a light inherent in their own structure, and, like our Sun, may possibly be the centre of a system of planets wholly invisible to us.

In order to assist the memory, the ancient astronomers divided the heavens into districts of various shapes, called *constellations*. These consist of a number of stars lying contiguously, which are supposed to be circumscribed by the imaginary outline of some animal or other figure. The stars

in the constellations are generally distinguished by letters of the Greek alphabet, and some of the principal have proper names.

The distances of a few of the fixed stars have been determined : of these distances none are less than 200,000 times the distance of the Earth from the Sun. Or, to put it in another form, light travels 186,700 miles per second, and comes to us from the Sun in 8 minutes : to come at the same rate from the nearest fixed star (α Centauri) $4\frac{1}{2}$ years would be required.

The *celestial objects* from which the *data* are derived, through reflecting instruments and the Nautical Almanac, for the various problems of Nautical Astronomy, are the Sun and Moon ; the planets Venus, Mars, Jupiter, and Saturn ; and the Fixed Stars of the first and second magnitudes.

The *symbols* of the Solar system are given on p. 206 ; the Greek alphabet by which the fixed stars are distinguished, is as follows—

α (alpha)	η (eta)	ν (nu)	τ (tau)
β (beta)	θ (theta)	ξ (xi or ksi)	υ (upsilon)
γ (gamma)	ι (iota)	o (omicron)	ϕ (phi)
δ (delta)	κ (kappa)	π (pi)	χ (khi)
ϵ (epsilon)	λ (lambda)	ρ (rho)	ψ (psi)
ζ (zeta)	μ (mu)	σ (sigma)	ω (omega)

The following works will be useful to the student who wishes to acquire a more complete and accurate knowledge of Astronomy: "Rudimentary Astronomy" (Weale's Series), by the Rev. R. Main, M.A., F.R.S., revised by W. T. Lynn, B.A., F.R.A.S. ; "Elements of Astronomy," by R. S. Ball, LL.D., F.R.S., Royal Astronomer of Ireland.

For the Stars and their use in navigation, *see* "How to Find the Stars, etc.," and "Stellar Navigation," by W. H. Rosser.

II.—PRELIMINARY DEFINITIONS

CIRCUMFERENCE is a general term denoting the line or lines bounding any figure.

A CIRCLE is a figure bounded by a curved line which is everywhere equally distant from a point within it called the *centre ;* the boundary of the circle is its circumference or periphery : the circle is the space contained within the circumference.

The RADIUS of a circle is a straight line drawn from the centre to the circumference ; and around which the circle is generated ; it is also the semi-diameter.

The DIAMETER of a circle is a straight line passing through the centre from one part of the circumference to another. The diameter = 2 radii (or twice the radius).

A *semicircle* is half a circle : or the space between a diameter and the semi-circumference.

A *quadrant* is the fourth part of a circle, being bounded by two radii perpendicular to each other.

A circle is divided into 360° ; a semicircle into 180° ; and a quadrant into 90°.

The ARC of a circle is any *part* or portion of the circumference *of a circle.*

A PLANE is a perfectly level surface, or extension, of indefinite

dimensions; and when the term is used to mark the separation of one part of a sphere from another, it is called a *cutting plane.*

A SPHERE or *Globe* is a body generated by the revolution of a semicircle about its diameter. A sphere is the surface in solid geometry which corresponds to the circle in plane geometry.

A sphere may be either solid or hollow. As regards Navigation, the sphere is the *surface* of the Earth; as regards Nautical Astronomy, it is an imaginary spherical surface in space, generally called the *celestial concave,* with the Earth as a point at the centre.

Every point on the surface of a sphere is equally distant from an interior fixed point, called the *centre;* a straight line drawn from the centre to any point on the surface is a constant distance, and is termed the *radius;* and if the line passes through the centre and is produced both ways to meet the surface, it is called the *diameter.*

NOTE.—In Nautical Astronomy the semi-diameter of a heavenly body, as the Sun, Moon, or any of the Planets, is expressed in angular measure.

Every section of the surface of a sphere made by a cutting plane is a *circle;* if the centre of the sphere be *in* the cutting plane, the section is a GREAT CIRCLE; if the centre of the sphere be *out* of the cutting plane, the section is a SMALL CIRCLE.

The plane of a *great circle* cuts the sphere into *two equal* parts: and all great circles have the same radius, which is also the radius of the sphere. The plane of a *small circle* cuts the sphere into *two unequal* parts

All great circles of a sphere are equal to each other, and bisect each other; *i.e.*, divide each other into two equal parts or semicircles.

A great circle may be drawn through any two points on the surface of a sphere, but not through more than two, taken at random.

If through the centre of a circle, whether great or small, a straight line be drawn perpendicular to its plane, the points in which the line meets the surface of the sphere are called the POLES of that circle. Such a line is a diameter of the sphere.

All PARALLEL CIRCLES are, at all points, equi-distant from each other, and have the same poles.

Every point on a great circle is 90° from either of its poles.

Every point on a small circle is less than 90° from its *adjacent* pole, and more than 90° from its *remote* pole.

If any great circle is taken as a *primary,* all the great circles that pass through its poles are *secondaries*, and all secondaries cut their primary at right angles. Thus, *vertical circles* are secondaries to the rational horizon as a primary, and *polar* circles are secondaries to the equinoctial (or equator) and the ecliptic, as the case may be.

The ARC OF A GREAT CIRCLE is measured by the angle subtended by it at the centre of the sphere, and is the same as the angle at its pole of two secondaries passing through the extremities of the arc.

The DISTANCE of any two points on the surface of a sphere is measured by an arc of a great circle joining them; and it is the *shortest* distance.

The AXIS of a sphere is any line passing through the centre, and terminating at the surface on both sides.

The AXIS OF ROTATION is the line or diameter around which a body revolves when put into motion.

If one axis is longer than the other, the *longer* is the *major axis*; the *shorter* is the *minor axis*.

An Ellipse is an oval figure, or flattened circle, in which the distance from the centre to the circumference varies in different parts; hence it has a major, and a minor, axis (diameter).

A Spheroid is a solid spherical body generated by the rotation of an ellipse around one of its axes. If the rotation is around the minor axis the body is an *oblate spheroid*, if around the major axis it is a *prolate spheroid*.

A Spherical Angle is an angle on the surface of the sphere contained between two great circles at their point of intersection.

A Spherical Triangle is that portion of the surface of a sphere contained by three arcs of great circles which cut one another, two and two.

The *Complement* of an Arc or Angle is what it differs from 90°.

The *Supplement* of an Arc or Angle is what it differs from 180°.

N.B.—For the Definitions relating to the Earth and Navigation, *see* p. 76

III.—Definitions appertaining to Nautical Astronomy

The Celestial Concave.—In nautical astronomy, which is an important subdivision of practical astronomy, we are no longer concerned with the *surface* of the earth, except in so far as we may be an observer on some given spot of that surface. Our observations are extended to the heavenly bodies—the sun, moon, planets, and fixed stars which *appear to revolve*, in definite positions, around the earth from east to west, owing to the rotatory motion of the earth on its axis in the opposite direction—namely, from west to east. Now space, in the abstract, has neither form nor dimensions and must ever remain to the human mind illimitable and immeasurable: yet the heavenly bodies, situated as they are in space at various (and many at incalculable) distances from us and from each other, and considered merely as they *appear* to us, may be conceived to occupy a vast *concave sphere*, extending around us in every direction. If we were placed in the centre of a glass sphere, through which we could see all the heavenly bodies, and were, *at a given instant*, to make marks on the glass at all the spots where those bodies appeared, the marks would become in some measure a record of the relative distances at which the bodies seem apart. Such is the principle of the concave sphere, which is usually called the celestial concave, the radius of which is infinite, and the centre of which is the eye of the observer. On the celestial concave we likewise conceive, together with the heavenly bodies, certain great circles of the sphere to be projected; then, knowing the positions of these bodies with respect to the great circles, also the positions of the circles themselves with respect to each other, and reducing the celestial observations made on the surface of the earth to such as they would be if taken at the earth's centre, the position of any given spot on the earth can be determined, together with all the fundamental elements necessary for conducting a vessel over the ocean.

And, although what we have been speaking of is a mere conception, it is mathematically as real as if the whole had an actual existence: for, though

the rotation of the Earth on its axis once in every 24 hours, producing day and night, and the annual progression of the Earth around the Sun, giving us the seasons, might seem to render our conception impossible ; yet those who are acquainted with the nature of relative motion will see that we might (not without inconvenience, but without inaccuracy) assume any one point of the universe we please for a fixed point, provided we give all other points, not their absolute motions but the motions which they have relatively to the centre chosen.

It is also necessary to take note that while in Navigation we deal with *actual* distances, so many miles North, South, East, or West, or in any intermediate direction between those cardinal points, in Nautical Astronomy we are concerned only with *angular* measures and distances,—the arcs of great circles, forming spherical triangles, being the arguments of the problems to be solved.

Some of the points and great circles of the celestial concave are synonymous, if not coincident, with those of the terrestrial sphere ; others are not common to the two.

Axis and Poles of the Celestial Sphere.—The Axis of the Celestial Sphere is the line formed by the prolongation of the axis of the Earth to the celestial concave. The two points in which the axis meets the celestial concave are called the Poles of the Celestial Sphere, or, as they are sometimes called, the Poles of the Heavens, and they are the only points of the sphere that appear fixed and immovable ; about them, the heavenly bodies, in their apparent daily revolution from east to west, seem to describe circles, which are greater or less, according as they are further from, or nearer to, their apparent centre of motion ; and the nearer a body is to either of these points, the slower is its motion. The name (North or South) of the Pole corresponds to that on the terrestrial sphere which it faces.

The celestial sphere has three primary great circles ; they are the *Equinoctial*, the *Ecliptic*, and the *Horizon*.

The Equinoctial.—A line parallel with the Earth's equator, hence perpendicular to the axis, will, owing to the diurnal motion, describe a plane whose intersection with the celestial sphere coincides with the great circle the poles of which are the Poles of the Heavens ; this is the Equinoctial or *celestial* Equator.

As the Equinoctial is in the same plane with the terrestrial Equator, so, in like manner, the terrestrial meridians become *celestial* Meridians or Hour Circles, in the same plane with the terrestrial Meridians, perpendicular to the Equinoctial, and passing through the Poles of the celestial sphere.

The Ecliptic.—The great circle which the Sun's *apparent*, but the Earth's actual path (or orbit) traces in the celestial concave in the course of a sidereal year is called the ecliptic : the name is derived from the fact that solar and lunar eclipses can only happen when the Moon is in or very near this circle. Owing to the Earth's axis of diurnal rotation being inclined at what may be considered as a constant angle to the plane of her orbit of annual revolution, it follows that the planes of the equator and ecliptic are not coincident ; and this nearly constant angle of inclination ($23^\circ\ 27\frac{1}{4}'$) is called the obliquity of the ecliptic. The two points of intersection between the celestial equator and ecliptic are called the *Equinoxes* or equinoctial points, because, when the Sun appears in either of them,

being then on the Equator, and as the equator and the horizon of every place bisect each other, he must remain as long above as below the horizon, producing everywhere day and night of equal length: thence the term EQUINOCTIAL for the celestial equator.

The ecliptic is divided by astronomers into *twelve* equal parts, called ***signs***, each sign being equivalent to 30° of arc. A zone or belt extending about 8° on each side of the ecliptic is called the Zodiac, in which are the orbits of the planets, with the exception of some of the Asteroids. The signs are common to the ecliptic and zodiac—hence the term SIGNS OF THE ZODIAC. Their names, in the direction of the Sun's apparent motion, with the days on which that luminary appears to enter them, are as follows—

NORTHERN SIGNS

Spring Signs

♈ Aries, March 20-21.
♉ Taurus, April 19-20.
♊ Gemini, May 20-21.

Summer Signs

♋ Cancer, June 21.
♌ Leo, July 22-23.
♍ Virgo, August 22-23.

SOUTHERN SIGNS

Autumnal Signs

♎ Libra, September 22-23.
♏ Scorpio, October 23.
♐ Sagittarius, November 22.

Winter Signs

♑ Capricornus, December 21-22.
♒ Aquarius, January 19-20.
♓ Pisces, February 18-19.

The point where the sun passes from the southern to the northern hemisphere, on March 21st, at the *vernal equinox*, is called the FIRST POINT OF ARIES; which, however, is no longer in the *constellation* Aries. So with regard to the other signs, they do not coincide with the constellations of the same name, though they did originally when the First Point of the sign Aries was in the constellation Aries. The change is owing to an extremely slow motion, at the rate of about 50″·1 annually, produced by the attractive force which the sun, moon, and planets exercise upon the protuberant parts of the Earth about the equator; causing the equinoctial points to move westward, and thus to continually *precede* (in the order in which the *signs* are read) the position occupied at any former epoch; hence the motion is known as the PRECESSION OF THE EQUINOXES, and it has brought the First Point of Aries into the constellation Pisces. The *autumnal equinox*, which the sun reaches on September 23rd, when crossing from the northern to the southern hemisphere, is the *First Point of Libra*, now in the constellation Virgo. Thus, it is to be observed that the signs retain their names, though they are no longer in the constellations after which they were originally called, which is a matter of no importance compared with choosing *a fixed initial point of reckoning*—and this is the point of the vernal equinox known as the FIRST POINT OF ARIES.

Note

Owing to the precession of the equinoxes, the pole of the celestial sphere completes a revolution around the pole of the ecliptic in about 26,000 years.

The points of the ecliptic 90° from the equinoxes are known as the SOLSTICES, or *solstitial points*, which are respectively in the signs Cancer and Capricorn; when in Cancer, the sun has reached its furthest limit in the northern hemisphere; when in Capricorn, its furthest limit in the southern hemisphere.

The *circles connected with the ecliptic* are those which determine the Latitude and Longitude of a heavenly body. Great circles which pass through the poles of the ecliptic, and which are perpendicular to its plane, are CIRCLES OF CELESTIAL LATITUDE; and the arc of such a circle between any given star and the ecliptic marks its LATITUDE, which is N. or S. according as the star is *north* or *south* of the ecliptic. The LONGITUDE of such a star is reckoned on the ecliptic, and is the arc between the circle of latitude of the star and the First Point of Aries, estimated in the order of the signs, from 0° to 360°.

It is necessary to note that *celestial* latitude and longitude have nothing in common with *terrestrial* latitude and longitude, as they are not referred to the same plane.

The COLURES are the two great circles passing through the celestial poles, dividing the ecliptic into four equal parts, and marking the seasons of the year. The circle of celestial latitude whose longitude is zero, is the *colure of the equinoxes;* and that whose longitude is 90° is the *colure of the solstices.* The arc of the latter colure between the equator and the ecliptic, as also the arc between the pole of the equator and the pole of the ecliptic, is equal to the obliquity of the ecliptic. The term "colure" is almost obsolete.

THE VISIBLE HORIZON.—Standing on the earth's surface our vision is bounded by the circumference of a small circle indicated by the *apparent* junction of land or water with the sky; this is the VISIBLE HORIZON, and our range of vision is greater or less according to the height at which we stand: from such a standpoint only a small part of the terrestrial hemisphere is visible, and always more than a hemisphere of the heavens.

SENSIBLE HORIZON.—The circle which bounds the horizontal plane extending from the spot where the observer stands is the SENSIBLE HORIZON; and this plane, which is tangential to the earth's surface, is perpendicular to the radius of the earth which extends from the centre to the observer's station. The plane of the *sensible horizon* may also be conceived as that which sensibly coincides with the surface of a fluid at rest: the reduction of an astronomical observation due to the difference between the visible and sensible horizons is called the *dip.*

The RATIONAL or *celestial* HORIZON, which determines the rising or setting of the heavenly bodies, is the great circle whose plane is parallel with the sensible horizon, and which, passing through the centre of the earth, extends to the celestial concave: this is the primary great circle to which all vertical circles are secondaries.

The CARDINAL POINTS OF THE HORIZON are the *North, South, East,* and *West.*

The *North point* is that point in which the meridian meets the horizon on that side of the prime vertical on which the N. pole is situated.

The *South point* of the horizon is that point where the meridian meets the horizon on that side of the prime vertical most distant from the N. pole.

The *East point* is that point of the horizon where the prime vertical meets it, on that side of the meridian on which the heavenly bodies rise.

The *West Point* is that point of the horizon where the prime vertical meets it, on that side of the meridian towards which the heavenly bodies appear to move, and in which direction they set.

DIP, OR DEPRESSION, OF THE HORIZON.—The angle between the visible and sensible horizons, depending on the height of the observer above the

horizontal plane: the quantity corresponding to the height of the eye is subtractive from an observed altitude.

ZENITH.—The point directly overhead, 90° from every point of the horizon. The zenith changes its position as the observer changes his station on the earth.

NADIR.—The point diametrically opposite the zenith.

The zenith is the upper pole of the observer's horizon, and is the point in the celestial concave to which the angular distances, called *zenith distances,* are referred: the nadir is the lower pole of the observer's horizon.

Since the horizon of an observer changes as he moves from station to station, so does his zenith: and the terms zenith and nadir are relative; to observers standing in opposite hemispheres, and feet to feet, the nadir of one would be the zenith of the other, and *vice versâ.*

VERTICAL CIRCLES.—The great circles which pass through the *zenith,* and are perpendicular to the horizon, are *vertical circles,* or *circles of altitude:* they are secondaries to the horizon.

POLE.—The pole of the heavens is the indefinite extension of the earth's axis—it is the point around which the whole heavens appear to revolve.

POLAR DISTANCE.—The polar distance of a heavenly body is the arc of the celestial meridian contained between the elevated pole and the object's centre. When the latitude and declination have the same name the polar distance is less than 90°, and when of opposite names it is greater than 90°.

The ELEVATED POLE of the observer is the pole which is nearer to him, and is always above his horizon. The elevation of the pole is equal to the latitude.

CIRCUMPOLAR.—A term applied to a heavenly body when it revolves round the pole and is above the horizon during the whole of its revolution. A circumpolar body crosses the meridian of an observer twice in each twenty-four hours; once above the pole and once below. When the polar distance of a celestial body is less than the latitude, and of the same name, the body is said to be circumpolar.

DECLINATION.—The declination of a heavenly body is the arc of a celestial meridian contained between the equinoctial and the centre of the body. It may here be remarked that the celestial meridians, hour circles, and declination circles are all synonymous.

CIRCLES OF DECLINATION or hour-circles are secondaries to the equinoctial.

PARALLELS OF DECLINATION are small circles parallel with the equinoctial.

RIGHT ASCENSION.—The right ascension of a heavenly body is the arc of the equinoctial between the first point of Aries and the celestial meridian which passes through the body: it is reckoned eastward (*i.e.* from west to east) in the direction of the apparent annual motion of the sun, through 360°, or 24h.; generally, however, expressed in hours, minutes, and seconds.

The *celestial* MERIDIAN of an observer's station is the vertical circle that passes through the celestial poles, over the N. and S. points of the horizon, and through the zenith, and is the actual projection of the terrestrial meridian on the celestial concave.

HOUR ANGLE is the arc of the equinoctial, or corresponding angle at the pole, contained between the observer's meridian and the meridian

through the object's centre. The westerly hour-angle is the apparent time at place.

ZENITH DISTANCE.—The zenith distance of a celestial object is the arc of a vertical circle contained between the zenith and the centre of the object. The altitude and zenith distance are each the complement of the other. When the latitude and declination are both 0°, the zenith distance turned into arc is the hour-angle. The altitude of a heavenly body alters most rapidly when it is on the prime vertical, that is, when it bears east or west.

The PRIME VERTICAL is the vertical circle passing through the zenith, and the *east* and *west* points of the horizon. As the plane of the meridian passes through the north and south points, and that of the prime vertical through the east and west points, of the horizon, and both planes through the zenith, they are perpendicular to each other. The meridian is the *twelve o'clock* (*noon*) *hour-circle*.

ALTITUDE.—The altitude of a heavenly body is the arc of a vertical circle contained between the horizon and the centre of the object.

AZIMUTH is the arc of the horizon contained between the observer's meridian and a vertical circle passing through the centre of the object. Azimuth is reckoned from the north and south points of the horizon towards east and west.

The *observed* altitude of a heavenly body is its altitude above the visible horizon, measured by a reflecting instrument.

The *apparent* altitude of a heavenly body is the arc of the circle of altitude between the sensible horizon and the object's centre; to obtain which the observed altitude must be corrected for index error of the instrument, dip, and semi-diameter (if any).

The *true* altitude of a heavenly body is the arc of the circle of altitude between the rational horizon and the object's centre; to obtain which the apparent altitude must be corrected for refraction in altitude, and parallax in altitude (if the object has any horizontal parallax).

SEMI-DIAMETER.—The radius, or *half* the diameter, of a body. The semi-diameter of the sun, moon, or any of the planets, as given in the Nautical Almanac, is the angle under which the semi-diameter of the body would appear if viewed from the centre of the earth.

THE AUGMENTATION OF THE MOON'S SEMI-DIAMETER is the apparent increase in her semi-diameter due to the observer being brought nearer to the moon as she rises in altitude. When the moon is in the zenith she is nearer to the observer by the amount of the earth's radius than when she was on the horizon.

THE AUGMENTATION OF THE SUN'S SEMI-DIAMETER:—Owing to the enormous distance of the sun is quite inappreciable.

REFRACTION, in nautical astronomy, is the change of direction which a ray of light, proceeding from a heavenly body to the eye of the observer, undergoes in passing through an atmosphere of increasing density; the body is seen in the direction in which the last part of the ray enters the eye, hence there is a difference between the real and apparent places of the body, the fact being that it appears higher than it really is: the correction, called the *refraction in altitude*, is subtractive from the apparent altitude.

PARALLAX is the difference between the altitude of a heavenly body above the rational horizon and its altitude above the sensible horizon; it is,

therefore, equal to the angle at the object subtended by the earth's radius in that latitude and is always additive. For navigational purposes none of the stars has any parallax, owing to their enormous distance from the earth.

The HORIZONTAL PARALLAX of any heavenly body (sun, moon, and planets), as given in the Nautical Almanac, is the *greatest* angle under which the earth's equatorial semi-diameter would appear if seen from the centre of the body; hence that of the moon, owing to her proximity to the earth, requires *reduction for the latitude* of the observer's station. The *parallax in altitude* is the correction (additive) by which the true altitude above the *sensible* horizon is reduced to the true altitude above the *rational* horizon.

PARALLELS OF ALTITUDE, formerly called ALMUCANTARS, are small circles whose planes are parallel to the horizon.

RISING AND SETTING.—The *rising* of a heavenly body takes place when its centre is on the *eastern* edge of the horizon; and its *setting* when its centre is on the *western* edge of the horizon. But since *refraction* causes bodies to appear higher than they really are, and its effect is greatest on the horizon, being then about 33′, it follows that a heavenly body is on the horizon (*rising* or *setting*) when its centre appears elevated rather more than half a degree above it. If it could be observed or recognised exactly on the horizon, it would then be more than half a degree below the horizon: thus, the heavenly bodies appear to rise earlier and set later than they should do, considered from an astronomical standpoint. The position of rising and setting is dependent upon the polar distance of the body.

The old terms *cosmical*, *achronical*, and *heliacal* rising and setting are nearly obsolete: the cosmical rising and setting of a star or planet takes place when it rises and sets with the sun; rising at sunset and setting at sunrise, it is said to rise and set achronically; rising a short time before the sun, or setting shortly after the sun, *i.e.*, being visible for a brief period in the morning or evening twilight, indicates heliacal rising or setting.

TWILIGHT is the effect of the illumination of the upper regions of the atmosphere by the sun, before he has risen or after he has set, at the place of the spectator. Twilight continues, generally, while the sun is less than 18° below the horizon.

DIURNAL ARCS are those parts of the parallels of Declination which are apparently described between the times of the rising and setting of a heavenly body; NOCTURNAL ARCS are the parts of those parallels described from the time of setting to the time of rising. SEMI-DIURNAL and SEMI-NOCTURNAL ARCS (or the halves of the preceding arcs) are those parts of the parallels intercepted between the Meridian and the Horizon. The corresponding part of the Equator answering to the Semi-Diurnal Arc, gives the time between noon and the rising or setting; and the equatorial part answering to the Semi-Nocturnal Arc, shows the time between midnight and the time of setting or rising.

ASCENSIONAL DIFFERENCE.—An arc of the Equinoctial intercepted between the Horizon and the Hour-Circle of the object at rising and setting; it is the time the *Sun* rises before or sets after 6 o'clock.

AMPLITUDE.—The Arc of the horizon between the *East* point and the centre of a heavenly body when *rising*, and between the *West* point and its centre when *setting*, is its amplitude. The term *amplitude* is only used in connection with the rising and setting of a heavenly body.

HELIOCENTRIC.—Position referred to the centre of the Sun.

GEOCENTRIC.—Position referred to the centre of the Earth.

The terms heliocentric and geocentric are applicable to latitude and longitude.

The REDUCED ZENITH, AND THE ANGLE AT THE VERTICAL OR REDUCTION OF LATITUDE.—Since the Earth is a spheroid and not a sphere, the *vertical line* perpendicular to the plane of the horizon at the position of the observer does not coincide with the prolongation of the Earth's radius, except at the poles and the equator; hence a line extending from the Earth's *centre*, through the place of the observer, and thence into space, terminates in what may be called the *Reduced* (or geocentric) *Zenith* to distinguish it from the *true* or *geographical* Zenith, already described (*see* p. 213). The intersection of the two lines is the *angle at the vertical*, and marks the difference between the *geographical* latitude and the *geocentric* latitude; and thus this difference is also called the *reduction of latitude*. The geocentric is always less than the geographical latitude, the greatest difference between the two being about $10\frac{1}{2}'$ in Lat. $45°$; at the equator and the poles the difference is *zero*. In Nautical Astronomy it is chiefly the Moon's elements and various lunar observations that are affected by the reduction of latitude.

DISC.—The apparent flat surface of the Sun, Moon, or any planet.

The CULMINATION of a heavenly body is its greatest altitude, which occurs at the time of its passage across the meridian.

TRANSIT.—The passage of a heavenly body across any great circle of the celestial sphere is its transit; the term is generally applied to that passage across the meridian which is nearest to the point of the horizon most remote from the elevated pole. The passage of one heavenly body across the disc of another is also called a *transit*; as the transit of Mercury and Venus across the disc of the Sun, or the transit of a satellite across the disc of Jupiter.

COMPRESSION.—The ratio which the difference between the equatorial and polar diameters bears to the equatorial diameter. The Earth's compression is something less than $\frac{1}{300}$.

ORBIT.—The path described by the revolution of a planetary body round the Sun is called its orbit: similarly the path of a moon or satellite around its primary is its orbit. The orbit of all the heavenly bodies is more or less *elliptical*, the object around which it revolves being in one of the *foci*,—not in the centre, though near it.

The PLANE OF THE ORBIT is the surface passing through the centres of the primary and secondary bodies, and the bounding line of which is the orbit.

The INCLINATION OF AN ORBIT, as of that of the Moon or of any of the planets, is the angle between its plane and the plane of the ecliptic.

PERIHELION.—The point in an orbit which is nearest to the Sun.

APHELION.—The point in an orbit which is farthest from the Sun.

The RADIUS VECTOR is the distance between the centre of the Earth, or any other planet, and the true place of the Sun's centre, on any given day.

PERIGEE.—The point of the Moon's orbit which is nearest to the Earth; also, the point in which the Sun or a planet is nearest to the Earth.

APOGEE.—The point of the Moon's orbit which is farthest from the Earth; also, the point in which the Sun or a planet is farthest from the Earth.

The LINE OF APSIDES is the line on the plane of the orbit joining the perihelion and aphelion points; with regard to the Earth and the Moon, it is the line joining the perigee and apogee: hence it is the major axis of an elliptical orbit.

NODES.—The two opposite points where the orbit of the Moon, or that of any of the planets, appears to intersect the orbit of the Earth—the planes of the two orbits being inclined to each other at an angle—are called the *nodes*. The line joining these two points is called the LINE OF THE NODES. The node at which the object ascends from the south to the north side of the ecliptic is the *ascending node;* that at which the object descends from the north to the south side of the ecliptic is the *descending node.*

ELONGATION.—The angle under which a planet would appear from the Sun, when reduced to the ecliptic; GREATEST ELONGATION is the greatest angular distance to which the planets recede *east* or *west* from the Sun; in respect to the Earth, the term is only applicable to the inferior planets, Mercury and Venus.

CONJUNCTION.—The position of two or more heavenly bodies when they have the same longitude or right ascension; with respect to the Sun and Earth, a planet is in SUPERIOR CONJUNCTION when in a line with the two bodies, but beyond the Sun: INFERIOR CONJUNCTION takes place when a planet is between the Sun and Earth. Planets more distant from the Sun than is the Earth can have no inferior conjunction.

OPPOSITION.—The position of two heavenly bodies whose longitudes differ by 180°. A superior planet is in opposition when the Earth is in the same straight line with, but between it and the Sun.

QUADRATURE.—The position of two heavenly bodies whose longitudes differ by 90°.

PLANETARY MOTION.—The inferior planets *retrograde* before and after inferior conjunction, and move *directly* in the rest of their orbits: the superior planets move directly before and after conjunction, and retrograde before and after opposition.

LUNATION.—The period the moon takes to complete her journey round the earth, viz., 29 days 12h. 44m. 2·8s.

PHASES OF THE MOON.—The aspect or form in which the Moon appears in a lunation: at *new moon* she is in *conjunction* with the Sun, and hence invisible; subsequently she takes a *crescent* form; at *first quarter* she is in *quadrature* with the sun, and *half* her disc is visible: when more than half the disc is visible her form is *gibbous:* at *full moon*, when the whole disc is seen, she is in *opposition* to the sun: she then proceeds to decrease her phases as she had previously increased them, and finally arrives again at new moon.

N.B.—The inferior planets have phases like the moon.

SYZIGIES.—The term used for New and Full Moon; *i.e.* when the Moon is in conjunction and opposition in respect to the Sun.

LIBRATION OF THE MOON.—An apparent irregular motion of the Moon about her own axis, whereby we see a little more than one-half of the lunar disc.

ECLIPSE.—When the disc of one heavenly body is partially or totally obscured by another, but opaque, body, or by the shadow of another body, the phenomenon is called an *eclipse.*

An ECLIPSE OF THE SUN is the passage of the opaque disc of the Moon across the Sun, being then between the Earth and Sun. A solar eclipse can happen only when the Moon is *new; i.e.*, in conjunction with the Sun, and in the nodes, or near them. According to the position and distance of the bodies the eclipse may be *partial, total,* or *annular;* in the latter case there is a ring of brilliant light round the opaque body of the Moon.

An ECLIPSE OF THE MOON is the passage of the Earth between the Sun and Moon, when the Earth's shadow is projected on the Moon. A lunar eclipse can happen only when the Moon is *full; i.e.*, in opposition with the Sun, and in or near the nodes: it may be *partial* or *total.*

An ECLIPSE of any of JUPITER'S SATELLITES takes place when the particular satellite is on that side of Jupiter which is opposite to the Sun; the satellite then passes into the shadow of Jupiter.

OCCULTATION.—When a fixed star or planet is obscured by the interposition of the Moon, or when one of Jupiter's satellites passes behind the body of the planet, the phenomenon is called an *occultation;* the elements connected with occultations by the Moon, and the occultations of Jupiter's satellites, are given in the Nautical Almanac to facilitate the determination of terrestrial longitude.

ABERRATION is an apparent motion of the heavenly bodies, occasioned by the progressive motion of light combined with the motion of the Earth in its orbit, and causing those bodies to appear in a different position from that which they really occupy—the true position being always in advance of the apparent.

NUTATION.—The oscillatory motion of the axis of the Earth, whereby its inclination to the plane of the ecliptic is not always the same, but varies backwards and forwards a few seconds. It is due to the action of the Moon on the Earth, and the position of every heavenly body as seen from the Earth is affected by it.

FIXED STARS.—This appears to be the place to introduce a matter of considerable importance in stellar observations, and for this purpose a catalogue of the stars of the first, second and third magnitudes is arranged in the *order of their Declinations* (*see* p. 219), so that you may understand, for any given place on the earth's surface, the range of their visibility.

With regard to an observer's position in either hemisphere the whole heavens may be divided into three parts—(1.) A part which is never below the horizon. (2.) A part which rises and sets. (3.) A part which never rises above the horizon.

FIXED STARS	DECL.	FIXED STARS	DECL.
	°		°
α Ursæ Minoris (*Polaris*)	N 88·7	α Aquilæ (*Altair*)	8·6
β Ursæ Minoris (*Kochab*)	74·6	α Orionis (*Betelegeux*)	7·4
β^2 Cephei	70·1	α Serpentis (*Unuk*)	6·8
α Ursæ Majoris (*Dubhe*)	62·4	α Canis Minoris (*Procyon*)	5·5
α Cephei (*Alderamin*)	62·1	α Ceti (*Menkar*)	3·6
η^2 Draconis	61·8	γ Ceti	N 2·8
α Cassiopeiæ	55·9	δ Orionis	S 0·4
γ Ursæ Majoris	54·3	α Aquarii	0·9
β Draconis	52·4	ε Orionis	1·3
γ Draconis	51·5	α Hydræ (*Alphard*)	8·2
η Ursæ Majoris (*Benetnasch*)	49·9	β Orionis (*Rigel*)	8·3
α Persei (*Mirfak*)	49·5	β Libræ	9·0
α Aurigæ (*Capella*)	45·9	α Virginis (*Spica*)	10·6
α Cygni	44·9	α Libræ	15·6
α Canum Venaticorum	38·9	α Canis Majoris (*Sirius*)	16·6
α Lyræ (*Vega*)	38·7	α Leporis	17·9
α^2 Geminorum (*Castor*)	32·1	β Ceti	18·6
β Tauri	28·5	β^1 Scorpii	19·5
α Andromedæ (*Alpheratz*)	28·5	α Scorpii (*Antares*)	26·2
β Geminorum (*Pollux*)	28·3	ε Canis Majoris	28·8
ϵ^2 Bootis	27·6	α Piscis Australis (*Fomalhaut*)	30·2
α Coronæ (*Alphacca*)	27·1	α Columbæ	34·1
α Arietis (*Hamel*)	22·9	α Gruis	47·5
γ^1 Leonis	20·4	α Argûs (*Canopus*)	52·6
α Bootis (*Arcturus*)	19·8	α Pavonis	57·1
α Tauri (*Aldebaran*)	16·3	α Eridani (*Achernar*)	57·8
β Leonis (*Denebola*)	15·2	ι Argûs	58·8
α Pegasi (*Markab*)	14·6	β Centauri	59·8
α^1 Herculis	14·5	α^2 Centauri	60·4
γ Pegasi (*Algenib*)	14·5	α^1 Crucis	62·5
α Ophiuchi	12·6	α Trianguli Australis	68·8
α Leonis (*Regulus*)	12·5	β Hydri	77·9
ε Pegasi	9·3	σ Octantis	S 89·3

1. *No star can set if its polar distance is less than the latitude* of the observer's position. Thus, for latitude 51½° N., all the stars in the upper part of the catalogue, as far as *Vega* (α Lyra) never set; the polar distance (90° less the declension) of all of them being less than 51½° they are *circumpolar*, and it is to be observed that all such stars pass the meridian twice in 24 hours,—once above, and once below, the pole.

2. Of the stars just indicated, some of them, at some hour of the day or night, must pass across the zenith, *i.e.*, directly overhead; *any star whose polar distance is exactly the same as the co-latitude* will do so. Now the co-latitude of a place in lat. 51½° N. is 90° less 51½° = 38½°; and the polar distance of γ Draconi, in the catalogue, is 38½° since its declination is 51½°; therefore this star must, at some hour, be on the zenith of the place in lat. 51½° N.

3. For the same position, viz., lat. 51½° N., it is to be noted that all the stars above γ Draconis, in the catalogue, *never come to the prime vertical, because* their *polar distance is less than the co-latitude;* when they pass the meridian above the pole they do so between the zenith and the pole; when

they pass the meridian below the pole they do so between the pole and the North point of the horizon.

Of the other stars that pass the meridian twice in 24 hours, those that lie between γ Draconis and *Vega* in Lyra, will, when they pass the meridian below the pole, do so between the pole and the North point of the horizon; but as their polar distance is greater than the co-latitude, they will pass the prime vertical in their revolution, and come to the meridian above the pole between the zenith and the South point of the horizon.

4. It remains to show how many of the stars in this catalogue below *Vega* in Lyra can be seen in lat. 51½° N. *No star can be visible if its declination and latitude have different names, and their sum exceeds* 90°; casting your eyes down the catalogue you will see that the declination of α Gruis is 47½° S., this added to lat. 51½° N. exceeds 90°, the star is consequently beyond the range of your vision; therefore you can see no bright star of the catalogue beyond α Columbæ when in 51½° N. You can put the range of visibility into the other hemisphere in another form; you can see no star the declination of which is greater than the co-latitude,—the declination differing in name from the latitude; α Columbæ nearly answers to this.

5. Thus then, in lat. 51½° N., night by night and according to the season, some or other of the stars between *Polaris* and α Columbæ will be visible; all those in the catalogue, from *Polaris* to *Vega* (in Lyra), with declination exceeding 38½° N., are *circumpolar*, being never below the horizon; all those below α Columbæ, with declination exceeding 38½° S., are never seen, for they never rise to the horizon; and all those from *Vega* (in Lyra) to α Columbæ, with declination less than 38½° N. and 38½° S. rise and set alternately.

6. What has already been stated may be put in another, and (to some) in a simpler form,—using the terms *Declination* of a star, and *Latitude* of a place—hence its *co-latitude*—

(*a*) With declination and latitude of the same name, both N. or both S., if the declination be *greater* than the co-latitude, such stars are never below the horizon of the observer, and are called *circumpolar* stars.

Example.—In lat. 49° N., and hence co-lat. 41°, all stars having greater declination than 41° N. never set, and appear to revolve round the N. pole of the heavens.

Example.—In lat. 42° S., and hence co-lat. 48°, all stars having greater declination than 48° S. never set, and appear to revolve round the S. pole of the heavens.

(*b*) With declination and latitude of different names, one N. and the other S., if the declination be *greater* than the co-latitude, such stars are never above the horizon of the observer.

Example.—In lat. 52° N., and hence co-lat. 38°, all stars having greater declination than 38° S. never rise above the horizon.

Example.—In lat. 40° S., and hence co-lat. 50°, all stars having greater declination than 50° N. never rise above the horizon.

(*c*) With declination and latitude of the same name, if the declination is *equal* to the latitude, such stars cross the zenith of the observer.

Example.—In lat. 50° N., all stars having declination 50° N. pass directly overhead, in the zenith.

Example.—In lat. 44° S., all stars having declination 44° S. pass directly overhead, in the zenith.

(*d*) Whether declination and latitude are of the same or of different names, all stars having declination *less* than the co-latitude rise and set alternately.

Example.—In lat. $51\frac{1}{2}°$ N., and hence co-lat. $38\frac{1}{2}°$, all stars having less declination than $38\frac{1}{2}°$ N. or $38\frac{1}{2}°$ S., or that range between decl. $38\frac{1}{2}°$ N. and decl. $38\frac{1}{2}°$ S., rise and set alternately.

Example.—In lat. 44° S., and hence co-lat. 46°, all stars that range between decl. 46° S. and decl. 46° N. rise and set alternately.

Thus far it has only been necessary to speak of the stars and their visibility in relation to their declination, but you must clearly recognise the fact that even of those that do come within range, some will be visible at one hour, others at another hour, some at one season, others at another season, for they will rise, culminate, and set in the order of their Right Ascension, and relatively according to the *right ascension of your own meridian.*

IV.—DEFINITIONS RELATING TO TIME AND ITS MEASURES

The ARTIFICIAL DAY is the time elapsed between sunrise and sunset; from sunset to sunrise is NIGHT; day and night vary in length with the latitude of the place, and the position of the sun in the ecliptic; together they constitute the CIVIL OR NATURAL DAY of 24 hours, which commences at midnight and terminates at the midnight following: the first twelve hours are designated A.M. (from *ante meridiem*), being the hours *before* noon; the second twelve hours are P.M. (from *post meridiem*), being the hours *after* noon. With this method of reckoning time Nautical Astronomy has little to do, except incidentally.

Primarily, however, the sun is our time-keeper, and gives us apparent time, but the intervals he marks are of unequal length.

APPARENT TIME.—When the sun is on the meridian of any given place it is *apparent noon*, and the sun's hour-circle coincides with the meridian. On the return to the same meridian the next day an interval called an APPARENT SOLAR DAY is marked out, and such a day may be characterised by the interval between two consecutive transits of the sun's hour-circle across the celestial meridian. Any portion of time computed from an altitude of the sun, in the interval between two transits, is APPARENT TIME, or *the hour-angle of the real sun*. But, as before said, the solar or apparent day is of variable length: (1) because the sun moves in the ecliptic and not in the equator, and (2) because the diurnal velocity varies with the Earth's distance from the sun.

MEAN TIME.—"Astronomers, with a view of obtaining a convenient and uniform measure of time, have recourse to a *mean solar day*, the length of which is equal to the mean or average of all the apparent solar days in a year. An imaginary sun, called the *mean* sun, is conceived to move uniformly in the Equator with the real sun's *mean* motion in Right Ascension, and the interval between the departure of any meridian from the *mean* sun and its succeeding return to it, is the duration of the *mean solar day*. Clocks and chronometers are adjusted to mean solar time; so that a complete revolution (through 24 hours) of the hour-hand of one of these machines should be performed in exactly the same interval as the revolution of the earth on its axis with respect to the *mean* sun. If the mean sun

could be observed on the meridian at the instant that the clock indicated oh. om. os., it would again be observed there when the hour-hand had returned to the same position."—*Nautical Almanac.* From this it follows that *mean noon* is the instant when the *mean sun's* hour-circle coincides with the meridian; and a MEAN SOLAR DAY is the interval between two consecutive transits of the mean sun's hour-circle across the celestial meridian. Also, MEAN TIME is the hour-angle of the mean sun.

EQUATION OF TIME.—As the time deduced from an observation of the *true* sun gives us *apparent* (or *true*) *time*, we cannot *immediately* obtain mean time therefrom, without the application of the *equation of time*, which is the difference, at any instant, between the apparent and mean times at that instant. Or, to define it more correctly, the equation of time is the angle at the celestial pole between the hour-circle of the true sun and that of the mean sun. This element is tabulated in the Nautical Almanac for every day, with instructions for its application. Four times in the year the equation is *zero*, viz., on a day in April, in June, in August, and in December, when the hour-circles of the true and the mean sun coincide: there must also be four *maxima* of divergence, and these occur on a day in February, in May, in July, and in November.

SIDEREAL TIME.—There is yet another measure of time of great importance to the Navigator, viz., Sidereal Time, without which no computations can be made which involve observations of the Moon, Planets, and Fixed Stars: in this case the First Point of Aries is the initial point of reckoning. It is *Sidereal Noon* at the instant when the hour-circle of the First Point of Aries coincides with the meridian; and a *sidereal day* is the interval between two consecutive transits of the hour-circle of the First Point of Aries across the celestial meridian. Also, SIDEREAL TIME is the hour-angle of the First Point of Aries; or, for any given instant, it is the number of hours, minutes, and seconds which have elapsed since the First Point of Aries was on the meridian of the place of observation.

A *Sidereal Day* is also the interval between the departure and the return of a star to the same meridian. Now, all the stars give the same exact duration of 23h. 56m. 4·09s. for the length of the sidereal day, instead of 24 hours: consequently, the sidereal day, which is subject to no variation, is the period in which the earth makes one revolution on its axis; and this acceleration of the sidereal on the solar day produces all the varied aspects we see in the heavens in the course of a year. The solar day would have the same length as the sidereal day if the earth stood still in space, and only turned upon its axis.

The RIGHT ASCENSION OF THE MERIDIAN is the *sidereal time at the place of observation.*

ACCELERATION OF SIDEREAL ON MEAN SOLAR TIME is the difference of time between a sidereal day and a mean solar day; it amounts to 3m. 55·9094s., and by this quantity the First Point of Aries anticipates the sun in a mean diurnal revolution.

Comparison of lengths of a mean solar day and a sidereal day—

24 hours of mean time = 24h. 3m. 56·5554s. sidereal time; and
24 hours of sidereal time = 23h. 56m. 4·0906s. mean time.

The CIVIL DAY AND THE ASTRONOMICAL DAY *compared.*—Both days consist of 24 hours; but the Civil Day commences at midnight, while the

Astronomical Day commences at noon; the noon of each having the same date.

Now, the Civil Day is divided into two portions of 12 hours each; that from midnight to noon being noted as A.M.; and that from noon to midnight P.M.; thus we say, 8 A.M. if an event occurred at 8 o'clock in the morning; and 8 P.M. if it occurred at 8 o'clock in the evening.

But the Astronomical Day is reckoned from noon to noon, without the distinction of A.M. and P.M. Thus we say, 1h., 2h., 3h., etc., 20h., 21h., 22h., 23h.,—prefixing the day to the hour. Hence, we should write—

> *Civil time:* May 8th at 3h. 40m. 20s. P.M.;
> and corresponding *Astronomical Date*, May 8d. 3h. 40m. 20s.
> Also *Civil time:* May 8th at 6h. 10m. 40s. A.M.;
> but corresponding *Astronomical Date*, May 7d. 18h. 10m. 40s.

as will be explained more fully in the sequel.

The GREENWICH DATE is the time at Greenwich which corresponds to any given time at any other place on the earth's surface; it is of the utmost importance, for the purposes of Nautical Astronomy, to know this *date* whenever an observation of a celestial object is made, since all the elements in the Nautical Almanac connected with the Sun, Moon, Planets, and Fixed Stars are given *for a definite time at the meridian of Greenwich*, which may not, and generally will not, correspond to the time of observation elsewhere.

The WEEK—a period of seven days—has been in use among Eastern nations from time immemorial; the same period was adopted by the Greeks and Romans, and the names of the days derived from the Sun, Moon, and five ancient planets, viz., Mercury, Venus, Mars, Jupiter, and Saturn. Similarly, our week-days are derived from the Saxon names of the same planets, as Woden, Friga, Tuisco, Thor, etc.

An ANOMALISTIC MONTH is the period of the moon's revolution round the earth from perigee to perigee, which occupies 27·5546 days or 27d. 13h. 18·624m.

A SYNODICAL MONTH, which is also a *lunation*, is the average time in which the moon goes through all her phases, from one new moon to another, or from one full moon to another, and consists of 29·5305888 days, or 29d. 12h. 44m. 2·87s.

The CALENDAR MONTH of civil reckoning is a period varying from 28 to 31 days, and was probably derived from the synodical month. There are 12 calendar months in a year, each consisting of an integral number of days—as January, 31 days; February, 28 (29 leap-year); March, 31; April, 30; May, 31; June, 30; July, 31; August, 31; September, 30; October, 31; November, 30; December, 31 days; making a total of 365 days for the length of the common year, and 366 days for leap-year.

A SIDEREAL YEAR is the time which elapses between the Sun's leaving a fixed star until his return to it—in fact, the period the earth takes to go round the sun; it consists of 365·25634 days, or 365d. 6h. 9m. 9·4s.

The ANOMALISTIC YEAR is the time the earth takes to return to perihelion, and consists of 365d. 6h. 13m. 45·7s.

The SOLAR YEAR—called also a TROPICAL or EQUINOCTIAL YEAR—is the year of the seasons, and marks the period in which the sun moves from the

vernal equinox to the vernal equinox again, *i.e.*, it marks the interval of time between two passages of the sun through the tropics or equinoctial points; owing to the *precession of the equinoxes* this period is *less* than a sidereal year, its average value—called a *Mean* SOLAR YEAR—being 365·242216 mean solar days, or 365d. 5h. 48m. 47½s.

The CALENDAR OR ORDINARY YEAR of civil reckoning consists of 365 days, and is derived from the *Mean Solar* (or *Tropical*) *Year;* the latter, however, being 5h. 48m. 47½s. (nearly a quarter of a day) in excess of 365 days, the excess is allowed to accumulate until every *fourth* year, by which time it has amounted to *nearly* a day, and thus every *fourth* year (with certain exceptions) consists of 366 days, and is called *Leap Year* or *Bissextile.* The additional day is given to the month of February.

But the increase of the length of the year by one day every fourth year would still produce an error in the lapse of time, as the excess of the mean solar year over 365 days is about 11m. 12½s. less than a quarter of a day; the error (within a very small amount) is corrected by the following *rule of intercalation*—

1. *For years that are not even centuries:* If the year, when divided by four, leaves a remainder, such year is ordinary, consisting of 365 days; if there be no remainder, the year is bissextile (of 366 days).

2. *For years that are even centuries:* If the number of centuries, when divided by four, leaves a remainder, the year is ordinary; if there be no remainder, it is bissextile.

Thus—1889, 1890 and 1891 are *ordinary* years (of 365 days).
1888, 1892, 1896 are *bissextile* years (of 366 days).
Also—1900, 2100, 2200, 2300 are ordinary years.
2000, 2400, 2800, 3200 are bissextile years.

According to this arrangement every period of 400 years consists of

97 bissextile years, or	35,502 days	
and 303 ordinary „ or	110,595 „	
	146,097 days	

the 400th part of which is 365·2425 days, or only 0·00028 of a day in excess of the mean solar year.

The error (*an excess*) will amount to a day in about 3,600 years, which the late Sir John Herschel proposed to correct by making every 4000th year lose its *leap* year, by which method there would not be an error of a day in 28,000 years.

The Bissextile Year was introduced into the calendar by order of Julius Cæsar, B.C. 45; and a subsequent adjustment was made by Pope Gregory XIII. in 1582, but not adopted in Great Britain until 1752, when the third day of September was (by Act of Parliament) called the 14th; and then commenced the *New Style.*

This part of the work may be aptly closed by introducing Professor Ball's note on the various Cycles.

Periods connected with the Sun and the Moon.—"If the nodes of the moon were fixed, then the period of revolution of the sun with regard to those nodes would be simply equal to the sidereal year. On account, however, of the regression of the moon's nodes, the sun returns to the same node in a period less than a single year. This period amounts to 346·619

NAUTICAL ASTRONOMY.

Fig 1.

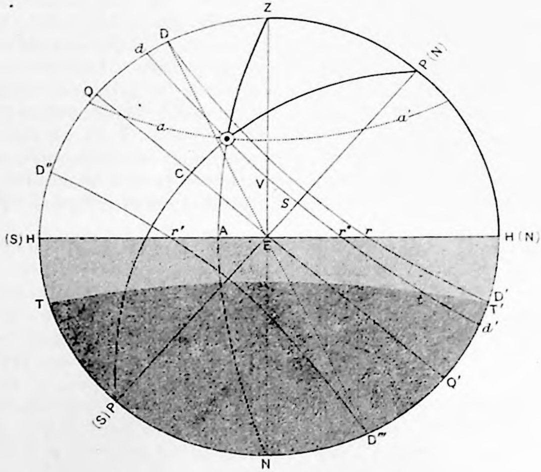
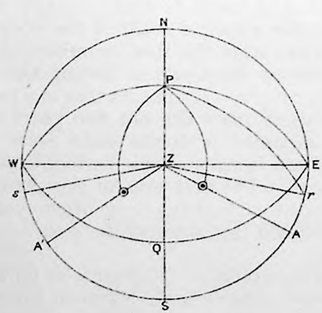


Fig. 2.



Imray, Laurie, Norie & Wilson.

days. From a comparison of this period with that of the moon itself a very remarkable result is obtained. If the moon revolved actually in the plane of the ecliptic, then the centre of the moon would in each revolution pass across the centre of the sun, and the moment of this occurrence is called the time of *new moon*. Owing, however, to the circumstance that the orbit of the moon is inclined to the plane of the ecliptic, the moon will not usually pass over the surface of the sun. It is therefore necessary to modify the definition of new moon accordingly. We define the time of new moon to be the moment when the *longitude* of the centre of the moon is equal to the *longitude* of the centre of the sun. The interval between two successive new moons is termed a *lunation*, and it is by this period that the successive phases of the moon are regulated. The length of the lunation is such that 223 lunations make 6585·32 days. Thus 19 periods of the revolution of the sun with respect to the nodes of the moon coincide very nearly with 223 lunations. This remarkable period, amounting to about 18 years 11 days, is of service in the prediction of eclipses. It is known as the *Saros*.

"Another very remarkable period arises from the circumstance that 235 lunations form 6939·69 days, while 19 years of 365·25 days amount to 6939·75 days. We therefore conclude that 19 years are nearly identical with 235 lunations. This is the *Cycle of Meton*. If the dates of new moon and full moon are known for a period of 19 years, they can be predicted indefinitely, for in each subsequent 19 years the dates are reproduced in the same manner. The number which each year bears in the Cycle of Meton is called the *golden number*. In 1888 the *golden number* is 8. In 1881 the *golden number* was 1, being the commencement of a new cycle.

"The period called the *Solar Cycle* is founded upon the recurrence of the day of the week upon the same day of the month. Owing to the complication produced by leap year, this period is 28 years. In the year 1888 the Solar Cycle is said to be 21. This signifies that 1888 is the twenty-first of one of these groups of 28 years. The cycle known as the *Roman Indiction* is a period of 15 years. Though this cycle is not connected with any astronomical phenomenon, it is still retained. Thus the year 1888 is the first year of the Roman Indiction and the commencement of a new cycle.

"In the Almanacs it is usual to find a certain number stated as the *Julian Period*. Thus, for example, 1888 is the 6601st year of the Julian Period. This cycle arises from the three numbers 19, 29, 15, which represent the entire periods of the Cycle of Meton, the Solar Cycle, and the Roman Indiction respectively. It appears that in a period of $19 \times 28 \times 15 = 7{,}980$ consecutive years there are not two years which have the same Golden Number accompanied with the same Solar Cycle and the same Roman Indiction. There is thus a new period, called the Julian Period, consisting of 7,980 years. The first year of this period is 4713 B.C., which has been adopted, because each of the three other cycles had the value 1 on that year. This period will continue till the year A.D. 3267."—R. S. Ball's *Elements of Astronomy*.

It remains now to illustrate by Diagrams, as far as it is possible to do so on a flat surface, some of the terms in Nautical Astronomy that have been defined in words; for this purpose Fig. 1, plate VII. is taken as a projection of the CELESTIAL SPHERE ON THE PLANE OF THE OBSERVER'S MERIDIAN in lat. 49° N.

The outer circle of Fig. 1 is the celestial meridian passing through the north and south points of the horizon, the zenith, and the pole. Other important great circles, as the horizon, the equinoctial, and the prime vertical are of necessity represented by *straight lines.* The observer is supposed to be standing in the centre of the projection with the various circles around him meeting in their respective poles ; then—

H H is the *rational* horizon, with H (N) the north point, and (S) H the South point ; E (in the centre of the diagram) will be the east point, and must also stand for the west point (which is represented only in imagination). The great circle of the horizon is divided into four quadrants by a plane passing through the north and south points, and another passing through the east and west points.

P (N) is the north pole, and (S) P the south pole of the celestial sphere, towards which the great circles called hour-circles and the circles of declination trend.

Z is the observer's zenith, or point directly overhead ; N is his nadir, or the point in the opposite hemisphere, beneath his feet.

Q Q′ is the equinoctial, or celestial equator, coincident with which great circle is the sun's path on the 20th of March and 23rd of September, giving equal day and night to all parts of the earth. The plane of the equinoctial passes through the E. and W. points of the horizon (represented at the centre of the diagram).

Z E N is the prime vertical, which passes through the E. and W. points of the horizon, and whose plane is perpendicular to the meridian, the latter passing through the N. and S. points of the horizon.

P E P is the six o'clock hour-circle.

H (N) P (N) is the arc of the meridian representing the altitude of the *elevated* pole, which is also the latitude of the observer's station—here projected for lat. 49° N.

P (N) Z is the co-latitude (*i.e.*, the *complement* of the latitude).

The line joining D D‴, cutting the plane of the equinoctial at E, represents the plane of the ecliptic whose *obliquity* in respect of the equinoctial is $23\frac{1}{2}°$.

D D′ is the small circle representing the parallel of the sun's *greatest northern* declination (23° 27′ N.) on the 21st of June.

(S) H D is the arc of the meridian representing the sun's altitude (*i.e.*, the meridian altitude) at noon on the 21st of June.

Z D is the arc of the meridian representing the sun's meridian zenith distance on June 21st.

H (N) D′ is the distance to which the sun descends below the horizon on June 21st.

r is the place of the sun's rising and setting, on June 21st, and E *r* is the sun's Amplitude, reckoned from the east point at rising, and from the west point at setting, towards north, because the declination is (in this case) N.

r D is the semi-diurnal arc representing half the length of the longest day, in lat. 49° N., and—

r D′ is the semi-nocturnal arc representing half the length of the shortest night for the same position.

Similarly, D″ D‴ is the parallel of the sun's *greatest southern* declination (23° 27′ S.), on the 21st of December.

(S) H D″ is the sun's meridian altitude on December 21st in lat. 49° N.

Z D″ is the sun's meridian zenith distance on December 21st.

H (N) D‴ is the distance to which the sun descends below the horizon on December 21st.

r′ is the place of the sun's rising and setting on December 21st; and E. *r′* is his amplitude, reckoned from the east point at rising, and from the west point at setting, towards south because the declination is (in this case) S.

r′ D″ is the semi-diurnal arc, representing half the length of the shortest day, and—

r′ D‴ is the semi-nocturnal arc, representing half the length of the longest night.

T T′ is the small circle, parallel with the horizon H H, and 18° below it, indicating the extent of twilight. The sun is on this circle before rising, at dawn, at the beginning of twilight, and on it again after setting, at the end of twilight.

It has been stated above that the projection of the diagram (Fig. 1, plate VII.) is for an observer's station in lat. 49° N.; it is further projected to illustrate an observation of the sun when the declination is 17° N., as on the 7th of May or 4th of August, 1888.

⊙ is the sun, and *d* ⊙ *d′* is the parallel of his declination (17° N.) on either day.

r″ will be the place of the sun's rising and setting; and E *r″* is the rising and setting amplitude, reckoned from East at rising, but from west at setting, toward north as the declination is N.

S is the sun's place on the six o'clock hour-circle, and V is his place on the prime vertical.

Taking ⊙ as the place of the sun at between 8h. and 9h. A.M. or between 3h. and 4h. P.M.; then A ⊙ is his altitude, and Z ⊙ his zenith distance. The dotted line *a a′* is the sun's parallel of altitude.

C ⊙ is the arc of the sun's circle of declination representing 17° N., and P (N) ⊙ is the sun's north polar distance, 73°.

Three sides of a spherical triangle (*viz.*, P (N) Z the co-lat., Z ⊙ the zenith distance of the sun, and P (N) ⊙ the Sun's N. polar distance) are given to find the hour-angle and azimuth; it is customary, however, to use the latitude and altitude instead of their complements.

⊙ P (N) Z is the sun's hour-angle, or meridian distance measured on the equinoctial by the arc C Q; ⊙ Z P (N) is the sun's azimuth from the *north* measured by the arc of the horizon H (N) A; and ⊙ Z (S) H is his azimuth from the *south*, measured by the arc of the horizon (S) H A.

(N) P ⊙ P (S) is the sun's hour-circle, and Z ⊙ A N his vertical circle for altitude and azimuth.

(S) H *d* is the sun's meridian altitude on the day, and Z *d* his Meridian Zenith distance.

r″ d is the semi-diurnal arc representing half the length of the day, and, as *d′* would be the sun's position at midnight below the horizon, *r″ d′* is the semi-nocturnal arc representing half the length of the night.

r″ t would be the limit of the duration of twilight, and *r″ s* the ascensional difference.

Having in Fig. 1, plate VII. illustrated the various circles of the sphere by means of a diagram drawn on the *plane of the observer's meridian*, it remains to depict them from another aspect—*viz.*, on a projection in which the observer is supposed to be surveying the entire plane of the horizon from the zenith as his standpoint—hence he sees the whole hemisphere which is above the horizon.

In such a projection (*see* Fig. 2, plate VII.) the horizon is the bounding circle, with the zenith as its centre; hence all circles that pass through the zenith, as the meridian, prime vertical, and other vertical circles, must appear as *straight lines.*

N E S W N is the horizon, with Z (the zenith) at the centre: Z, and its opposite point at the nadir, are the poles of the horizon.

The cardinal points of the horizon are: N, the north; E, the east; S, the south; and W, the west.

P is the *elevated* pole; and N P the *elevation of the pole*—equal to the latitude of the observer's station.

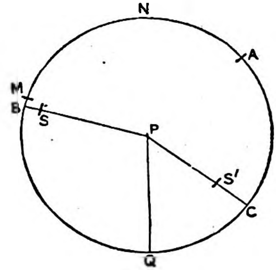
N Z S is the meridian of the observer's station, cutting the horizon in the N. and S. points; E and W are the poles of the meridian, and 90° from every point on that circle. Also, all circles that cut the meridian at right angles meet at the points E and W.

E Z W is the prime vertical, at right angles to the meridian, cutting the horizon in the E. and W. points; N and S are the poles of the prime vertical.

E P W is the six o'clock hour-circle.

E Q W is the equinoctial or celestial equator.

Supposing ☉ to be a heavenly body; then P ☉ is its polar distance; ☉ A is its altitude, *east* of the meridian; and ☉ A′ its altitude, *west* of the meridian; Z P ☉ (on the right of the diagram) is its *easterly* hour-angle or meridian distance, and Z P ☉ (on the left of the diagram) is its *westerly* hour-angle, or meridian distance; S Z A (on the right of the diagram) is its azimuth between south and east, and S Z A′ (on the left of the diagram) is its azimuth between south and west; supposing it to have risen at *r*, it would set at *s*; having E *r* for its amplitude at *rising*, and W *s* for its amplitude at *setting*.



Again, let the circle represent the equinoctial with the pole as its centre, and all the celestial meridians must appear as *straight lines.* P Q is the meridian of the place, P S B is the meridian passing through the object S, and P S′ C is the meridian through S′. Let A be the position of the first point of Aries, M that of the mean sun, S that of true sun, and S′ that of a star; then Q M is the mean time at place, Q B is the apparent time at place, M B is the equation of time, Q M A is the sidereal time at place or right ascension of the meridian, A M is the right ascension of the mean sun, A B is the right ascension of the true sun, A M Q C is the right ascension of the star, Q B A C is the westerly hour-angle, and Q C is the easterly hour-angle of the star S′.

PREPARATORY PROBLEMS

LONGITUDE AND TIME

The circumference of every circle is divided into 360 equal parts, called degrees; now the earth, whose circumference as a sphere is 360°, turns on its axis in the direction of this circumference once in 24 hours; therefore the 360° of circular measure are the equivalent of 24 hours; dividing 360 by 24 we get 15, and hence 15° of circular measure are the equivalent of 1 hour. We say, astronomically, that as a complete rotation of the earth on its axis is performed in 24 hours, meridians 15° as under are thus brought to the sun at regular intervals of one hour. This gives us the following Table :—

In Angular Measure (Arc).		In Time.
360°	=	24 hours.
15°	=	1 hour.
1°	=	4 minutes (*i.e.*, the 15th part of an hour).
15′ (*i.e.*, ¼ of a degree)	=	1 minute.
1′	=	4 seconds (*i.e.*, the 15th part of a minute).
15″ (*i.e.*, ¼ of 1′)	=	1 second.

This Table furnishes the following rules for converting longitude into time, or time into longitude:

I. TO CONVERT LONGITUDE (OR ARC) INTO TIME, the shortest method is to multiply the degrees and minutes (° and ′) of arc by 4, then the minutes (′) of arc become seconds of time, and the degrees become minutes of time, which reduce to hours by dividing by 60.

Or the conversion may be made at sight by Table (*Norie's Tables*, *p.* 198).

Example.—Convert 137° 26′ into its equivalent time.

```
     137°  26'          By Tab.  137° = 9  8
             4                    26' =    1 44
 6,0)54,9m. 44s.                         9  9 44
     9h. 9m. 44s.
```
(H. M. S.)

Example.—Convert 107° 11′ 40″ into time.

```
   107°   11'  40"        By Tab.  107° = 7  8
                4                   11' =    0 44
 6,0)42,8  46  40(= ·7)             40" =    0  2·67
     7h. 8m. 46·7s.                          7  8 46·67
```
(H. M. S.)

II. To Convert Time into Longitude (Arc), multiply the time by 15 (or by 3 and 5 successively, since $3 \times 5 = 15$); then the seconds of time become seconds (″) of arc, the minutes of time become minutes (′) of arc, and the hours become degrees.

Or, if we turn the hours into minutes, and divide by 4, then the minutes of time give degrees, the seconds of time give minutes (′) of arc, and the remainder (if any) multiplied by 60 and divided by 4 gives seconds (″) of arc.

Or the conversion may be made at sight by Table (*see Norie's Tables p.* 198).

Example.—Convert 8h. 40m. 34s. into longitude (or arc).

H.	M.	S.
8	40	34
		3
26	1	42
		5
130°	8′	30″

Or thus,

H.	M.	S.
8	40	34
60		
4)520m. 34s.		
130°	8′	30″

By Tab.

8h.	= 120°		
40m.	= 10	0′	
34s.	=	8′	30″
	130°	8′	30″

By these Rules we get the following results, which you can work out and prove:

Long.		Time. H. M. S.	Time. H. M. S.		Long.
0° 47′ 0″	=	0 3 8	0 2 40	=	0° 40′
12 49 0	=	0 51 16	1 53 20	=	28 20
108 53 30	=	7 15 34	5 21 46	=	80 26 30″
169 20 45	=	11 17 23	9 37 22	=	144 20 30
137 43 15	=	9 10 53	11 59 59	=	179 59 45
97 23 36	=	6 29 34·4	7 49 29	=	117 22 15

Comparison of Civil Time and Astronomical Time

Civil Time.—For business purposes and the registry of the common transactions of daily life, it has been deemed more convenient to begin the day at *midnight*, that is when the sun is on the meridian at its *lower transit*.

But the civil day—consisting of 24 hours—is divided into two periods of twelve hours each, *viz.*, from midnight to noon, marked a.m. (*ante meridiem*), and from noon to midnight, marked p.m. (*post meridiem*).

Astronomical Time.—A *solar day* at any place is the interval of time between two successive transits (passages) of the sun's centre over the meridian of that place: and *solar time* at any instant is the westerly hour angle of the sun at that instant.

The solar day (it may be *apparent*, or it may be *mean* time) is conceived by astronomers to commence at *noon* (apparent or mean), and is divided into twenty-four hours, numbered successively from 0 to 24.

Astronomical time (apparent or mean) is, then, the hour angle of the sun (apparent or mean), reckoned on the equator *westward* throughout its circumference from 0h. to 24h.

a.m. and p.m. never appear in astronomical time.

To *convert civil time into astronomical time*.—Since, from what has been said above, civil time agrees with astronomical time from noon to midnight, while from midnight to noon they differ, hence the *rules*—

1. If the civil time is p.m., and thus referring to the hours from noon to midnight, it agrees with the astronomical time; thus—

Civil T., May 4th 5h. 20m. p.m. = Astronomical T., May 4d. 5h. 20m.
Astronomical T., Sep. 20d. 10h. 42m. = Civil T., Sep. 20th 10h. 42m. p.m.

2. If the civil time is a.m. and thus referring to the hours from midnight to noon, add 12h. to the civil time and write the date a day back, to get astronomical time.

Or, to get civil time from astronomical time, subtract 12 hours from the astronomical time and write the date a day forward.

Civil T., Jan. 19th 5h. 17m. 4s. p.m. = Astron. T., 19d. 5h. 17m. 4s.
Civil T., May 4th 5h. 42m. a.m. = Astron. T., May 3d. 17h. 42m.
Civil T., Aug. 1st 9h. 20m. a.m. = Astron. T., July 31d. 21h. 20m.
Astron. T., Dec. 3d. 15h. 5m. = Civil T., Dec. 4th 3h. 5m. a.m.
Astron. T., Jan. 31d. 18h. 3m. = Civil T., Feb. 1st 6h. 3m. a.m.
Astron. T., Dec. 31d. 18h. 10m. = Civil T., Jan. 1st 6h 10m. a.m.

Time at Different Meridians, and Greenwich Date

Solar or Sun Time is measured by the daily motion of the real or true sun; it is called the *apparent time*.

An *apparent solar day* is the interval of time between two successive transits of the sun's centre over the same meridian. It is *apparent noon* when the real sun's hour circle coincides with the meridian of the place, that is, when the sun's meridian altitude is observed for the latitude; it is the readiest and most natural measure of time, and hence the *unit* of time adopted by the navigator at sea is the apparent solar day.

But these intervals, though convenient for sea purposes, are not equal, and therefore cannot be taken as a standard, since the sun describes in a day an arc of 57′·2 of the ecliptic in July, and of 61′·2 in December. This want of uniformity in the real sun's motion is obviated by the adoption of a fictitious, or *mean sun*, which is supposed to move in the equator with a uniform velocity, which is the mean velocity of the true sun in the ecliptic.

Hence *mean time* is the westerly hour angle of the *mean sun*; a *mean day* is the interval between two successive transits of the mean sun over the meridian; and *mean noon* is the instant when the mean sun's hour circle coincides with the meridian.

Mean time, being perfectly equable, and lapsing uniformly, is measured by the chronometer when the correct rate is applied from day to day.

The difference between apparent and mean time is the *equation of time*. Four times in the year—in April, June, August, and December—apparent and mean time coincide; in the intervals they differ, and their greatest difference occurs in February, May, July, and November.

The *equation of time* is used to convert apparent time into mean time, and mean time into apparent time. Thus, if the apparent time be given,

the corresponding mean time will be found by adding or subtracting the equation of time according to the precept at the head of the column in which it is found on p. I. of the month in the Nautical Almanac. If the mean time be given, the apparent time is found by applying the equation of time as directed by the precept on p. II. of the month in the Nautical Almanac.

The westerly hour angle of the sun at any meridian is the *local* (solar) time at that meridian ; *i.e.*, the time at place which may be apparent or mean.

The westerly hour angle of the sun at the Greenwich meridian at the same instant is the corresponding *Greenwich time.*

The difference between the local time at any meridian and the Greenwich time is equal to the longitude of that meridian from Greenwich, expressed in time, on the basis of 1h. = 15°.

The difference between the local time of any two meridians is equal to the difference of longitude of those meridians.

In comparing the corresponding times at two different meridians, the most easterly meridian may (owing to the earth rotating on its axis from west to east) be distinguished as that at which the time is *greatest*, or *most advanced ;* inasmuch as the sun will rise, culminate, and set at any given meridian earlier than at any meridian to the westward of it : for example, at a given instant of absolute time on March 20th, the sun (by local time) will be setting at 6h. p.m. in the middle of the Bay of Bengal, it will be on the meridian (noon) at Greenwich, and be rising at 6h. a.m. in the middle of the Gulf of Mexico.

N.B.—The *local time*, at sea, is usually called the *time at ship.*

The Greenwich date, either exact or approximate, is essential for the correction of the data from the Nautical Almanac ; and the first step in any computation is as follows—

Given the Time at Ship and Longitude, to find Greenwich Date

1. Express the ship date as astronomical time.
2. Convert the longitude into time, and write it under the ship date.
3. If the longitude is *west*, add it to the ship time.

N.B.—If the sum of the hours is more than 24h., reject 24h., and carry one day to the date.

4. If the longitude is *east*, subtract it from the ship time.

N.B.—If the longitude exceed the hours at ship, add (mentally) 24h. to the ship time, and put the date one day back.

Find the Greenwich date in each of the following examples—

Example 1.—May 10th, time at ship being 4h. 42m. 20s. p.m., in long. 48° 32′ W.

Long.	48° 32′
	4
	6,0)19,4 8s.
	3h. 14m. 8s.

	D.	H.	M.	S.	
Ship date; May	10	4	42	20	
Long. in time		3	14	8	W.
Green. date, May	10	7	56	28	

Example 2.—August 4th, time at ship 8h. 50m. 12s. a.m., in long. 134° 12′ W.

		D.	H.	M.	S.	
Long. 134° 12′ W.	Ship date, August	3	20	50	12	
In time 8h. 56m. 48s.	Long. in time......		8	56	48	W.
	Green. date, August	4	5	47	0	

Example 3.—August 1st, time at ship 9h. 40m. p.m., in long. 49° 15′ E.

	D.	H.	M.	
Ship date, August	1	9	40	
49° 15′ E. long., in time		3	17	E.
Green. date, August	1	6	23	

Example 4.—December 5th, time at ship 4h. 32m. p.m., in long. 127° 30′ E.

	D.	H.	M.	
Ship date, December	5	4	32	
127° 30′ E. long., in time...........		8	30	E.
Green. date, December	4	20	2	

	Approx. Ship Time.	H.	M.	S.		Long.			Approx. Green. Date.	D.	H.	M.	S.
Ex. 5.	April 1st	3	46	0	p.m.	90°	4′	E. =	March	31	21	45	44
Ex. 6.	May 10th	7	40	20	a.m.	64	55	W. =	May	10	0	0	0
Ex. 7.	Feb. 18th	8	7	0	a.m.	110	30	W. =	Feb.	18	3	29	0
Ex. 8.	Sept. 30th	8	10	0	a.m.	175	30	W. =	Sept.	30	7	52	0
Ex. 9.	Feb. 1st	7	11	50	p.m.	165	0	E. =	Jan.	31	20	11	50
Ex. 10.	June 1st	0	0	0	(noon)	64	39	E. =	May	31	19	41	24

These examples show you how, through an *approximate* time at ship, and longitude by dead reckoning, you can get an *approximate* Greenwich date. The method of proceeding is essential, inasmuch as all observations at sea are referred to chronometers regulated to Greenwich mean time, and as these instruments are only marked on the face or dial from 0h. to 12h., it becomes necessary to distinguish whether it is a.m. or p.m. at Greenwich, and hence when it is necessary to increase the chronometer time by 12h.

To find the correct Greenwich Date by Chronometer

Proceed as by previous rule to get the *approximate* Greenwich date, then apply the error to the chronometer time, and give the result a date corresponding to the approximate date, adding 12 hours to the time by chronometer when necessary to make them agree.

Example 1.—May 10th, at about 4h. 42m. p.m. at ship, in long. 48° W., the chronometer read 7h. 56m. 22s., which was slow on Greenwich mean time 3m. 25s. Find the correct Greenwich date.

	D.	H.	M.
Ship date ....May	10	4	42
Long. 48° W.		3	12
Approx. Gr. date, May	10	7	54

	D.	H.	M.	S.
Time by chron.		7	56	22
Slow		+	3	25
Cor. Gr. date, M.T. May	10	7	59	47

Example 2.—July 10th, at about 8h. 50m. a.m. at ship, in long. 134° W., the chronometer read 6h. 0m. 10s., which was fast on Greenwich mean time 6m. 11s. Find the correct Greenwich date, mean time.

		D.	H.	M.
	July	10	8	50 a.m.
Ship date....	July	9	20	50
Long. 134° W.			8	56
Approx. Gr. date,	July	10	5	46

	D.	H.	M.	S.
Time by chron.....		6	0	10
Fast..........		—	6	11
Cor. Gr. date, M.T. July	10	5	53	59

Example 3.—August 6th, at about 4h. 32m. p.m. at ship, in long. 127° E., the chronometer read 8h. 0m. 35s., which was fast on Greenwich mean time 1m. 40s. Find the correct Greenwich date, mean time.

		D.	H.	M.
Ship date	August	6	4	32
Long. 127° E.............			8	28
Approx. Gr. date,	August	5	20	4

	D.	H.	M.	S.
Time by chron (+ 12h.)		20	0	35
Fast		—	1	40
Cor. Gr. date, M.T. Aug.	5	19	58	55

Example 4.—April 1st, at about 8h. 7m. a.m. time at ship, long. 45° W., the chronometer read 11h. 5m. 49s., which was 6m. 50s. fast on Greenwich mean time. Required the correct Greenwich date, mean time.

		D.	H.	M.
	April	1	8	7 a.m.
Ship date	Mar.	31	20	7
Long. 45° W.			3	
Approx. Gr. date,	Mar.	31	23	7

	D.	H.	M.	S.
Time by chron. (+12h.)		23	5	49
Fast		—	6	50
Cor. Gr. date, M.T. Mar.	31	22	58	59

Verify the following examples for getting the correct Greenwich date—

	Approx. Ship Time.				Long.	Chron.			Error.			Cor. Green. Date.				
			H.	M.		H.	M.	S.	M.	S.			D.	H.	M.	S.
Ex. 5.	Aug.	3rd	3	2 P.M.	75° W.	8	11	7	6	10	fast =	Aug.	3	8	4	57
Ex. 6.	Dec.	7th	1	3 A.M.	150° E.	3	14	14	25	19	fast =	Dec.	6	2	48	55
Ex. 7.	Feb.	13th	8	7 P.M.	135° E.	10	37	13	30	30	slow =	Feb.	12	23	7	43
Ex. 8.	Jan.	15th	3	30 P.M.	75° W.	8	25	2	3	15	fast =	Jan.	15	8	21	47
Ex. 9.	Sept.	1st	8	0 A.M.	75° E.	3	15	20	3	15	fast =	Aug.	31	15	12	5
Ex. 10	May	20th	7	56 A.M.	154° W.	6	0	20	11	32	slow =	May	20	6	11	52
Ex. 11.	Nov.	6th	3	46 P.M.	129° E.	7	0	26	4	39	slow =	Nov.	5	19	5	5
Ex. 12.	Dec.	1st	0	6 P.M.	179° E.	0	0	12	6	58	slow =	Nov.	30	12	7	10
Ex. 13.	Feb.	4th	8	22 A.M.	96° W.	2	50	27	8	37	fast =	Feb.	4	2	41	50

1. N.B.—*When it is noon at ship in west longitude*, the Greenwich date is the longitude in time.

Example.—May 15th, noon at ship in long. 95° W. (= 6h. 20m.), the Greenwich date is May 15d. 6h. 20m.

2. N.B.— *When it is noon at ship*, in *longitude east*, subtract the longitude in time from 24h., and the remainder will be the Greenwich date of the *preceding day.*

Example.—May 15th, noon at ship in long. 152° E. (=10h. 8m.), the Greenwich date is May 14d. 13h. 52m.

NOTE.—This method clears up the ambiguity that sometimes exists in connection with the indicated hour by chronometer and its relation to the correct Greenwich date.

Given the Greenwich Date, Mean Time, Longitude, and Daily Reckoning, to find the Ship Date

Under the circumstances it is most appropriate, in connection with the time at Greenwich and the time at ship, to use the terms " Greenwich date " and " Ship date " inasmuch as *time* is too often taken to imply merely hours, minutes, and seconds, to the neglect of the day to which those data belong. It is only by using the term *date*, which implies day, hours, minutes, and seconds, that a proper comparison between the times at the various meridians can be made ; and in the absence of a due appreciation of this fact it is by no means certain that the student will have obtained correct elements from the Nautical Almanac, or a correct determination of *longitude*, as arising out of the *difference between* the *ship date* and *Greenwich date*. This is the reverse process to that given in the preceding examples ; and the Greenwich date is taken to be the time by chronometer corrected for error and daily rate, and is hence always Greenwich *mean time*.

1. To the Greenwich date add the longitude in time, if *longitude* is *east*.
2. From the Greenwich date subtract the longitude in time, if *longitude* is *west*.

If the ship date exceeds 12h., subtract 12h. from it, for civil time a.m., and add 1 to the astronomical day for the ship day.

	D.	H.	M.	S.
Ex. 1. Green. date, Nov.	7	7	56	40
Long. 48° 32′ E. +		3	14	8
Ship date, Nov.	7	11	10	48
Or, ,, ,, Nov. 7th		11	10	48 p.m.

	D.	H.	M.	S.
Ex. 2. Green. date, May	6	8	40	56
Long. 49° 15′ W. —		3	17	0
Ship date, May	6	5	23	56
Or, ,, ,, May 6th		5	23	56 p.m.

	D.	H.	M.	S.
Ex. 3. Green. date, June	8	22	4	12
Long. 62° E. +		4	8	0
Ship date, June	9	2	12	12
Or, ,, ,, June 9th		2	12	12 p.m.

	D.	H.	M.	S.
Ex. 4. Green. date, Feb.	5	2	0	21
Long. 90° 4′ W. —		6	0	16
Ship date, Feb.	4	20	0	5
Or, ,, ,, Feb. 5th		8	0	5 a.m.

	D.	H.	M.
Ex. 5. Green. date, Jan.	6	14	0
Long. 52° E. +		3	28
Ship date, Jan	6	17	28
Or, ,, ,, Jan. 7th		5	28 a.m.

	D.	H.	M.
Ex. 6. Green. date, Aug.	20	0	0
Long. 104° W. —		6	56
Ship date, Aug.	19	17	4
Or, ,, ,, Aug. 20th		5	4 a.m.

N.B.—When questions are set with time shown by chronometer, the ship time with the longitude in time applied to it will show whether the chronometer is showing astronomical time or whether 12 hours are to be added to it.

THE NAUTICAL ALMANAC AND ASTRONOMICAL EPHEMERIS

The first "Nautical Ephemeris" for the year 1767 was projected on the basis proposed by Dr. Nevil Maskelyne—the Astronomer-Royal of that day—and was published by order of the Commissioners of Longitude, together with the "Tables requisite to be used with the Nautical Ephemeris." The Ephemeris took its present form as "The Nautical Almanac and Astronomical Ephemeris" in 1834, and an alteration in the "difference columns" was made at a later date.

It is well here also to notice that the term "difference" as "Diff. for 1 hour," etc., which was used in the earlier editions of the Almanac, has been changed into the term "variation," as "Var. for 1 hour," "Var. for 10 m.," etc.

The Nautical Almanac is computed for the meridian of Greenwich, and, consequently, it contains the right ascensions and declinations of the sun, moon, planets, and fixed stars, the equation of time, lunar distances, and various other solar, lunar, and stellar elements, for given instants of Greenwich time.

Eighteen pages (I. to XVIII.) of the Almanac are given to each month for various elements or quantities relating to the sun and moon. It is probable that pp. I. and II. are more used than any other parts of the work, since those pages relate to the sun—its right ascension, declination, and semi-diameter—together with the equation of time; p. I. being adapted to Greenwich *apparent* noon, and p. II. to Greenwich *mean* noon; the "Var. in 1 hour" given on p. I. of the month is the variation (that is difference) at *noon* that the quantity is undergoing at that instant; but this "variation" is applicable to its *proper* quantity taken from either page, as required.

Before the student begins to use practically the various elements or quantities set down in the pages of the Nautical Almanac, it is earnestly recommended that he should attentively read, and become perfectly familiar with, the "explanation of the articles" given at the end of that work.

In this Epitome the Nautical Almanac can only be referred to incidentally, in relation to the necessary elements or quantities that are taken from it for the purpose of working the various problems in navigation.

Before we can find from the Almanac the values of any of these quantities for a given *local time*, we must invariably find the *corresponding Greenwich date* (*see* pp. 233 to 234). When this time is exactly one of the instants for which the required quantity is put down in the Almanac, nothing more is necessary than to transcribe the quantity as there set down. But when, as is mostly the case, the time falls between two of the times of the Almanac, the required quantity must be found by *interpolation*. To facilitate this interpolation, the Almanac contains the *rate of change*, or *variation* (that is *difference*) of each of the quantities in some unit of time.

To use the *variation* (difference) columns with advantage, the Greenwich

time must be expressed in that unit of time for which the variation (difference) is given: thus, when the variation is for one hour, the time must be expressed in hours and decimal parts of an hour; when the variation is for one minute, the time must be expressed in minutes and decimal parts of a minute, etc.

Simple Interpolation.—In the greater number of cases *in practice* it is sufficiently exact to obtain the required quantities by *simple* interpolation; that is, by assuming that the variations of the quantities are proportional to the differences of the times, which is equivalent to assuming that the variations given in the Almanac are constant. This, however, is never the case; but the error arising from the assumption will be smaller the less the interval between the times in the Almanac; hence, those quantities which vary most irregularly, as the moon's right ascension and declination, are given for every hour of Greenwich time; others, as the moon's semi-diameter and horizontal parallax, are given for every twelfth hour, viz., for noon and midnight; others, as the right ascension and declination of the sun, are given for each noon, as are also most of the planetary elements; while others, as the right ascensions and declinations of the fixed stars, are given for every tenth day.

The following examples illustrate *simple* interpolation when the Greenwich date mean time is determined or given—.

RULE.—When the quantity, as the sun's declination or the equation of time, has the " Var. in 1 hour " given.

1. Take from the Nautical Almanac for the nearest *preceding* mean time date the required quantity, and the corresponding " Var. in 1 hour."

2. Multiply the " Var. in 1 hour " by the hours and decimal of an hour of the Greenwich date; the product is the *correction*.

3. *Add* this correction (properly reduced) to the Nautical Almanac quantity, if that quantity is *increasing*, but *subtract* it if *decreasing*.

4. Also, note that if the Greenwich date is nearer to a *subsequent* than to a *preceding* Almanac date it will be more accurate to interpolate back from the subsequent date, in which case *subtract* the correction if the quantity is *increasing*, but *add* it if the quantity is *decreasing*.

NAUTICAL ALMANAC, 1914 EDITION

The " Abridged Edition for the use of Seamen " of the Nautical Almanac supplies the seaman with all the astronomical data he requires for finding his position, etc., at sea by observations of the sun, moon, planets, or stars; or for rating his chronometers on shore by observations of the sun or stars.

Quantities are given to a degree of accuracy comparable with that obtainable in the data by sextant observations; as a general rule to 0′·1 of arc and 0·1s. of time. The Almanac gives, at Greenwich mean noon throughout the year and in certain cases for greater convenience at every even hour of Greenwich mean time, the positions, with reference to the equator, of all the heavenly bodies the seaman makes use of, in terms of declination and right ascension, together with the equation of time. The values at any other Greenwich mean time may be found, either by the ordinary methods of interpolation or by making use of certain auxiliary tables, or in certain cases by inspection only.

The chief alterations are—

Page I. Apparent time no longer given: Greenwich mean noon takes

its place. The declination is given in degrees, minutes, and tenths, and the acceleration from 1 to 24 hours is given on each page in a column parallel and adjacent to the right ascension of the mean sun.

Page II. is occupied by the moon and the transit of the first point of Aries.

Pages III., IV., V., VI. tabulate the right ascension mean sun, declination, and equation of time for every even hour, therefore the necessary correction can be found at sight.

Pages VII., VIII., IX., and X. are given to the moon's right ascension and declination for every even hour, with the difference for two hours between.

Pages XI. and XII. tabulate the data relating to the four planets, Venus, Mars, Jupiter, and Saturn.

Pages 146 to 153 are given to stars.

The right ascension and declination are given of all stars of magnitude 3·0 and upwards, at intervals of ninety days. In most cases the values for any day can be taken out by inspection.

The Pole Star Tables are much extended, and a table of Rising and Setting is added (pp. 160-161).

The rearrangement has been made to cut out such Tables and ephemeris as were not used by sailors, and to simplify the remainder.

Correct the Following Elements

Given the following: October 5d. 8h. 42m. 2s. Greenwich date, mean time; correct for that date the sun's declination, the equation of time, and the sun's right ascension.

Here the numbers appertaining to each element are taken from Nautical Almanac, p. II. of October, because the time given is *mean* time, but the "Var. in 1 hour" must be taken from p. I. under the heading "Var. in 1 hour."

Var. in 1h. = 57″·77	Sun's decl.	4°	49′	14″·3 S.
8h. 42m. = 8·7	Cor. for 8·7h.	+	8	22 ·6
40439	Corrected sun's decl.	4	57	36 ·9 S.
46216				
6,0)50,2·599				
8′ 22″·6				

		M.	S.
Var. in 1h. = 0·74 s.	Equat. of time	11	36·94
8·7	Cor. for 8·7h.	+	6·44
6·438	Cor. equat. of time	11	43·38

		H.	M.	S.
Var. in 1h. = 9·114 s.	Sun's right ascension.......	12	44	50·09
8·7	Cor. for 8·7h.	+	1	19·29
6,0) 7,9·2918	Cor. sun's R.A.	12	46	9·38
1m.19s.29				

NOTE.—In the first example the method of obtaining the correction is given in full; in the subsequent examples the method is abbreviated; but the student can verify the result by multiplying as required.

When the given Greenwich time is nearer to a *subsequent* than to a *preceding* Almanac date, it will be *more accurate to interpolate back* from the subsequent date.

Example.—February 22d. 18h. 42m. 3s. Greenwich date, mean time.

Here it is preferable, for accuracy, to refer the time to noon of the next day, and reckon it as 5h. 18m. = 5·3h. before noon of the 23rd ; and for that time find the correction, which is to be added if the element is decreasing, but to be subtracted if increasing.

Var. in 1h. = 55″·16	Sun's decl.	9°	43′	53″·9 S.
5h. 18m. = 5·3	Cor. for 5·3h.	+	4	52·3
6,0)29,2·348	Corrected sun's decl.	9	48	46·2 S.
4′ 52″·3				

s.		M.	s.
Var. in 1h. = 0·357	Equat. of time	13	33·04
5·3	Cor. for 5·3h.	+	1·89
1·8921	Cor. equat. of time	13	34·93

s.		H.	M.	s.
Var. in 1h. = 9·498	Sun's right ascension	22	26	51·73
5·3	Cor. for 5·3h.	—		50·34
50·3394	Cor. sun's R.A.	22	26	1·39

To find the Declination of the Sun at the time of its transit over a given meridian, also the Equation of Time at the same instant

When the sun is on the meridian at any place in *west* longitude the Greenwich *apparent* time is precisely equal to the longitude. That is, the Greenwich apparent time is *after* the noon of the same date as the local date, by a number of hours (and decimal of an hour) equal to the longitude.

When the sun is on the meridian at any place in *east* longitude, the Greenwich apparent time is *before* the noon of the same date as the local date, by a number of hours (and decimal of an hour) equal to the longitude.

Hence, to obtain the sun's declination and the equation of time for apparent noon at any meridian, take these elements from the Nautical Almanac (p. I. of the month) for Greenwich apparent noon of the same date as the local date, and apply a correction equal to the " Var. in 1 hour " multiplied by the number of hours (and decimal of an hour) in the longitude, observing to *add* or *subtract* the correction thus obtained, according as the element in the Nautical Almanac may indicate, for a time *before* or *after* noon.

Rule for Sun's Declination at noon, local date

Turn the longitude into time, as hours and decimal of an hour.

From Nautical Almanac, p. I. of the month, take out the sun's declination for the ship date, and also the " Var. in 1h." for the same date.

Multiply the " Var. in 1h." by the longitude in time, and the product will be the *correction*, to be applied to the declination as follows—

W. long.	Decl. increasing, add correction.
	Decl. decreasing, subtract correction.
E. long.	Decl. increasing, subtract correction.
	Decl. decreasing, add correction.

NOTE.—In March the sun's declination changes from S. to N., and in September it changes from N. to S.; if the correction is subtractive and greater than the declination, subtract the declination from the correction, and give the remainder the *contrary* name to the declination; these cases must be specially noted.

Example.—Find the sun's declination from the Nautical Almanac for apparent noon at place on the following dates, the longtitude being 100° 30′ W. = 6h. 42m. = 6·7h.—

Var. in 1h. = 57″·56	*April* 2d. decl.	5°	1′	19″	·2	N.
Long. in time 6·7	Cor. for 6·7h.	+	6	25	·7	
40292	Corrected decl. ...	5	7	44	·9	N.
34536						
6,0)38,5·652						
Cor. 6′ 25″·7						
Var. in 1h. = 53″·56	*Feb.* 19d. decl.	11°	10′	42″	·3	S.
Long. in time 6·7	Cor. for 6·7h.	—	5	58	·9	
6,0)35,8·852	Cor. decl.	11	4	43	·4	S.
Cor. 5′ 58″·9						
Var. in 1h. = 59″·26	*March* 20d. decl.	0°	3′	31″	·1	S.
Long. in time 6·7	Cor. for 6·7h.	—	6	37	·0	
6,0)39,7·042	Cor. decl.	0	3	5	·9	N.
6′ 37″·0						

Example.—Find the sun's declination from the Nautical Almanac for apparent noon at place on the following dates, the longitude being 154° 30′ E. = 10h. 18m. = 10·3h.—

Var. in 1h. = 56″·26	*April* 7d. decl.	6°	55′	11″	·9	N.
Long. in time 10·3	Cor. for 10·3h. ...	—	9	39	·5	
16878	Cor. decl.	6	45	32	·4	N.
56260						
6,0)57,9·478						
Cor. 9′ 39″·5						
Var. in 1h. = 53″·81	*Aug.* 30d. decl.	8°	56′	14″	·4	N.
Long. in time 10·3	Cor. for 10·3h.	+	9	14	·2	
6,0)55,4·243	Cor. decl.	9	5	28	·6	N.
9′ 14″·2						
Var. in 1h. = 58″·48	*Sept.* 23d. decl.	0°	9′	16″	·1	S.
Long. in time 10·3	Cor. for 10·3h.	—	10	2	·3	
6,0)60,2·344	Cor. decl.	0	0	46	·2	N.
10′ 2″·3						

N.B.—If the declination at Greenwich noon is 0° 0′ 0″; in *east* longitude the correction will be the declination of the same name as that of the day *before*; in *west* longitude the correction will be the declination of the same name as that of the day *after*.

To find the Declination and Right Ascension of the Moon, at any given Greenwich date

The declination and right ascension of the moon are given for every hour of Greenwich date, together with the "Var. for 10 minutes." Consequently by removing the decimal point of the variation one place to the left, you have the "Var. for 1 minute"; then—

RULE.—Multiply the "Var. for 1m." by the minutes and decimal of a minute of the Greenwich time; this gives the correction required, which is to be added to, or subtracted from the quantity, according to whether it is increasing or decreasing.

Example.—Given the Greenwich date May 5d. 9h. 8m. 30s., required the moon's right ascension and declination.

For the right ascension the variation in 10m. is 23·503s., hence for 1m. it is 2·3503s.; also 8m. 30s. of Greenwich time = 8·5m.

	s.		H. M. S.
Var. in 1m. =	2·3503	Moon's R.A. *May* 5d. at 9h. =	15 45 48·34
8m. 30s. =	8·5	Cor. for 8·5m.	+ 19·98
	117515	Moon's corrected R.A.	15 46 8·32
	188024		
	19·97755		

For the declination the variation in 10m. is 105″·08, hence for 1m. it is 10″·508.

Var. in 1m. =	10″·508	Moon's decl. *May* 5d. at 9h. =	17° 32′ 13″·3 S.
	8·5	Cor. for 8·5m.	+ 1 29 ·3
	6,0)8,9·3180	Moon's corrected decl.	17 33 42 ·6 S.
	1′ 29″·3		

Here, also, as in the case of the sun, if the Greenwich time is nearer to a *subsequent* than to a *preceding* Almanac date, it will not only be more convenient, but more accurate, to interpolate back from the subsequent date

Example.—Given the Greenwich date May 5d. 9h. 47m. 42s., required the moon's right ascension and declination.

The variation for the right ascension in 10m. at 10 hours is 23·563s.; for 1m. = 2·3563s.

The variation for the declination in 10m. at 10 hours is 104″·11; for 1m. = 10″·411.

Also 9h. 47m. 42s. is 12m. 18s. (or 12·3m.) from 10 hours; hence—

	s.		H.	M.	S.
Var. in 1m. =	2·3563	Moon's R.A. *May* 5d. at 10h. =	15	48	9·54
	12·3	Cor. for 12·3m.		—	28·98
	70689	Moon's cor. R.A.	15	47	40·56
	282756				
	28·98249				

Var. in 1m. =	10″·411	Moon's decl. *May* 5d. at 10h. =	17°	42′	40″·9 S.
	12·3	Cor. for 12·3m.	—	2	8 ·1
	6,0)12,8·0553	Moon's cor. decl.	17	40	32 ·8 S.
	2′ 8″·1				

Interpolation by Second Differences.—This method is wholly unnecessary for sea computations, which assume the *first differences* variation in 1h. or variation in 10m., to be constant; but for great accuracy interpolation is required by *second differences.*

NOTE.—The differences between successive first differences are called the second differences.

To Correct the Sun's Declination by Second Differences.—The Nautical Almanac says the "Var. in 1 hour" (formerly called the *hourly difference*) is intended to facilitate the reduction of the quantities from noon to any other time; but *it is the variation at noon,* and requires to be reduced to midway between noon and the time at which the declination is required.

The simplest and surest way to make this correction is to take the *difference* between the "Var. in 1 hour" for the given Greenwich day and that for the following day; divide this difference by 4, and the quotient by 12 (since 4 times 12=48), then multiply the last quotient by the Greenwich time and apply the product to the "Var. in 1 hour" for the given day, adding it if the "Var. in 1 hour" is increasing, otherwise subtracting it.

Suppose the sun's declination and variation in 1 hour to be as follows—

	DECL.	VAR. IN 1h.
December 19	23° 25′ 56″·2 S.	2″·78
20	23° 26′ 48″·8	1″·60
21	23° 27′ 13″·1	0″·43
22	23° 27′ 9″·1	0″·75
23	23° 26′ 36″·9	1″·93

Let it be required to find the sun's declination for Greenwich date mean time, December 19d. 14h. 24m.

The difference between 2″·78 and 1″·60 is 1″·18, which divided by 4 gives ·295, and ·295 divided by 12 gives ·0246; then ·0246 multiplied by 14·4 gives ·35424; and ·35 (using only two decimals) subtracted from 2″·78 (since variation in 1 hour is decreasing) leaves 2″·43 for the correct "Var. in 1 hour."

Finally, in the usual way, multiply 2″·43 by 14·4 (the Greenwich time), and 34″·99 is the correction of declination, to be added because declination is increasing; hence, declination for December 19d. 14h. 24m.=23° 26′ 31″·2 S.

Remember that twice in a year, viz., once in June, and once in December, the sum of the "Var. in 1 hour" for the two days will be their difference

because *in the interval* the sun has attained its greatest declination and has begun to decrease. In the present case this occurs between the 21st and 22nd of December, when ·43 + ·75 = 1·18 is the difference of the two variations. Then, proceeding as already directed, if the *correction* of variation is less than the "Var. in 1 hour" the correction of declination will be additive to the declination; but if the *correction* of variation is *greater* than the "Var. in 1 hour," subtract the variation from its correction, and the correction of declination when obtained must be subtracted from the declination, for the sun has passed its *maximum* declination.

When the declination is about to change from S. to N. or from N. to S. the "Var. in 1 hour" is large and the difference of two successive variations very small; but when the declination is near a maximum the "Var. in 1 hour" is small, and the difference of two successive variations large; at the maximum the sum of two variations will probably be their difference.

To Correct the Equation of Time by Second Differences.—The remarks already made in respect to the sun's declination equally apply to the correction of the equation of time.

Suppose, for two consecutive days, the equation of time and variation in 1 hour to be—

	Eq. of T.	Var. in 1h.
Day	M. S.	S.
15	13 46·78	0·585
16	14 0·55	0·562

and the equation be required for 15d. 14h. 24m. or 14·4h.

Then, ·023 is the difference of variation, which divided by 4 gives ·00575, and this divided by 12 gives ·00048; multiply ·00048 by 14·4 and you get ·006912; or, say ·007 to be subtracted from ·585, which leaves ·578 for the corrected "Var. in 1 hour"; finally ·578 multiplied by 14·4 gives correction of equation 8·32s. to be added to 13m. 46·78s.: hence, corrected equation of time will be 13m. 55·1s.

Remember that four times in the year, viz., once in February, once in May, once in July, and once in November, the sum of the "Var. in 1 hour" for the two days will be their difference.

When the equation of time is about to change from *sub. to add.* or from *add. to sub.* the "Var. in 1 hour" is large and the difference of two successive variations very small; but when the equation is near a maximum, the "Var. in 1 hour" is small, and the difference of two successive variations large; at the maximum the sum of the two variations will probably be their difference, as already explained in connection with the sun's declination.

To Correct the Moon's Declination and Right Ascension by Second Differences.—The moon's declination and right ascension are given for every hour of the day, Greenwich date, with the variation in 10 minutes; but this "Var. in 10m." must be reduced to midway between the time for which the declination or right ascension is required and the preceding hour.

Rule.—Take the difference between the "Var. in 10m." for the given Greenwich hour and that for the next hour; multiply this difference by the minutes (and decimal of a minute) of Greenwich time, and divide the product by 120 (twice 60); apply the result to the "Var. in 10m." for the

given hour, adding it if the variation is increasing, otherwise subtracting it.

Suppose the moon's right ascension and declination to be as follows—

Hour	Moon's R.A.			Var. in 10m.	Moon's Decl.			Var. in 10m.
	H.	M.	S.	S.	°	′	″	″
0	16	1	58·48	25·587	S 22	49	34·1	40·66
1	16	4	32·17	25·642	22	53	33·6	39·17
2	16	7	6·18	25·696	22	57	24·1	37·67

Suppose it is required to find the declination and right ascension for Greenwich time, 1h. 40m. 42s., or 1h. 40·7m.

For the declination the difference between 39″·17 and 37″·67 is 1″·5, which multiplied by 40·7 gives 61·05; then 61·05 divided by 120 gives ·509, or say ·51 to be subtracted from 39″·17, leaving 38″·66 as the *proper* variation in 10m. for the declination.

Proceeding in a similar manner for the variation in 10m. for the right ascension you get 25·660s. as the *proper* variation in 10m.

Having obtained the variation in 10m. you can understand that by removing the decimal point one figure to the left you get the " Var. in 1 minute."

Thus 3″·866 will be the Var. in 1 min. for the Decl.;
And 2·566s. will be the Var. in 1 min. for the R.A.

For the Correction of the Declination or Right Ascension.—Multiply the variation in 1m. by the given minutes (and decimal of a minute) of Greenwich time, and apply this correction in the usual way.

Thus, for the declination, 3″·866 multiplied by 40·7 gives 157″·3462, or 2′ 37″·3 to be added to the declination for 1 hour, since the declination is increasing; and the corrected declination for 1h. 40m. 42s. is 22° 56′ 10″·9 S.

When the correction is subtractive and exceeds the declination, take the declination from the correction, and change the name of the declination.

Similarly, for the right ascension, 2·566s. multiplied by 40·7 gives 104·4362s. or 1m. 44·44s. to be added to right ascension for 1 hour, since the right ascension is increasing; and the corrected right ascension fo 1h. 40m. 42s. is 16h. 6m. 16·61s.

The correction of right ascension is always additive, and when the application of the correction causes the right ascension to exceed 24 hours, reject the 24 hours, leaving only the minutes and seconds.

When the moon's declination is about to change from N. to S., or from S. to N., the " Var. in 10m." is large and the difference between two successive variations small; but when the declination is near a maximum (N. or S.) the " Var. in 10m." is small but the difference between two successive variations is large. At the maximum the sum of two variations will be their difference; for example, suppose the " Var. in 10m." to be as follows—

Var. in 10m.	
2″·79	diff. by subtraction is 1″·51
1 ·28	diff. by addition is . . 1 ·52
0 ·24	diff. by subtraction is 1 ·53
1 ·77	

To find the local time of the Moon's Meridian Passage (or transit over a given meridian)

The mean time of transit of the moon over the Greenwich meridian on each day is nothing more than the hour angle of the mean sun at the instant, or the difference between the right ascension of the moon and the right ascension of the mean sun. If this difference did not change, the mean *local* time of the moon's transit would be the same for all meridians; but as the moon's right ascension increases more rapidly than the sun's, the moon is apparently *retarded* from transit to transit.

Page IV. of the Nautical Almanac gives the mean time of the moon's "Meridian Passage" (or transit) over the meridian of Greenwich for each day; and the difference between two successive "Passages" is the *retardation* of the moon (varying from 44m. to 66m. according to the rate of the moon's motion) in passing over 24 hours of longitude.

RULE.—Take the moon's "meridian passage" (*upper*) from page IV. of the Nautical Almanac for the given astronomical day.

If *in west longitude* take the difference between the time of "passage" on the given and following day. If *in east longitude* take the difference between the time of "passage" on the given and the previous day.

The difference being a 24-hour difference, hence, multiply it by the *longitude in time*, and divide the product by 24 (or by 2 and 12 successively); the result will be the correction to be applied to the Greenwich meridian passage as follows:

If *in west longitude* add this correction to the Greenwich meridian passage for the given day. If *in east longitude* subtract the correction from the Greenwich meridian passage for the given day. The result will be the meridian passage at ship.

NOTE.—In dealing with the day, it may be *civil time* a.m., for during half the lunar month the moon passes the meridian after midnight, but the time in the Nautical Almanac is astronomical time. Take an example: On October 4th the moon (by Nautical Almanac) passes the meridian of Greenwich at 17h. 26m.; now this is the 5th a.m. at ship; therefore, for W. long. the day and day after would be the 4th and 5th; for E. long. the day and day before would be the 4th and 3rd; in each case the actual astronomical day is the 4th, since it corresponds to the civil day 5th a.m.

The Rule just given applies equally to *planets*, which have the *mean times of transit* given in the Nautical Almanac.

For the Greenwich mean time corresponding to the time at ship.—To the time of the meridian passage at ship apply the longitude in time; add if west; subtract if east.

Then correct the moon's declination for the Greenwich time (*see* p. 241).

Example.—January 29th; long. 82° 30′ W., find the local time of the moon's meridian passage, and the corresponding Greenwich date—

	H.	M.
Jan. 29d.	7	26·7
,, 30d.	8	13·9
Diff.		47·2
Long. in time		5·5
		2360
		2360
		2)259·60
		12)129·8
Cor. for 5·5h. =		10·8

Long. 82° 30′ = 5h. 30m. = 5·5h.

	H.	M.
Green. Mer. Pass. Jan. 29d.	7	26·7
Cor.	+	10·8
Mer. Pass. at Ship	7	37·5
Long. in time	5	30
Green. Date, Jan. 29d.	13	7·5

Example.—February 2nd; long. 97° 30′ E., find the local time of the moon's meridian passage, and the corresponding Greenwich date—

	H.	M.
Feb. 2d.	10	40·6
„ 1d.	9	51·3
Diff.		49·3
Long. in time		6·5
		2)320·45
		12)160·22
Cor. for 6·5h. =		13·35

Long. 97° 30′ = 6h. 30m. = 6·5h.

	H.	M.
Green. Mer. Pass. Feb. 2d.	10	40·6
Cor.	—	13·4
Mer. Pass. at Ship	10	27·2
Long. in time	6	30
Green. Date, Feb. 2d.	3	57·2

Example.—April 10th a.m. at ship, in long. 105° W., find the local time of the moon's meridian passage, and the corresponding Greenwich date—

	H.	M.
April 9d.	16	5·2
„ 10d.	17	4·4
		59·2
Long. in time		7
		2)414·4
		12)207·2
Cor. for 7h. =		17·3

Long. 105° = 7h.

	H.	M.
Green. Mer. Pass. April 9d.	16	5·2
Cor.	+	17·3
Mer. Pass. at Ship	16	22·5
Long. in time	7	0
Green. Date, April 9d.	23	22·5

In this *example* the moon passes ship's meridian on April 10th at 4h. 22·5m. a.m.

N.B.—See Correction of Moon's Mer. Pass., Norie's Tables.

To find a Planet's Right Ascension and Declination at the time of transit over a given meridian

Venus, Mars, Jupiter and Saturn are the only planets the navigator uses to determine the latitude by the meridian altitude (*i.e.*, at transit). The Nautical Almanac, for the purpose of facilitating the reduction of the right ascensions and declinations of these planets, gives their "Variations in 1 hour of longitude," which can consequently be used in the same manner as in the "Var. in 1 hour" of the sun, and be similarly applied as regards E. and W. long. (*see* also Nautical Almanac explanations, "Planetary Ephemerides at Transit").

Moon's Semi-diameter and Horizontal Parallax

These elements are given in Nautical Almanac, p. III. of the month, for noon and midnight, consequently they require to be interpolated for the given Greenwich time; if that time exceeds 12 hours the interpolation will naturally fall between midnight and the succeeding noon.

The corrected semi-diameter will require the *augmentation* due to the apparent altitude (*see* Table, Augmentation of the moon's semi-diameter [Table D]).

The corrected horizontal parallax will require the *reduction* for latitude (see Table E. Reduction of the Moon's Horizontal Equatorial Parallax for the figure of the Earth). Both these tables are in Norie's Tables.

Planet's Right Ascension and Declination for a given Greenwich Date, mean time

These elements come from the Nautical Almanac under the head of the *given planet* "mean time," not "at transit at Greenwich," *unless* for "latitude by the planet's meridian altitude." They are given from day to day for Greenwich mean noon, and the difference of each element for two successive days is a 24-hour difference; this difference must be taken as the basis of the correction for the given Greenwich date, mean time.

Thus, suppose the daily (or 24-hour) difference of declination to be 47′38″·3 and you require the correction for 8h. of Greenwich time; then 47′ 38″·3 multiplied by 8 and the product divided by 24 (or by 2 and the resulting quotient by 12), the proportional part for 8 hours will be 15′ 52″·8, to be added to, or subtracted from. the declination of the given day, according as the declination is increasing or decreasing.

The correction for the right ascension is obtained in a similar manner.

To find the Right Ascension of the Mean Sun, for a given time and place

The *Sidereal Time of Mean Noon* is also *the Right Ascension of the Mean Sun* at Greenwich *mean noon*, and may be reduced by interpolating for the constant hourly difference 9·8565s., but preferably by a table, as follows—

RULE.—From the Nautical Almanac, p. II. of the given month, column headed "sidereal time," take out the sidereal time for Greenwich noon of the given day, and accelerate it for the Greenwich mean time, using the table near the end of the Nautical Almanac entitled "Table for Converting INTERVALS OF MEAN SOLAR Time into EQUIVALENT INTERVALS OF SIDEREAL Time."

Example.—February 1d. 15h. 40m. 52s. M.T. at Green.; required the Mean Sun's R.A.—

	H.	M.	S.
Naut. Alm., p. II. *Feb.* 1; Sid. Time ..	20	46	34·50
Acceleration for 15h. ..		2	27·847
,, 40m. ..			6·571
,, 52s. ..			·142
Mean Sun's R.A. ..	20	49	9·06

Example.—March 22d, at 9h. 48m. p.m. local mean time in Long. 47° 12′ W.; required the Mean Sun's R.A.—

	H.	M.	S.
Local Mean Time, *March* 22d. ..	9	48	0
Long. 47° 12′ W. =	3	8	48
Green. Mean Time 22d. =	12	56	48

	H.	M.	S.
Naut. Alm., p. II. *March* 22d.; Sid. Time ..	23	59	45·63
Acceleration for 12h. ..		1	58·278
„ 56m. ..			9·199
„ 48s. ..			·131
Mean Sun's R.A. ..	0	1	53·238

The right ascension of the mean sun is also equal to the right ascension of the true sun + the equation of time, using the sign for the equation of time indicated for its application to mean time.

The Right Ascension and Declination of the Fixed Stars

The elements of the fixed stars are given in the Nautical Almanac for every *tenth* day, and can be taken out, or interpolated, at sight. The hours and minutes of right ascension, and degrees and minutes (′) of declination, are placed at the head of the columns as constants, and belong equally to all the numbers below them; hence the seconds sometimes exceed 60; in which case increase the minutes by 1 and write down the number of seconds in excess of 60; thus on May 1, the declination of α Argus (*Canopus*) is given as 52° 37′ 84″, which is to be read as 52° 38′ 24″.

CORRECTIONS OF OBSERVED ALTITUDES

The *altitude* of a heavenly body is its distance in arc from the horizon measured on a vertical circle.

The *true altitude* is the altitude of the object's centre above the rational horizon, as if it were measured by an observer at the centre of the earth.

The *apparent altitude* is the altitude of the object's centre above the sensible horizon.

The *observed altitude*, measured at sea by a reflecting instrument without index error, is the altitude above the visible horizon, and is reduced to the apparent, or true altitude, as may be required, by the application of certain corrections, which should be taken in the following order:

For the apparent altitude of the object's centre, apply the *index error* (if any), the *dip*, and the *semi-diameter* (if any).

For the true altitude of the object's centre: Having applied the corrections as specified for the apparent altitude, next apply the *refraction* and *parallax;* and the result will be the *true* or *geocentric* altitude of the heavenly body's centre. The sun, moon, and planets have parallax, but not the fixed stars.

It is not absolutely necessary in ordinary navigation to observe the order just indicated, but when intent on attaining to the nearest amount of precision use correct methods. There are Tables in Norie's which combine most of, or all the corrections, and these are near enough for sea practice; never attempt such slipshod work as getting the sun's zenith distance by using 89° 48′ in connection with the sun's altitude.

Index Error.—Reference has already been made to the correction for index error in the quadrant or sextant: it arises from defect of parallelism

between the index-glass and horizon-glass when the index is at 0° ; it is, therefore, the first correction to be applied to the altitude as obtained by the instrument, to get the *correct* observed altitude.

Dip or Depression of the Horizon.—Dip of the horizon is the angle of *depression* of the visible sea-horizon below the true horizon, arising from the elevation of the eye of the observer above the level of the sea.

In the Fig. 1 suppose A to be the position of the observer's eye, the height of which above the level of the sea is B A ; and S the position of the star whose altitude is to be found by the sextant. Then A H being the *sensible* horizontal line, the angular measure required is S A H. Draw A T H′ as a tangent to the earth's surface at T ; then, disregarding refraction, T will be the most distant point of the surface visible from A, and the altitude of S, as obtained by the sextant, would be the angle S A H′, instead of the angle S A H. The angle H A H′, by which the angle S A H is increased, is the *dip*, or depression of the visible horizon, which must be subtracted from the observed altitude to give the apparent altitude S A H.

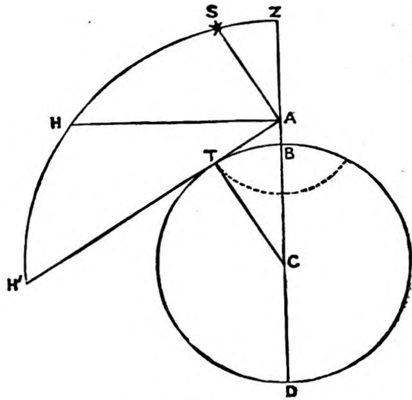


Fig. 1

The sensible horizon is strictly a tangent plane touching the earth's surface extending from B, but owing to the distance of the heavenly bodies the angle at S subtended by A B (the height of the eye) is immeasurably small.

To find the Dip.—Draw C T from the centre of the earth to the point of contact T ; then H A H′ and the angle at C are each the complement of C A T, and are therefore equal ; that is, the angle at C is equal to the angle of the *dip*.

Let h = the height of the eye = A B
r = the radius of the earth (in feet) = C T
d = the dip, or depression of the horizon.

Then, in the triangle C A T, we have A C T = H A H′ = d (the dip), and hence—

$$\text{Tan. } d = \frac{\text{A T}}{\text{C T}}$$

By Euclid III. 36, we have—

$$\text{A T} = \sqrt{\text{A B} \times \text{A D}} = \sqrt{h\,(2r + h)}$$

whence—

$$\text{Tan. } d \text{ (dip)} = \frac{\sqrt{2r\,h + h^2}}{r} = \sqrt{\frac{2h}{r} + \left(\frac{h}{r}\right)^2}$$

and since h is always very small compared with r, the square of the fraction $\frac{h}{r}$ is quite inappreciable; therefore, tan. d (dip) $= \sqrt{\frac{2h}{r}}$ very nearly, which is the basis of the " dip " tables.

The dip of the horizon given in the Dip Table for every probable height of the observer's eye, expressed in feet, is calculated with regard to the effect of *terrestrial refraction.*

The dip of the horizon, having regard to the atmospheric refraction.—The curved path of a ray of light from the point T (*see* Fig. 2) to the eye at A, is the same as that of a ray from A to T; and this is a portion of the whole path of a ray of light (as from a star S) which passes through the point A, and being bent from its right-lined course, is a tangent to the earth's surface at T; hence the direction in which the observer at A sees the point T will be A H′, the tangent at A to the curved path A T; the true dip is therefore H A H′, which is less than that shown in Fig. 1. It is also evident that the most distant visible point of the earth's surface is more distant from the observer than it would be if the earth had no atmosphere.

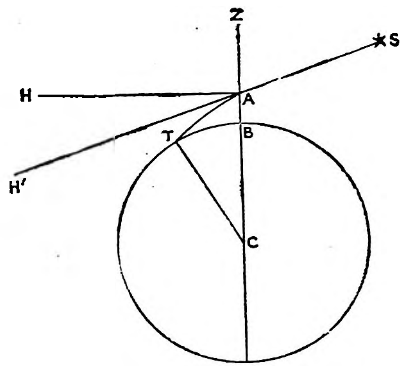


Fig. 2.

Excluding the effect of refraction—

$$\text{Dip in minutes} = 1'{\cdot}063\sqrt{h} \text{ (in feet) very nearly;}$$

but various scientists assume that the correction for refraction diminishes the amount of dip by $\frac{1}{10}$ to $\frac{1}{15}$ of itself; hence the co-efficient of $\sqrt{h}$ varies; generally, however—

$$\text{Dip (in minutes)} = 0'{\cdot}98\sqrt{h} \text{ (in feet);}$$

Chauvenet, and U.S. authorities, make—

$$\text{Dip} = 58''{\cdot}82\sqrt{h} \text{ (in feet),}$$

assuming the mean value of earth's radius = 20888625 feet.

Hence it may be taken that *the dip in minutes very nearly equals the square root of the height of the eye in feet.*

Dip at different Distances from the Observer.—What has been heretofore said about the dip of the horizon supposes it free from all incumbrances of land or other objects; but, as it often happens, when ships are sailing along a shore, or are at anchor in harbour, that an observation is desired when the sun is over the land, and the shore nearer to the ship than the visible sea-horizon would otherwise be, in this case the dip will be different

from that of the Dip Table, and greater the nearer the ship is to that part of the shore to which the sun's image is brought down. For this reason a Dip Table has been inserted, which gives the dip at different heights of the eye at different distances of the ship from the land.

Semi-diameter.—In order to obtain the altitude of the centre of a heavenly body which has a well-defined disc, the upper or lower limb must be observed, and the position of the centre deduced by the suitable application of the angular semi-diameter of the body.

The angular *semi-diameter* of a heavenly body is the angle subtended by the radius (or half the diameter) of the visible disc at the eye of the observer.

The semi-diameter, whether of the sun, moon, or any of the planets, varies with the distance from the earth.

The semi-diameter of the sun and moon is a considerable quantity—a quarter of a degree, or more—and cannot be estimated, or omitted, but must be taken from the Nautical Almanac. With regard to the planets, it is different: the semi-diameter of each planet is given in the Almanac, but it is so small a quantity that the general method of observing the altitude is to bring the planet's centre (approximately) down to the horizon. If the limb of Venus or Jupiter be observed, the semi-diameter must be used.

If the object is in the horizon of the observer, the distance from him is *nearly* the same as from the centre of the earth, and hence the *geocentric* is frequently called the *horizontal* semi-diameter, though this designation is not strictly exact.

If the object is in the zenith, its distance from the observer is less than its geocentric distance by a radius of the earth, and the apparent semi-diameter of the object has then its greatest value. Hence arises the correction called the *augmentation* of the semi-diameter.

For the sun's semi-diameter (which is always found in the Nautical Almanac on p. II. of the given month) no appreciable correction is required for that given in the Nautical Almanac, since the distance of the sun from the earth (93 millions of miles) admits of none.

For the moon's semi-diameter (found in the Nautical Almanac, p. III. of the given month) besides the interpolation for Greenwich time, there is an *augmentation* due to the altitude. The distance of the moon from the earth is only about 60 times the earth's semi-diameter; the moon's horizontal semi-diameter will therefore be increased one-sixtieth part when in the zenith, and hence the amount of augmentation for any altitude is found (approximately) by multiplying one-sixtieth of the moon's horizontal semi-diameter by the *sine* of the altitude. The Augmentation Table in Norie's Tables (D) gives this correction.

The *fixed stars* have no appreciable semi-diameter.

The semi-diameter is the correction to be applied, *after* that for the dip, to get the apparent altitude of the object's centre.

Correction for Refraction

General laws of Refraction.—The path of a ray of light is a straight line so long as the ray is passing through a medium of uniform density. But when a ray passes obliquely from one medium into another of different

density, it is bent or *refracted*. The ray before it enters the second medium is called the *incident* ray; after it enters the second medium it is called the *refracted* ray; and the difference between the directions of the incident and refracted rays is called the *refraction*.

Refraction.—The rays of light from a star in coming to the observer must pass through the atmosphere which surrounds the earth. The atmosphere, however, is not of uniform density, but is most dense near the surface of the earth, and gradually decreases in density to its upper limit, where it is supposed to be of such extreme tenuity that its first effect upon a ray of light may be considered as infinitesimal. The ray is, therefore, *continually* passing from a rarer into a denser medium, and hence its direction is continually changed, so that its path becomes a curve which is concave towards the earth.

The last direction of the ray, or that which it has when it reaches the eye, is that of a tangent to its curved path at this point; and the difference of the direction of the ray before entering the atmosphere and this last direction is called simply the *refraction*, or occasionally the refraction in altitude.

In Fig. 3, let A T represent the strata of the earth's atmosphere. S R, a ray of light, entering the atmosphere at R, is bent into the curve R O; the apparent direction in which the star is seen is O S′, which is the tangent to the curve at O; and the refraction is the difference of directions of the lines R S and O S′.

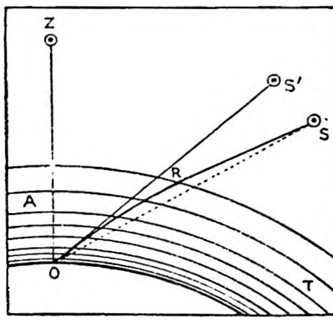


Fig. 3.

If O Z is the vertical line of the observer, by a law of optics the vertical plane of the observer which contains the tangent O S′ must also contain the whole curve R O and the incident ray S R; hence refraction increases the apparent altitude of a star without affecting its azimuth.

At the zenith the refraction is nothing; the less the altitude the more obliquely the rays enter the atmosphere and the greater will be the refraction. At the horizon the refraction will be greatest.

" Refraction is of all astronomical corrections the most difficult to determine with accuracy. The refracting power of the atmosphere varies with its density, and this is affected, in any particular stratum, not only by the superincumbent *pressure*, but also by its *temperature*, and its degree of *moisture*; and we are not definitely acquainted with the laws of their distribution."—Harbord.

The Mean Refraction Table is constructed on the supposition that at the level of the sea the barometer stands at 29·6 inches, and the thermometer at 50° Fahr., and there is an auxiliary Table (Correction of the Mean Refraction Table) for varying heights of the barometer and thermometer. All observed altitudes of the heavenly bodies must be decreased by the quantity taken from Table of Mean Refraction, the entry being by the apparent altitude.

There are various Tables of Refraction, as Ivory's and Bessel's, etc.; those by Bessel are now generally used by astronomers. All the tables have

a general agreement at mean altitudes, and differ most at low altitudes, so much so, indeed, that little dependence can be placed on altitudes less than 12° or 10°.

To find the Refraction (approximately) *by the Traverse Tables:* with the altitude as course, and departure 58, take out the difference of latitude, which is the refraction in seconds.

The *co-efficient of refraction* at the zenith distance of 45°, with the barometer at 30 inches and the temperature (Fahr.) 50°, may be taken as 58″·2. At moderate zenith distances *the amount of refraction is proportional to the tangent of the zenith distance*, hence the expression 58″·2 × tan. Z for the value of the refraction at the zenith distance (Z); which is, however, obviously incorrect near the horizon, where the tangent Z approaches to infinity, while the actual value of the refraction is only 33′ 46″.—R. S. Ball's "Astronomy."

Parallax.—The *parallax* of a heavenly body is, in general, the difference of the directions of the straight lines drawn to the object from two different points. In Nautical Astronomy *geocentric parallax* is the difference between the positions of a heavenly body as seen from the centre of the earth and from a point on its surface at the same instant.

The observed altitude when corrected for dip, semi-diameter, and refraction will be measured by the angle S′ O S, which is the altitude of the object's centre above the sensible horizon O S A; and S′ C H will be the altitude of the centre above the rational horizon C H.

" In this case the directions of the object from C and from O are compared with each other by referring them to two lines which have a common direction, *i.e.* parallel lines. But a still more direct method of comparison is obtained by referring them to one and the same straight line, as Z O C, in which Z is the zenith. We then call Z C S′ the true, and Z O S′ the apparent zenith distance; and these are evidently the complements of the true and apparent altitudes respectively.

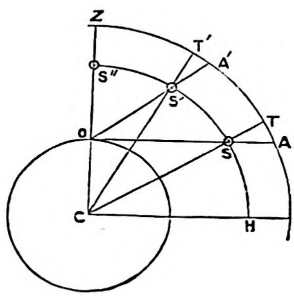


Fig. 4

In Fig. 4 we have at once

Z O S′ — Z C S′ = O S′ C, that is—the parallax in zenith distance or altitude is the angle at the celestial object subtended by the radius of the earth."—Chauvenet.

Or to be more explicit: Let C be the centre of the earth, and O the place of an observer on its surface; C H the rational horizon, and O S A the sensible horizon; Z the zenith of the observer; also H S S′ S″ a vertical circle whose radius is the distance of a heavenly body from the earth's centre.

If the object S is in the sensible horizon O S A, its *apparent* place would be as if seen at A, but its *true* place (as seen from C, the centre of the earth) would be in the direction C T; here the earth's radius O C being at right angles to O S subtends the angle O S C, which is the greatest possible angle at the object for the same distance, and this angle is called the *horizontal parallax*. When the object has moved to S′ its *apparent* place as viewed from the earth's surface is in the direction O A′, while its *true* place as viewed

from the centre is in the direction C T′, and the angle O S′ C is the *parallax in altitude*, which decreases as the altitude increases, until reaching S″ the object appears in the zenith Z, where there is no angle subtended by the earth's radius O C, and consequently no parallax.

The parallax in altitude *for the sphere* is computed from the horizontal parallax as follows—

In the triangle S′ O C, we have—

$$\frac{\text{O C}}{\text{S}' \text{C}} = \frac{\sin.\ \text{O S}' \text{C}}{\sin.\ \text{S}' \text{O C}}$$

Now $\quad$ S C = S′ C; and S′ O C = 180° — S′ O S″ (or S′ O Z)

Therefore $\dfrac{\text{O C}}{\text{S C}} = \dfrac{\sin.\ \text{O S}' \text{C}}{\sin.\ \text{S}' \text{O S}''}$; and sin. O S′ C $= \dfrac{\text{O C}}{\text{S C}}$ cos. S′ O S

But $\quad \dfrac{\text{O C}}{\text{S C}} =$ sin. O S C (the Hor. Par.) ; and cos. S′ O S = cos. Alt.

And since the parallax in altitude and horizontal parallax are always small the sines are nearly proportional to the angles, thus we have—

Parallax in altitude (in ″) = horizontal parallax (in ″) × cosine altitude (corrected for refraction) ; a quantity, which, from the fig. is *additive* to the apparent altitude.

The parallax in altitude is deduced from the *equatorial horizontal parallax* which is given in the Nautical Almanac for the sun, moon, and planets respectively ; and it is evident from the fig. that the latter must vary with the distance ; thus, in the triangle C O S, if C O be the equatorial radius—

$$\text{Sin. Equatorial Hor. Par.} = \frac{\text{Equatorial radius of earth}}{\text{Dist. of object from earth's centre}}$$

hence the fixed stars, millions of millions of miles distant, have no horizontal parallax ; the sun, 93 millions of miles distant, has the horizontal parallax 8″·8 ; and the planets have also a small horizontal parallax ; while the moon, distant only 60 semi-diameters of the earth, has a mean horizontal parallax of 57′ 3″·7, which in an observation for ship's position has a very sensible effect, and gives at times a large parallax in altitude.

But this horizontal parallax of the moon also requires a correction due to the earth as a *spheroid*—a reduction which varies as the *radius vector* of the spheroid ; this reduction is given in the Table of Reduction of the Moon's Horizontal Equatorial Parallax.

The "correction of the moon's apparent altitude" in the Moon's Correction Table is the difference between the parallax in altitude and the refraction.

The sun's parallax in altitude is given in Sun's Parallax in Alt. Table, or the *correction* (difference between the parallax in altitude and refraction) in Table Sun's Correction ; the parallax in altitude for planets will be found in Table of Parallax in Altitude for Planets.

Summary of the Corrections of Observed Altitudes

The corrections of an altitude observed by the sea-horizon are to be taken in the following order : The index error of the instrument (if any), the

dip (for height of eye), the semi-diameter (if any), the refraction, and parallax (if any).

Then, these corrections being applied, if the zenith distance is required, proceed as follows—

For the Zenith Distance

For the Apparent Zenith Distance: Subtract the apparent (central) altitude from 90°.

For the True Zenith Distance: Subtract the true (central) altitude from 90°.

Then, if the observed object bears N., the zenith distance will be S.; if it bears S. the zenith distance will be N.

Or, otherwise, if the observer is N. of the object the zenith distance will be N.; if observer is S. of the object the zenith distance will be S.

Corrections for the Sun: (1) Apply to the observed altitude the ***index error***, additive (+) or subtractive (—), as the case may be.

(2) *Dip* (for height of eye), subtractive (—), Dip Table; gives the apparent altitude of the limb observed.

(3) *Semi-diameter* (from Nautical Almanac, p. II. of month); additive (+) when lower limb is observed; subtractive (—) if upper limb is observed. The application of these corrections gives the apparent altitude of the centre.

(4) *Refraction*, subtractive (—), Mean Refraction Table; enter with the apparent altitude of limb observed.

(5) *Parallax*, additive (+), Table of Sun's Parallax in Altitude.

Note.—These corrections when applied give the ***true altitude of the centre***.

N.B.—Refraction and parallax are combined in Table of Sun's Correction of Apparent Altitude.

All the corrections, excepting index error, are combined in Table of Sun's Total Correction of Observed Altitude, which is sufficiently accurate for sea purposes.

For the Zenith Distance *see* Rule above.

Example.—January 10th: the observed altitude of the sun's lower limb was 37° 24′ 30″ bearing north; index error of the sextant 1′ 42″ to subtract; height of the eye 19 feet. Required the true altitude, and thence the zenith distance—

Obs. alt. sun's L.L.	37°	24′	30″	N.
Index error	—	1	42	
	37	22	48	
Dip 19 ft.	—	4	16	
App. alt. sun's L.L.	37	18	32	
Sun's semi-d. (N.A. p. II.)	+	16	18	
App. alt. sun's centre	37	34	50	
Refraction	—	1	15	
	37	33	35	
Par. in alt.		+	7	
True alt. sun's centre	37	33	42	N.
	90			
Sun's true zen. dist.	52	26	18	S

Whole Cor. from Table of Sun's Total Correction

Sun's obs. alt.	37°	24′·5
Ind. err.	—	1·7
	37	22·8
Sun's Cor.	+	10·9
True alt. sun's centre	37	33·7
	90	
Sun's true zen. dist.	52	26·3

Corrections for the Moon : (1) To observed altitude apply *index error*, + or — as the case may be.

(2) *Dip*, subtractive (—), Dip Table ; gives the apparent altitude of limb observed.

(3) *Semi-diameter* (from Nautical Almanac, p. III. of month) to be corrected for Greenwich time, and *augmented* from the Moon's Augmentation Table ; apply *augmented semidiameter*, additive (+) if lower limb is observed, subtractive (—) if upper limb is observed.

(4) *Refraction*, subtractive (—), Mean Refraction Table ; enter with apparent altitude of limb observed.

(5) *Parallax*, additive (+) ; to be computed as follows—

Correct the horizontal parallax (from Nautical Almanac, p. III. of month) for Greenwich time, and *reduce* it by Table E : then, to the Proportional Logarithm of the corrected horizontal parallax add the L *secant* of the apparent (central) altitude, corrected for refraction ; the sum will be the proportional logarithm of the *parallax* in altitude.

Or the parallax in altitude may also be found as indicated on p. 254.

Convert the horizontal parallax into seconds, then to the logarithm of the horizontal parallax (in seconds) add the L *cosine* of the apparent (central) altitude, corrected for refraction ; the sum will be the logarithm of the parallax in altitude in seconds, which take out and reduce.

These corrections, when applied, give the *true altitude of the centre.*

N.B.—Refraction and parallax in altitude are combined in Moon's Correction Table, sufficiently accurate for sea purposes.

For the Zenith Distance *see* Rule, p. 255.

Example.—August 28th, about 2h. at a.m. at ship, in lat. 46° 4′ S., long. 165° E. ; mean time at Greenwich by chronometer (corrected) August 27d. 3h. 1m. 54s., the observed altitude of the moon's upper limb was 32° 45′ 40″ bearing north ; error of sextant 2′ 7″ to add ; height of eye 21 feet.

Required the true altitude, and thence the zenith distance—

Moon's Semi-diameter and Horizontal Parallax corrected for Greenwich time.

☾'s Semi-d., noon	16′ 34″·0	☾'s H.P. noon	60′ 41″·9
,, midnight	16 38 ·0	,, midnight	60 56·3
Diff.	4	Diff.	14·4
	3h.		3h.
	12) 12		12)43·2
Cor. for Gr. date	+ 1	Cor. for Gr. date	+ 3·6
	16 34		60 41·9
	16 35		60 45·5
Augment. (Tab. D)	+ 9·6	Cor. for lat.(Tab.E)	— 6·3
☾'s Correct semi-d.	16 44·6	☾'s Correct H.P.	60 39·2
		H.P.	3639″·2 in secs.

Altitude and Zen. Dist.

Obs. alt. ☾'s U.L.	32°	45′	40″	N.
Index error	+	2	7	
	32	47	47	
Dip 21 ft.	—	4	29	
App. alt. ☾'s U.L.	32	43	18	
☾'s Semi-d.	—	16	45	
App. alt. ☾'s centre	32	26	33	
Ref.	—	1	29	
	32	25	4 .... Secant 0·0736	
Par. in alt	+	51	12	
True alt. ☾'s centre	33	16	16	N.
	90			
☾'s True zen. dist.	56	43	44	S.

Correction by Moon's Cor. Tables

App. alt. (centre)	32°	26′	33″
Cor. (par.—ref.)	+	49	42
True alt. (centre)	33	16	15

H.P. 60′ 39″	Prop. log.	·4724
	Secant	0·0736
Par. in alt.	Prop. log.	0·5460

Or, otherwise

H.P. 3639″·2	Log.	3·561006
Alt. 32° 25′ 4″	Cos.	9·926426
6,0)307,2″·1	Log.	3·487432

Par. in alt. 51′12″·1 as above.

Corrections for a Planet.—You may follow the instructions as given for the Moon, but ordinarily it will be sufficient to use the usual Nautical Tables, and the quantities as given in the Nautical Almanac under the head of the given planet.

(1) To the observed altitude apply the index error, + or —, as the case may be.

(2) Dip, subtractive (—).

(3) It is usual to estimate the centre in bringing it to the sea-horizon, so that semi-diameter is not required.

(4) *Refraction,* subtractive (—).

(5) Parallax, additive (+), to be found as follows: Enter the Nautical Almanac with the heading the name of the planet and transit at Greenwich. Opposite the day of the month in the last column take out the horizontal parallax. Enter Table of Parallax in Altitude for Planets with the horizontal parallax at the top and altitude at the side; the quantity found is the parallax in altitude. These corrections give the ***true altitude of the centre.***

For the Zenith Distance *see* Rule, p. 255.

Example.—December 7th: the observed altitude of the centre of the planet Venus was 53° 14′ 20″; observer N. of the planet; index error + 3′ 46″; height of eye 22 feet. Required the apparent and true altitudes, and the apparent and true zenith distances—

By Nautical Almanac the Horizontal parallax is 33".

Obs. alt. Venus	53° 14′ 20″ S.		
Index error	+ 3 46		
	53 18 6		
Dip 22 ft.	— 4 36		
App. alt.	53 13 30	....	53° 13′ 30″
Ref.	— 43		90
	53 12 47	App. zen. dist.	36 46 30
Par. in alt.	+ 20		
True alt. (centre)	53 13 7 S.		
	90		
True zen. dist.	36 46 53		

Corrections for a Fixed Star: (1) Apply to the observed altitude the *index error* + or — as the case may be.

(2) *Dip*, subtractive (—).

The application of these corrections gives the *apparent* altitude of the star. A star has no semi-diameter.

(3) *Refraction*, subtractive (—), Mean Refraction Table; this correction, applied to the apparent altitude, gives the *true altitude*, since a fixed star has no parallax.

N.B.—The dip and refraction are combined in Star's Correction Table; accurate enough for sea purposes.

For the Zenith Distance *see* Rule, p. 255.

Example.—February 10th: the observed altitude of α Canis Minoris (*Procyon*) was found to be 18° 46′ 30″ bearing south; index error 3′ 17″ to subtract; height of eye 18 feet. Find the apparent and true altitudes of the star, and the apparent and true zenith distances—

*'s Obs. alt.	18° 46′ 30″ S.
Ind. err.	— 3 17
	18 43 13
Dip 18 ft.	— 4 9
*'s App. alt.	18 39 4
Refraction	— 2 47
*'s True alt.	18 36 17 S.
	90
*'s True zen. dist.	71 23 43 N.

Whole Correction by Star's Total Correction Table

*'s Obs. alt.	18° 46′·5
Ind. err.	— 3·3
	18 43·2
Star's Cor.	— 6·9
*'s True alt.	18 36·3
	90
*'s True zen. dist.	71 23·7
*'s App. alt.	18°39′4″
	90
*'s App. zen. dist.	71 20 56

Corrections of Altitude observed by an Artificial Horizon on Shore

The altitude having been observed by an artificial horizon, each angle, of whichever part of the object observed, will be double what it would have been if taken by the sea-horizon; and as it is customary to take several altitudes (3, 5, or 7) in succession, the sum of the altitudes must (in

the first place) be divided by the number observed ; this will be the *mean* of the observations, to which the next operation is to apply the *index error*, + or —, as the case may be ; and the result will be the *correct observed angle.*

But this angle being double what it would have been if the altitude had been taken direct, the *correct mean observed angle* must be divided by 2, and the result will be the *apparent altitude* of the part of the object observed, since there is *no correction* required *for* the height of the eye, or *dip*, the effect of the elevation of the observer being insensible.

The subsequent corrections then fall in the natural order, already given.

(1) To the apparent altitude apply the semi-diameter if any, + or — as the case may be.
(2) Refraction (—).
(3) Parallax (+) if any.

And the result will be the *true central altitude.*

Example.—November 7th, the altitudes of the sun's lower limb observed by artificial horizon were—

52° 0′ 45″
51 41 30
51 20 45

index error of sextant 2′ 50″ to subtract. Find the true altitude of the sun's centre.

Obs. altitudes	52°	0′	45″
" "	51	41	30
" "	51	20	45
	3)155	3	0
	51	41	0
Ind. err.	—	2	50
	2)51	38	10
App. alt. L.L.	25	49	5
Semi-diam. (N.A.)	+	16	11
App. alt. centre	26	5	16
Ref.	—	1	57
	26	3	19
Par.		+	8
True alt. (centre)	26	3	27

N.B.—If several altitudes are taken at about equal intervals, the *difference* between the altitudes will indicate if there be any serious errors in the observations.

If the sun is the object observed, and the altitude is *increasing* (or *rising*) ; when taking the *lower* limb, the images are continually *separating ;* but when taking the *upper* limb, the images are continually *overlapping.* The contrary effect is observed with the sun's *decreasing* (or *falling*) altitude. This leaves no doubt as to the limb observed.

For a Fixed Star : Divide by the number of observations ; to the result apply the *index error ;* then divide by 2 for the *apparent altitude ;* from the apparent altitude subtract the *refraction*, and the result will be the *true altitude.*

Example.—January 20th : the following altitudes of *Procyon* (α Canis Minoris) were observed by artificial horizon—

47° 0′ 15″
47 17 45
47 35 30

index error of sextant 3′ 10″ to subtract. Find the apparent and true altitudes.

Obs. altitudes	47°	0′	15″
" "	47	17	45
" "	47	35	30
	3)141	53	30
	47	17	50
Ind. err.	—	3	10
	2)47	14	40
*'s App. alt.	23	37	20
Ref.	—	2	10
*'s True alt.	23	35	10

Reduction of Altitude for Change of Ship's Position.—When the altitude of an object has been taken, and at a subsequent time and in an altered position of the ship a second altitude of the same object is observed as in the double altitude problem and in the "New Navigation," if we wish to combine these two observations it frequently happens that the first altitude has to be reduced to what it would have been if it had been taken at the altered position of the ship; in this case the azimuth or *bearing* of the object at the first altitude must be taken, and the course made good and distance run in the interval of the observations carefully noted.

(1) If the ship's course is *directly towards* a heavenly body, the effect is to *raise* the body 1′ for each mile of distance made good.

(2) If the ship's course is *directly away from* a heavenly body, the effect is to *depress* the body 1′ for each mile of distance made good.

(3) If the ship's course is *at right angles* to the bearing of the object, the ship neither approaches, nor recedes from, the body.

(4) But the object may bear *obliquely* from the course the ship makes good; and thus we have an angle between the bearing and the course, which, with the distance run, gives us a plane triangle corresponding to—

$$\text{diff. lat.} = \text{dist.} \times \text{cos. course}$$

that is—

$$\left.\begin{array}{l}\textit{reduction of altitude} \text{ or } \textit{correction}\\ \textit{for run} \text{ (as it is sometimes called)}\end{array}\right\} = \left\{\begin{array}{l}\text{distance} \times \text{cosine of angle between}\\ \text{bearing of object and ship's course}\\ \text{in the interval}\end{array}\right.$$

to obtain which the Traverse Tables are generally used, as follows—

(A) Find the angle between the bearing of the sun (or star) at the first observation and the ship's course made good: enter the Traverse Table with this angle as a course, and the distance run as a distance; the difference of latitude corresponding thereto is the *reduction* or *correction* of altitude for the run, to be *added* when the angle is *less* than 8 points or 90°, but to be *subtracted* when the angle is *more* than 8 points or 90°.

For example, taking the sun for illustration—

The sun's bearing at the first observation being S. 53½° E., and the ship's course in the interval of the observations being N. 64½° E., the angle is 62°; and supposing the run to be 19 miles—

$\left.\begin{array}{l}\text{Course } 62°\\ \text{Dist. 19m.}\end{array}\right\}$ Trav. Tab. Diff. lat. 9′ = cor. of alt., to be *added*.

The sun's bearing at the first observation being S. 45° E., and the ship's course in the interval of the observations being N. 28° E., the angle is 107°, in which case, deducting it from 180°, there remain 73°, and supposing the run to be 24 miles—

$\left.\begin{array}{l}\text{Course } 73°\\ \text{Dist. 24m.}\end{array}\right\}$ Trav. Tab. Diff. lat. 7′ = cor. of alt., to be *subtracted*.

N.B.—When the angle exceeds 8 points or 90°, enter the Traverse Table with its supplement or what it wants of 16 points or 180°.

(B) If the ship's course during the interval is *directly towards* the sun's bearing at the first observation, the distance run is the correction, to be *added* to the first altitude; if *directly from* the sun, the distance run is the correction, to be *subtracted* from the first altitude.

(C) If the course is exactly 8 points or 90° from the sun's bearing, the correction is 0.

If, in the interval of the two observations, the ship makes but one course, the difference between the compass bearing of the sun (or star) and the compass course will be the angle; when the ship makes more than one course in the interval, the compass course made good may be also used, provided there be no *deviation* on the courses steered, in which latter case it would be better to find the angle between the *true* course made good and the *true* bearing of the sun (or star).

(5) If it is necessary to reduce the *second altitude to the position at which the first altitude was observed*, you require the ship's run (*i.e.*, course and distance in the interval, as before) and the sun's (or star's) *bearing* at the *second* observation; the reduction or correction of altitude is obtained as already indicated, but it must be *applied to the second altitude the reverse way*; thus, where paragraph (A) says add, you must subtract; and where it says subtract, you must add.

When the course at the *second* observation is *directly towards* the sun (or star), the distance run is the correction, to be *subtracted* from the second altitude; but if *directly from* the sun (or star), the distance run is the correction, to be *added*.

In all cases the correction for run is to be applied to the *true* altitude.

It will have been observed that the rule just given has been determined on the principle of a right-angled plane triangle. This is not strictly correct, but as the error will not exceed 1′ in a run of 75 miles and with an altitude less than 50°, the navigator can consider it sufficiently near for ordinary navigation.

CONVERSION OF TIMES

I.—Intervals of Time, Mean, Apparent, or Sidereal

(1) *To convert an Interval of Mean Time into an Interval of Apparent Time:* From Nautical Almanac, p. I. of the month, take out the "Var. in 1h." of the equation of time, and multiply it by the hours and decimal of an hour, of the given *mean* interval; the result, which is the *correction,* is to be applied to the mean interval as follows—

Look to p. II. of Nautical Almanac to see whether the equation of time is to be added or subtracted, and again whether the equation is increasing or decreasing; then—

If	equat. be	*additive* to	mean time and	*increasing,*	*add* correction.
or,	,,	*subtractive* from	,, ,,	*decreasing,*	
But,	,,	*additive* to	,, ,,	*decreasing,*	*sub.* correction.
or,	,,	*subtractive* from	,, ,,	*increasing,*	

Example.—January 10th; convert 5h. 6m. 2s. of mean time into apparent time.

	s.		H.	M.	S.
Var. in 1h.	1·003	Mean interval	5	6	2
Mult. by int.	5·1	Cor.		—	5·1
Correction	5·1153	App. interval	5	5	56·9

Note.—To convert an apparent into a mean interval the application of the correction would be the reverse of the above; or use p. I. instead of p. II.

(2) *To convert an Interval of Mean Time into an Interval of Sidereal Time:* To the mean interval *add* the *acceleration* corresponding to the given hours, minutes, and seconds (*see* Nautical Almanac Table, "Intervals of Mean Solar Time into intervals of Sidereal Time," etc. or Accelaration Table in Norie's Tables.

Example.—Convert an interval of 6h. 12m. 42s. mean time into an equivalent interval of sidereal time.

	H.	M.	S.
Mean interval	6	12	42
6h.			59·14
12m.			1·97
42s.			0·11
Sidereal interval	6	13	43·22

(3) *To convert an Interval of Apparent Time into an Interval of Sidereal Time:* From the Nautical Almanac, p. I. of the month, and day, take out the "Var. in 1h." of the sun's right ascension, and multiply it by the interval (hours and decimal of an hour); the result is the correction to be added to the apparent interval.

Example.—December 11th; the apparent interval being 7h. 42m. 3s. Required the sidereal interval.

				H.	M.	S.
	S.					
R.A. var. in 1h.	11·02		App. interval.	7	42	3
Mult. by int.	7·7		Cor.		1	24·9
6,0)	84·854		Sidereal int.	7	43	27·9
Correction	1 24·9					

(4) *To convert an Interval of Sidereal Time into an Interval of Mean Time:* Subtract from the sidereal interval the ***retardation*** given in Retardation Table in Norie's Tables. Or convert the interval by the Nautical Almanac, "Table for converting Intervals of Sidereal Time into equivalent Intervals of Mean Solar Time."

Example.—Convert 8h. 18m. 14s. of sidereal time into mean time.

By Nautical Almanac

		H.	M.	S.
8h.	....	7	58	41·36
18m.	....		17	57·05
14s.	....			13·96
	Mean int.	8	16	52·37

The foregoing Rules (1, 2, 3, 4) relate exclusively to ***intervals*** of time, and the following Rules appertain to problems connected with ***absolute time.***

II.—To convert Apparent Time into Mean Time

This conversion is commonly required in the problem of finding *time* at ship by the sun, and thence the longitude through the Greenwich date.

Note.—For this conversion the Greenwich date must either be given, or otherwise found from the ship's time and longitude.

Rule.—The Greenwich date mean time being given, correct the equation of time from p. II. of Nautical Almanac by the "Var. in 1h." and the given Greenwich time (*see* p 237); then apply the *corrected* equation of time to the apparent time at ship according to the precept standing above the equation of time on p. I. of Nautical Almanac.

Example.—Given apparent time at ship 3h. 4m. 23s. p.m., and the Greenwich date January 5d. 9h. 42m. 2s. mean time. Required the mean time at ship.

				H.	M.	S.
		S.				
Eq. of T. var. in 1h. =		1·11	App. T. at ship	3	4	23
Gr. time		9·7	Correct eq. of T. +		5	54·45
Correction +		10·767	Mean T. at ship	3	10	17·45 p.m.
Eq. of T. (N.A. p. II.)	5	43·68				
Correct Eq. +	5	54·45 to be added to App. T.				

Note.—If required, the Green. Date M.T. could, in this case, be converted into Green. App. T. by applying the Eq. of T. to it, according to precept in Naut. Alm. p. II: thus—

	D.	H.	M.	S.
M.T.G. Jan.	5	9	42	2
Eq. of T.		—	5	54·45
App. T. at Green.	5	9	36	7·55

Example.—Given apparent time at ship 20h. 30m. 42s. (*i.e.* 8h. 30m. 42s. a.m.), and Greenwich date November 23d. 6h. 12m. 4s. mean time. Required the mean time at ship.

	M.	S.	
Equat. of T.	13	25·3	
·705 × 6·2 =	—	4·3	
Cor. eq. —	13	21	(sub. from App. T.)

	H.	M.	S.
App. T. at ship	20	30	42
Cor. eq. of T.	—	13	21
Mean T. at ship	20	17	21

In this case the ship time expressed as *civil time* would be apparent time 8h. 30m. 42s. a.m., and mean time 8h. 17m. 21s. a.m.

III.—To convert Mean Time into Apparent Time

This conversion is required in several problems, as in the "Reduction to the Meridian," and for "Time Azimuths of the Sun," etc.

The Greenwich date must either be given or otherwise found from the ship's time and longitude.

Rule.—Correct the equation of time for the Greenwich date, and apply the corrected equation of time to the given mean time, according to the precept standing above the equation of time on p. II. of Nautical Almanac.

Example.—Given mean time at ship 6h. 4m. 42s. p.m., and the Greenwich date January 31d. 8h. 48m. 10s. mean time. Required the apparent time at ship and at Greenwich.

		M.	S.
Equat. of T. (N.A. p. II.)		13	43·54
·354 × 8·8 = Cor.	+		3·11
Corrected Eq. (sub. from M.T.)	—	13	46·65

	H.	M.	S.
Ship M.T.	6	4	42
Eq. of T.	—	13	46·6
App. T. at ship	5	50	55·4

	H.	M.	S.
Gr. M.T. Jan. 31d.	8	48	10
Eq. of T.	—	13	46·6
App. T. at Green.	8	34	23·4

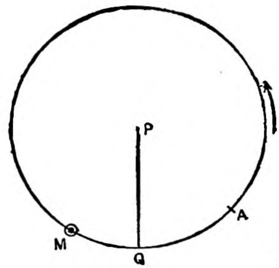
IV.—To convert Greenwich Mean Time into Sidereal Time at Greenwich

Rule.—Accelerate the sidereal time (Nautical Almanac, p. II.) for the Greenwich mean time (*see* p. 247); the result is the mean sun's right ascension. To the Greenwich mean time add the mean sun's right ascension, the sum (rejecting 24h if necessary) will be the sidereal time at Greenwich.

Formula— G. Sid. T. = G. M. T. + M. ☉'s R. A.

Example.—January 4d. 2h. 4m. 50s. mean time at Greenwich. Required the sidereal time at Greenwich.

	H.	M.	S.
Sid. T. (N.A. p. II.)	18	56	10·90
Accel. for 2h.			19·71
„ 4m.			·66
„ 50s.			·14
Mean sun's R.A.	18	56	31·41



	H.	M.	S.
Green. M.T.	2	4	50
Mean sun's R.A.	18	56	31·4
Sid. T. at Green.	21	1	21·4

The circle is the Equinoctial, P the Pole, P Q the Meridian of Greenwich, A the first point of Aries, and M the Mean Sun. A M Q is the G. Sid. T., Q M the G. M. T., and A M the M. S. R. A. The right ascension is always reckoned in the direction of the arrow from A.

V.—To convert Mean Time at a given Place into Sidereal Time at that Place

The Greenwich mean time must be known, or found through the mean time at place and the longitude.

Rule.—Find the mean sun's right ascension as in the previous case, or by p. 247. To the mean time at place add the mean sun's right ascension, and the sum will be the sidereal time at place, or right ascension of the meridian.

Formula— Sid. T. obs. or R. A. Mer. = M. T. S. + M. ☉'s R. A.

Example.—January 18th, at 8h. 40m. 24s. p.m. mean time at place of observation, the Greenwich date being January 18d. 10h. 50m. 38s. Find the sidereal time at place of observation.

	H.	M.	S.
Sid. T. (N.A. p. II.)	19	51	22·71
Accel. for 10h.		1	38·56
„ 50m.			8·21
„ 38s.			·10
Mean sun's R.A.	19	53	9·58

	H.	M.	S.
M.T. at place	8	40	24
Mean sun's R.A.	19	53	9·58
Sid. T. at place	4	33	33·58

Note.—The diagram is similar to that in IV., P Q being the meridian of the place.

VI.—To convert Apparent Time at a given Place into Sidereal Time at that Place

Rule.—Find the Greenwich date (apparent time); from the Nautical Almanac, p. I., take out the sun's right ascension for the Greenwich date, and correct it by the " Var. in 1 hour " for the Greenwich Apparent time.

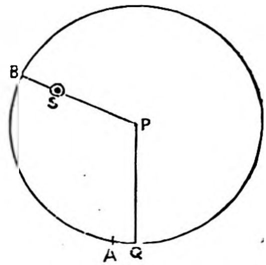
Then, to the apparent time at place add the sun's right ascension (corrected); the result will be the right ascension of the meridian, or sidereal time of observation at place.

Formula— R. A. Mer. or Sid. T. obs. = A. T. S. + App. ☉'s R. A.

Example.—December 3d., at 7h. 53m. 46s. p.m. apparent time at place, in long. 30° W. Find the right ascension of the meridian, or sidereal time at place.

	D.	H.	M.	S.
Dec.	3	7	53	46
W. long. 30°	=	2	0	0
Green. date, A.T.	3	9	53	46

			S.
Var. in 1h.	=		10·856
			9·9
	6,0)10,7·4744		
	+	1	47·4
Sun's R.A. (N.A. p. I.)	16	39	2·2
Sun's R.A. (corrected)	16	40	49·6



	H.	M.	S.
App. T. at place	7	53	46
Sun's R.A. (corrected)	16	40	49·6
R.A.M. or Sid. T. at place	0	34	35·6

The circle is the Equinoctial, P Q the Meridian of the place, A the first point of Aries, and S the sun. Q A is the R.A. mer., Q B the A.T.S, and A Q B the sun's R.A.

If preferred, the apparent time at place might be converted into mean time at place by II., and then into sidereal time at place by V.

VII.—To convert Sidereal Time at Place (or the Right Ascension of the Meridian) into Mean Time at Place

You will require a Greenwich date by which to get the mean sun's right ascension, for which *see* Case IV. p. 264, or p. 247. Then,

Rule.—From the sidereal time at place (*i.e.* the right ascension of the meridian), subtract the mean sun's right ascension, borrowing 24h. if required, before which write the date at place.

N.B.—This case is the reverse of the preceding case V.

Formula— M. T. S. = Sid. T. obs. — M. ☉'s R. A.

Example I.

	H.	M.	S.
Suppose, Sid. T. at place	14	10	40
Mean sun's R.A.	4	50	48
M.T. at place	9	19	52

Example II.

	H.	M.	S.
....	3	36	11
....	10	54	37
M.T. at place	16	41	34

With regard to *all problems* in which the right ascension of any of the heavenly bodies is introduced, attend to the following remark :

Caution.—Remember that whenever we speak of one right ascension as being subtracted from another, for the purpose of obtaining a third right

ascension, it is always tacitly supposed that 24 hours is added (if necessary) to the one from which the subtraction is to be made; and whenever one right ascension is to be added to another, to get a third right ascension, 24 hours is always suppressed from the sum if it exceeds that quantity. There is no displacement of a celestial object by increasing its right ascension by 24 hours, or by 360°, if the right ascension be expressed in angular measure. Also—

Hour Angle of any Heavenly Body.—Remember that whenever ***time***, or an object's ***hour angle***, is considered ***astronomically***, it is reckoned from oh. om. os. when on the meridian, thence ***westerly*** through 24 hours, to the meridian again. Hence when an object is ***east of the meridian*** we can convert the ***easterly*** into a ***westerly*** hour angle, thus—

Westerly H. A. = 24h. — Easterly H. A.

i.e., by subtracting the easterly hour angle from 24 hours, we get the westerly hour angle.

VIII.—Meridian Passage of the Sun

The *apparent time* of the sun's meridian passage is oh. om. os., except below the pole, when it is 12h. om. os.

For the *mean time* of the meridian passage :—Take the equation of time from Nautical Almanac, p. I., and reduce it for the longitude as a Greenwich date (*see* p. 239); then apply equation of time according to precept on Nautical Almanac, p. I.

Example.—March 2nd; long. 135° W. Find the mean time of sun's meridian passage.

	H.	M.	S.
(N.A.) Eq. of T.	0	12	18·6
·522 × 9h. =		—	4·7
Cor. eq. of T. +	0	12	13·9
App. noon.	0	0	0
M.T. of mer. pass.	0	12	13·9 p.m.
Or, March 2d.	0	12	13·9 M.T.

Example.—Nov. 30th; long. 30° E. Find the mean time of sun's meridian passage.

	H.	M.	S.
(N.A.) of Eq. of T.	0	11	8·9
·916 × 2h. =	0	+	1·8
Cor. eq. of T. —	0	11	10·7
App. noon	0	0	0
M.T. of mer. pass	11	48	49·3 a.m.
Or, Nov. 29d.	23	48	49·3 M.T.

IX.—Meridian Passage of a Fixed Star

The approximate *apparent time* of a star's meridian passage is given in Table of Star's Mer. Passage, and for the correction *see* "Explanation."

For the *mean time* of a star's meridian passage.

Rule.—Enter Table of Star's Mer. Pass. and note whether the time there given is more or less than 12 hours. From the star's right ascension taken from the "Apparent Places of the Stars" in the Nautical Almanac (increased by 24 hours if necessary) subtract the sidereal time (Nautical Almanac, p. II.) at mean noon of the astronomical day (this is the same as the civil day if the time in Table of Star's Mer. Pass. is less than 12h. or p.m., but is one less than the civil day if the time in the Table is more than 12h. or a.m.); the remainder is the approximate *mean time* of transit, civil time. From the

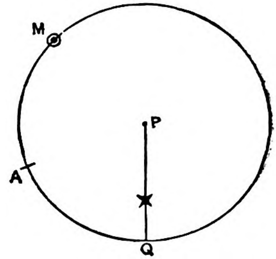
Retardation Table take out the *retardation* corresponding to this remainder and subtract it from the approximate mean time of transit. Also take from the same Table the *retardation* corresponding to the longitude in time, and *add* for E. longitude, but *subtract* for W. longitude. The result is the *mean time* of the star's meridian passage.

Formula.— M. T. S. = *'s R.A. — M. ⊙'s R. A.

Example.—January 11th; long. 45° W. Find the mean time of the meridian passage of Aldebaran.

The time in Table of Star's Mer. Pass. is less than 12h. Astronomical date same as Civil date.

	H.	M.	S.
Aldebaran R. A.	4	29	36·5
Sid. T. mean noon 11th	19	23	46·8
	9	5	49·7
Ret. 9h. = 1 M. 28·5 S.; 5m. ·8; 50s. ·1	—	1	29·4
	9	4	20·3
Ret. 45° W. or 3h.		—	29·5
M.T. mer. pass. 11th	9	3	50·8 p.m.

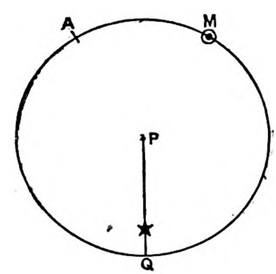


The circle is the equinoctial, * the star on the meridian P Q, A the first point of Aries, and M the mean sun. Q M is the M. T. S., A Q the Star's R. A., and also the R. A. Mer., and A Q M the M. ⊙'s R. A.

Example.—January 21st, civil time; long. 120° E. Find the mean time of the meridian passage of Regulus.

The time in Table of Star's Mer. Pass. is more than 12h. Astronomical date goes back one day.

	H.	M.	S.
Regulus R.A.	10	2	31·1
Sid. T. mean noon 20th	19	59	15·8
	14	3	15·3
Ret. 14h. = 2 M. 17·5 S.; 3m.	—	2	18·1
	14	0	57·2
Ret. 120° E. or 8h.	+	1	18·6
M.T. mer. pass. 20th	14	2	15·8
Civil time 21st	2	2	15·8 a.m.



The explanation of the diagram is the same as in the previous example.

For the *apparent time* of a star's meridian passage.

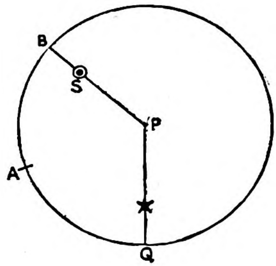
RULE.—Proceed as in finding the mean time of meridian passage, using apparent sun's right ascension (Nautical Almanac, p. I.) instead of sidereal

time, and the two corrections will be found by using " Var. in 1 hour" of sun's right ascension.

Formula.— A. T. S. = *'s R. A. — App. ☉'s R. A.

Example.—January 11th; long. 45° W. Find the apparent time of meridian passage of Aldebaran.

	H.	M.	S.
Aldebaran R.A.	4	29	36·5
Sun's R.A. apparent noon 11th	19	32	2·7
s.	8	57	33·8
" Var. in 1h." 10·84 × 8·96	—	1	37·1
	8	55	56·7
Long. 45° W. = 3h. 10·84 × 3	—		32·5
App. T. mer. pass. 11th	8	55	24·2



The circle is the equinoctial, * the star on the meridian P Q, A the first point of Aries, and S the sun. Q B is the A. T. S., A Q the star's R. A., and also the R. A. of the meridian, and A Q B the sun's R. A.

X.—Given the Mean Time at Place to find what Principal Stars are East and West of the Meridian

Rule.—Find the sidereal time at place, or right ascension of the meridian by Case V., p. 265; then, since the stars pass the meridian in the order of their right ascension, and they are catalogued in the Nautical Almanac in that order, turn to Nautical Almanac " Mean Places of Stars," or to Table of Mean Places of Stars in Norie's Tables, and find the right ascension corresponding nearest to the sidereal time at place. All the stars having less right ascension than the sidereal time are *west* of the meridian; all with greater right ascension are *east* of the meridian.

For the possible *visibility of the stars*, remember that if declination of star and latitude of place are of *different* names, one N. and the other S., and the declination be greater than the co-latitude, such stars are never *above* the horizon of the observer, or if declination and latitude exceed 90° the star will be below the horizon.

Also, with declination and latitude of the *same* name, when the polar distance is less than the latitude the star is circumpolar.

Example.—January 19th, at 1h. 20m. a.m. mean time in lat. 49° N., the Greenwich date being Jan. 18d. 16h. 7m.; what principal stars are visible E. and W. of the meridian?

	H.	M.	S.	
Ship M.T. Jan. 19d.	1	20	0	a.m.
„ 18d.	13	20	0	
(N.A.) Sid. T. mean noon	19	51	22·7	
Accel. for 16h. 7m.		2	38·8	
Sid. T. at place, or R.A. mer.	9	14	1·5	

Ans.—By Nautical Almanac or Table of Mean Places of Stars in Norie's Tables.—

The stars with less right ascension, and visible, are Pollux, Procyon, Castor, Sirius, *a* Orionis, Rigel, Capella, and Aldebaran, W. of the meridian.

The stars with greater right ascension, and visible, are *a* Hydræ, Regulus, *a* Ursæ Maj., Spica, *n* Ursæ Maj., and Arcturus, E. of the meridian.

The bright stars Canopus, *n* Argus, *a* Crucis, *B* Centauri, and *a* Centauri are never visible in lat. 49° N.; the declination of each being greater than co-lat. 41°.

N.B.—It is often convenient to know *what bright stars will pass the meridian between any given hours.*

Rule.—To each of the given times add the sidereal time (Nautical Almanac, p. II.) for Greenwich noon of the day; thus you have two *approximate* sidereal times at place corresponding to the given hours. Now, turn to Nautical Almanac for "Mean Places of Stars," or Table of Mean Places of Stars in Norie's Tables and all stars whose right ascension lies between the two sidereal times at place will pass the meridian in the given interval.

Example.—What bright stars will pass the meridian between 6h. p.m. and 11h. p.m. mean time, on November 7th?

	H.	M.	S.		H.	M.	S.
Ship time	6	0	0	....	11	0	0
(N.A.) Sid. T.	15	6	33	....	15	6	33
Approximate R.A. mer.	21	0	33	....	2	6	33

Ans.—Turn to Nautical Almanac or Table of Mean Places of Stars in Norie's Tables and find right ascension 21h.; all stars with less right ascension will have already passed the meridian; the stars with greater right ascension are *a* Gruis, Fomalhaut, and Markab; but the stars with less right ascension, at the beginning of the Catalogue, will follow Markab; these are the stars in Andromeda, Pegasus, Cassiopeia, Eridanus, and Aries; but which of all these will be seen depends on the latitude of the place, and the declination of the star.

XI.—To find the Hour Angle (H. A.) of a Star at a given Time and Place

Rule.—To the mean time at place add the mean sun's right ascension, and from the sum (which is the sidereal time or right ascension of the meridian) subtract the star's right ascension (borrowing 24 hours if necessary); the remainder will be the westerly hour angle or meridian distance; if the W. hour angle exceeds 12h. subtract it from 24h. for the less or E. hour angle, if required.

Formula.— $$\text{H. A.} = \overbrace{\text{M. T. S.} + \text{M. ⊙'s R. A.}}^{\text{R. A. M.}} - *\text{'s R. A.}$$

If the apparent time at place is given, find the sidereal time or right ascension of the meridian by VI., p. 265 and subtract the star's right ascension.

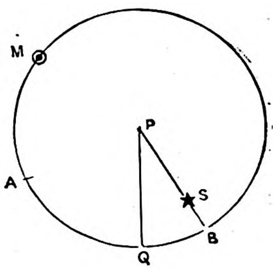
Formula.— $$\text{H. A.} = \overbrace{\text{A. T. S.} + \text{App. ⊙'s R. A.}}^{\text{R. A. M.}} - *\text{'s R. A.}$$

N.B.—"Star" may here denote *any* heavenly body, except the sun.

Example.—January 18th, at 8h. 40m. 24s. p.m. mean time at place; the Greenwich date being Jan. 18d. 10h. 50m. 38s. Find the hour angle of Sirius.

N.B.—The elements of the problem are brought forward from p. 265.

	H.	M.	S.	
M.T. at ship 18d.	8	40	24	
Mean sun's R.A.	19	53	9·6	
R.A.M. or Sid. T. obs.	4	33	33·6	
(N.A.) Sirius R.A.	6	40	18·5	
H.A. of Sirius	21	53	15·1	W.
	24			
H.A. of Sirius	2	6	44·9	E.



N.B.—The E. H. A. could have been found in a direct manner by subtracting the Sid T. from the *'s R. A.

The circle is the equinoctial, P Q the meridian of the place, A the first point of Aries, M the mean sun, S the star, Q A M B is the W. H. A., and Q B the E. H. A., Q M the M. T. S., A Q M the M. S. R. A., A Q the Sid. T., or R. A. mer., and A Q B the *'s R. A.

XII.—To find the Hour Angle (H.A.) of a Star, through the Hour Angle of another Star which has been previously found

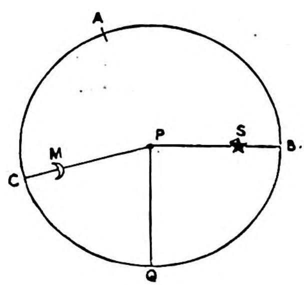
Rule.—To the given star's *westerly* hour angle add its right ascension; the sum will be the sidereal time at place, from which subtract the right ascension of the star for which the hour angle is required; the remainder will be the *westerly* hour angle which, if more than 12h., can be subtracted from 24h. for the *easterly* hour angle.

Formulæ— Sid. T. = (given) *'s W. H. A. + R. A.
W. H. A. (required *) = Sid. T. − R. A.

N.B.—All the necessary data must be corrected for the Greenwich date, astronomical time.

Example.—

	H.	M.	S.	
Suppose, *'s H.A.	6	14	20	E.
	24			
*'s H.A.	17	45	40	W.
*'s R.A.	17	0	20	
R.A.M. or Sid. T.	10	46	0	
☽'s R.A.	5	47	23	
☽'s H A.	4	58	37	W.



The circle is the equinoctial, P Q the meridian of the place, A the first point of Aries, S the star, and M the moon. Q B is the E. H. A., and Q A B the W. H. A. of the star, Q C the W. H. A. of the moon, A C Q B the R. A. of the star, and A C the R. A. of the moon, A C Q the Sid. T. or R. A. meridian.

XIII.—The Hour Angle (H. A.) of a Star having been found at a given Place and Date, to Find the Mean Time at Place

Rule.—To the star's *westerly* hour angle add the star's right ascension; the sum will be the sidereal time at place or right ascension of the meridian (R. A. M.), from which subtract the mean sun's right ascension; and the remainder will be the mean time at place.

N.B.—If the star be *east* of the meridian, subtract its hour angle from 24h. for the *westerly* hour angle; and then proceed as above.

Formula— $\overbrace{\text{*'s W. H. A.} + \text{R. A.}}^{\text{R. A. M.}} - \text{M. ⊙'s R. A.} = \text{M. T. S.}$

Example.—January 4th a.m. at ship; when the mean time at Greenwich was Jan. 4d. 3h. 22m. 24s.; the hour angle of Regulus W. of the meridian was 2h. 14m. 49s. Find the mean time at place.

	H.	M.	S.
Sid. T. (N.A. p. II.)	18	56	10·9
Accel. for Gr. date		+	33·2
Mean sun's R.A.	18	56	44·1

	H.	M.	S.	
Regulus H.A.	2	14	49	W.
,, R.A.	10	2	31	
R.A.M. or Sid. T.	12	17	20	
Mean sun's R.A.	18	56	44	
M.T. at ship, Jan. 3rd	17	20	36	

Example.—Or suppose a star's hour angle *east* of the meridian, then—

	H.	M.	S.	
*'s H.A.	5	15	17	E.
	24			
*'s H.A.	18	44	43	W.
*'s R.A.	11	24	54	
R.A.M. or Sid. T.	6	9	37	
Mean sun's R.A.	0	1	47	
M.T. at ship	6	7	50	

The diagram is similar to that in XI.

SPECIFIC GRAVITY AND MISCELLANEOUS CALCULATIONS

The specific gravity of a body may be defined as the ratio which exists between the weight, or density, of any substance when compared with another substance taken as a standard of comparison. For solids, distilled water is taken as the standard to which all other solids are referred when finding their specific gravity. The specific gravity of distilled water is taken as 1, or, as it is called, unity.

Whilst specific gravity does not tell you the absolute weight of an element, it conveys to you a clear idea of its weight compared with distilled water and when the volume of the element is known its weight can readily be found, as will be seen in the following calculations.

A cubic foot of distilled water weighs 1,000 oz., but it would be incorrect to say that the specific gravity of distilled water is 1,000 oz.

It is necessary to know the weight of a cubic foot of distilled water in order to find the weight of any other body when its specific gravity and volume are given.

A cubic foot of the Atlantic weighs about 1,025 oz., but its specific gravity is 1·025, or twenty-five thousandths heavier than distilled water. If you were to evaporate a cubic foot of the Atlantic water you would find about 25 oz. of solid matter, principally salt, lying at the bottom of the receptacle in which it was boiled.

We shall now show how to find the specific gravity of a solid heavier than water. First weigh the element, whose specific gravity is required, in water; then weigh it in air; divide its weight in air by the loss of weight when weighed in water; the result is its specific gravity.

wt in air wt in water
wt in

Example.—A piece of iron when weighed in water was found to 24 lbs., and when weighed in air was 28 lbs. Required its specific gravi

Weight in air	28 lbs.	= W
,, ,, water	24 ,,	
Loss of weight	4 ,,	= w

$$\frac{W}{w} = \text{sp. gr.} \quad \therefore \frac{28}{4} = 7 = \text{sp. gr. required.}$$

To find the Specific Gravity of a Log of Square Timber of Uniform Sectior throughout

Put the log into the water and note the depth to which it is immerse divide the depth, to which it is immersed, by the sectional depth of t log, and the result is its specific gravity.

Example.—A log of wood 70 feet long and 2 feet square was floating a draft of 18 inches. Required its specific gravity.

$$\frac{18}{24} = \frac{3}{4} = \cdot 75.$$

·75

Answer.—Specific gravity ·75.

If this log were constrained to stand in a vertical position it woı float at a draft of $70 \times \frac{3}{4} = 52\frac{1}{2}$ feet.

Let it now be required to find the weight of the log and the pressı per square inch on the immersed end.

To find the Weight of the Log

Multiply the number of cubic feet in the log by its specific gravity a then by 62·5 lbs., and the result is the weight required.

	$70 \times 2 \times 2 = 280$ cubic feet in log.
Then	$280 \times \cdot 75 = 210$ cubic feet of water displaced.
and	$210 \times 62\cdot 5 = 13{,}125$ lbs. = weight of log.

To find the Pressure per Square Inch on Lower End

Divide the weight of the log by the area of end section, in inches, a the result is the pressure per square inch.

Area of end $= 2 \times 2 \times 144 = 576$ square inches,

and $\frac{13125}{576} = 22\cdot 8$ lbs. nearly, the pressure per square inch.

If the spar were floating in a horizontal position the pressure per squ: inch would be ·65 lb.

To find the Cubic Capacity of a Circular Tank 6 feet in diameter and 15 feet deep, also the Number of Gallons it would contain, and the Weight

Area of circular section equals diameter squared multiplied by ·7854.

Thus	$6 \times 6 \times \cdot 7854 = 28\cdot 2744$ ft. area required.
and	$28\cdot 2744 \times 15 = 424\cdot 1$ ft. cubic capacity required.
and	$424\cdot 1 \times 6\cdot 25 = 2650\cdot 6$ gallons
and	$\frac{424\cdot 1}{36} = 11\cdot 8$ tons nearly.

35 cubic feet of salt water, and 36 cubic feet of fresh water, weigh 1 ton. Hence if the tank were full of salt water of specific gravity 1·025, divide by 35.

To find the Draught to which a Vessel may be loaded in Water which differs from Salt Water whose Specific Gravity is 1·025

Find the specific gravity (by hydrometer) of the water in which the ship is loading, and take the difference between this and the specific gravity of salt water and form a fraction; multiply the allowance for fresh water by the fraction just found, and the result is the amount the vessel's draught may be increased.

Example.—A vessel is loading in water whose specific gravity is 1·015. How much may the salt water loadline be immersed, the allowance for fresh water being 6 inches?

$$1\cdot 025 - 1\cdot 015 = 10$$
$$\frac{10}{25} \times 6 = 2\frac{2}{5} \text{ inches.}$$

The salt water loadline may, therefore, be immersed $2\frac{2}{5}$ inches. On going into salt water she would rise $2\frac{2}{5}$ inches.

The above method, although not absolutely correct, is sufficiently correct for all practical purposes.

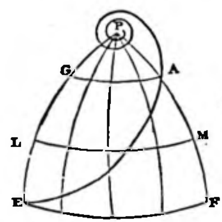
PLANE SAILING

Plane Sailing is the art of navigating a ship upon principles deduced from the supposition of the earth being an extended plane. On this supposition the meridians are considered as being all parallel to each other, the parallels of latitude at right angles to the meridians, and the length of a degree on the meridian, equator, and parallels of latitude everywhere equal. In this sailing there are four principal parts, *viz.*, the course, distance, difference of latitude, and departure. Difference of longitude is not considered. The methods of calculation by plane sailing are sufficiently accurate in low latitudes, but only partially accurate in the higher latitudes, unless the distance sailed be small and the course near a meridian.

The Course is the angle which a ship's track or path makes with the terrestrial meridian, and is hence the *true* course, always reckoned from the north or south point of the horizon, towards east or west, either in points of the compass, or in degrees.

Distance is the number of *nautical* miles between any two places reckoned on the arc of the *rhumb curve;* or it is the length of a ship's track when she sails on a direct course, in a given time, from one place to another.

Rhumb Line.—Looking from the centre of the compass towards any point on the horizon, the direction is the *rhumb line* as indicated by the compass; but the *continued* rhumb line which a ship traces out when sailing from E to A (*see* Fig.), without altering the course, is the *rhumb curve,* which may be defined as *the shortest line which can join two points on the globe, cutting all the meridians which it crosses at the same angle.* The length of any portion of the rhumb curve is the nautical distance.



This line, also known as the *loxodromic curve*, is in its continuity an unending spiral, winding round the pole, but never reaching it.

Difference of Latitude is the distance which a ship makes in a north or south direction from one place to another, and is reckoned on a meridian.

Departure is the true east or west distance (in nautical miles) that a ship makes when sailing on an oblique course. It receives its name from being taken to (approximately) measure how far a ship has *departed* from the meridian; it must not be confounded with *difference of longitude*, although it is the basis whence longitude by dead reckoning is derived.

If a ship sail due north or south, she sails on a meridian, makes no departure, and her distance and difference of latitude are the same.

If a ship sail due east or west, she runs on a parallel of latitude, makes no difference of latitude, and her departure and distance are the same.

But when a ship sails in any other direction than on a meridian or on a parallel, she makes both difference of latitude and departure, and these, with the distance, form a right-angled triangle, the hypotenuse of which is the distance sailed, the perpendicular is the difference of latitude, the base is

the departure, the angle opposite to the base is the course, and the angle opposite to the perpendicular is the complement of the course ; hence, any two of these parts being given, the rest may be found by plane trigonometry.

When a ship's course is 4 points, or 45 degrees, the difference of latitude and departure are equal.

When the course is less than 4 points, or 45 degrees, the difference of latitude is greater than the departure.

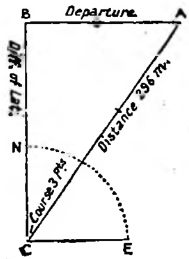
When the course is more than 4 points, or 45 degrees, the departure is greater than the difference of latitude.

Given the True Course and Distance, to find the Difference of Latitude and Departure

Example.—A ship from lat. 48° 40′ N., sails N.E. by N. 296 miles: required her present latitude, and the departure made good.

BY CONSTRUCTION

Draw the line B C to represent the meridian the ship sailed from ; with the chord of 60° in the compasses, and one foot in C, describe the arc N E ; from N to E lay off the chord of 90°, and draw the line C E : then will N C E represent the N.E. quarter of the compass. Take the course, three points, in the compasses from the line of rhumbs, which lay off from N towards E, and through the point where it cuts the arc draw the line C A, which make equal to the distance 296, taken from a scale of equal parts ; through A draw the line B A parallel to C E (Prob. IV. Geom.) ; then will B A represent the departure equal to 164 miles, and C B the difference of latitude 246 miles.



BY CALCULATION

To find the Departure

$$\frac{\text{Dep.}}{\text{Dist.}} = \text{sin. co.},$$

$$\therefore \text{dep.} = \text{dis.} \times \text{sin. co.}$$

$$\text{Log. dep.} = \text{log. dist.} + \text{L. sin. co.} - 10$$

Distance 296 miles	log.	2·471292
Course 3 pts.*	sin.	9·744739
Departure 164·4 miles	log.	2·216031

To find the Diff. Latitude.

$$\frac{\text{D. lat.}}{\text{Dist.}} = \text{cos. co.},$$

$$\therefore \text{D. lat.} = \text{dis.} \times \text{cos. co.}$$

$$\text{Log. D. lat.} = \text{log. dist.} + \text{L. cos. co.} - 10$$

Distance 296 miles	Log.	2·471292
Course 3 pts.*	Cos.	9·919846
Diff. lat. 246·1 miles	log.	2·391138

NOTE.—This case is the basis on which the *Traverse Tables* are constructed, and to which constant reference is made in navigation.

* When the courses are given in points of the compass, the log. sine, cosine, tangent, etc., are to be taken from the Table of Log. sines, etc., for points and quarter points.

To find the Latitude in

Latitude left	48° 40′ N.
Diff. of latitude 246 miles, or	4 6 N.
Latitude in	52 46 N.

Ans. Lat. in 52° 46′ N., departure 164·4 miles.

BY INSPECTION

Previous to resolving the cases by Inspection, read very carefully the Explanation of the Traverse Tables. If the course is less than 4 points, or 45°, it is taken from the top of the page; if more than 4 points, or 45°, take it from the bottom of the page, being careful to find the d. lat. and dep. in their proper columns. When the larger of the two given numbers is the d. lat., seek it in the column marked "d. Lat." at the top of the page until the smaller number is found by its side in the column marked "Dep." at the top; then the course corresponding to that d. lat. and dep. will be found at the top of the page, and the distance will be found on the left of the above numbers, in heavy type, in the column marked "Dist." at the top and bottom. If the dep. is more than the d. lat. the course will be at the bottom of the page and the d. lat. and dep. columns are reversed in position as compared with the top.

Enter the Traverse Table, and find the course 3 points (at the top of the page), and in one of the columns marked "Dist." find the distance 296; then opposite to this, in the columns marked "Lat." and "Dep.," will be the difference of latitude 246·1, and the departure 164·4.

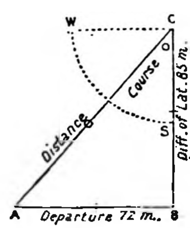
Given the Difference of Latitude and Departure, to find the True Course and Distance

Example.—A ship from Funchal, in Madeira, in lat. 32° 38′ N., sails a direct course between the south and west until she is in lat. 31° 13′ N. by observation, having made 72 miles of departure: required her true course, and distance run.

Latitude of Funchal........	32° 38′ N.
Latitude in, by observation	31 13 N.
Difference of latitude	1 25 S. = 85 miles.

BY CONSTRUCTION

Draw the meridian line C B, and from C describe the quadrant S C W; from C to B lay off the difference of latitude 85; through B draw A B parallel to C W, and equal to the departure 72; and join A C; then the course A C B will measure 40¼°, and the distance A C 111 miles.



BY CALCULATION

To find the Course

$$\frac{\text{Dep.}}{\text{D. lat.}} = \text{tan. co.}$$

Log. dep. + 10 — log. D. lat.
= L. tan. co.

Departure 72m. log. (+10)		11·857332
Diff. lat. 85 miles	log.	1·920419
Course S. 40° 16′ W.	tan.	9·927913

To find the Distance

$$\frac{\text{Dist.}}{\text{D. lat.}} = \text{sec. co.},$$

$$\therefore \text{Dist.} = \text{D. lat.} \times \text{sec. co.}$$

Log. dist. = log. D. lat.
+ L. sec. co. — 10.

Diff. lat. 85 miles	log.	1·929419
Course 40° 16′	sec.	10·117450
Distance 111·4 miles	log.	2·046869

By Inspection.—Seek in the Traverse Table till the difference of latitude 85, and the departure 72, or the nearest thereto, are found side by side in their respective columns, which in this case will be under the course 40 degrees; and the distance answering thereto will be 111 miles.

Ans. Course S. 40° 16′ W., distance 111·4 miles.

Obs.—For all practical purposes these Examples might be computed with sufficient accuracy, using only four places of decimals in the logarithms.

Examples for Practice

1. A ship from lat. 36° 30′ N., sails S.W. by W. (true) 420 miles: what is her latitude in, and what departure has she made?

Ans. Lat. in 32° 37′ N., and departure 349·3 miles.

2. A ship from lat. 3° 54′ S., has sailed N.W. $\frac{3}{4}$ W. (true) until she arrives at lat. 2° 14′ N.: required her distance run, and departure made good.

Ans. Distance 617·8, and departure 496·2 miles.

3. A ship from St. Helena, in lat. 15° 55′ S., sails S.S.E. $\frac{1}{2}$ E. (true) until she has made 115 miles of departure: required her latitude in, and the distance she has run.

Ans. Lat. in 19° 30′ S., and distance 244 miles.

4. A ship from lat. 28° 20′ N., sails north-easterly 486 miles, and finds by observation that she is in lat. 32° 17′ N.: what true course and what departure has she made?

Ans. Course N. 60° 49′ E., or N.E. by E. $\frac{1}{2}$ E. nearly, and departure 424·3 miles.

5. A ship sails between the north and west 170 leagues from a port, in lat. 38° 42′ N., until her departure is 98 leagues: required her true course and latitude in.

Ans. Course N. 35° 12′ W., or N.W. $\frac{3}{4}$ N. nearly, and lat. in 45° 39′ N.

6. A ship from the Lizard, in lat. 49° 58′ N., sails to the westward on a direct course, till she arrives in lat. 48° 11′ N., and finds she has made 87 miles of westing: required her true course, and distance run.

Ans. Course S. 39° 7′ W., or S.W. ½ S. nearly, and distance 137·9 miles.

7. A ship from Ascension Island, in lat. 7° 56′ S., sails N.N.W. ¾ W. (true) 244 miles: required her latitude in, and departure made good.

Ans. Lat. in 4° 27′ S., and departure 125 4 miles.

8. A ship from Cape St. Vincent, in lat. 37° 3′ N., sails between the south and west, till her difference of latitude is 69 miles, and her departure 215 miles: required her true course, distance and latitude in.

Ans. Course S. 72° 12′ W., or W.S.W. ½ W. nearly, distance 225·7 miles, and lat. in 35° 54′ N.

9. A ship from the Lizard, in lat. 49° 58′ N., sails 456 miles to the westward, and then finds she is 360 miles to the southward of the Lizard: required her true course, departure, and latitude in.

Ans. Course S. 37° 52′ W., or S.W. ¾ S. nearly, departure 279·9 miles, and lat. in 43° 58′ N.

10. A ship from Cape Hatteras, in lat. 35° 15′ N., sails in the north-east quarter 226 miles, and then finds that she is 198 miles to the eastward of the Cape: required her true course, and latitude in.

Ans. Course N. 61° 11′ E., or N.E. by E. ½ E., and lat. in 37° 4′ N.

11. If a ship take her departure at 6h. p.m. from Cape Verd, in lat. 14° 45′ N., and sails W.S.W. ¼ W. (true) at the rate of 7 miles an hour until the next day at noon: what will be her distance run, departure, and latitude in?

Ans. Distance 126 miles, departure 120·6, and lat. in 14° 8′ N.

12. A ship from lat. 55° 30′ N. sails S. 33° 45′ W. (true) during 20 hours, and then finds by observation she is in lat. 53° 17′ N.: required her rate of sailing per hour, and the departure she has made.

Ans. Departure 88·87 miles, distance run in 20 hours 160 miles, and therefore her rate per hour 8 miles.

13. Find the true course, supposing a ship to sail in the north-east quarter, until her departure is twice her difference of latitude.

Ans. Course N. 63° 26′ 6″ E.

TRAVERSE SAILING

When a ship, either from contrary winds or other causes, is obliged to sail on different courses, the irregular or zigzag track she makes is called a Traverse, or Compound Course; and the method of reducing these courses and distances into a *resultant* course and distance is called *resolving a traverse.*

To resolve a Traverse, make a Table (as in Example I.), and divide it into six columns; in the first of these set down the various courses, and opposite to them, in the second column, the corresponding distances: the third and fourth columns are to be marked, one N. and the other S. at the top, and are to contain the differences of latitude; the fifth and sixth are to be marked E. and W., to contain the departures.

The difference of latitude and departure corresponding to each course and distance can be found by plane sailing, as already indicated, but this is needless.

The most common method of resolving the traverse is by *inspection,* using the Traverse Table in Norie's Tables, to which you must now turn; set the difference of latitude and departure down opposite the distance in their proper columns, observing that the difference of latitude must be placed in the N. column if the course be northerly, and in the S. column if the course be southerly; and that the departure must be placed in the E. column if the course be easterly, and in the W. column if it be westerly. When the course is due north, south, east, or west, set down the whole distance in the column answering to its proper heading. This done, add up the columns of northing, southing, easting, and westing, and set down the sum of each at the bottom; then the difference between the sums of the north and south columns will be the difference of latitude *made good,* of the same name as the greater; and the difference between the sums of the east and west columns will be the departure *made good,* also of the same name as the greater.

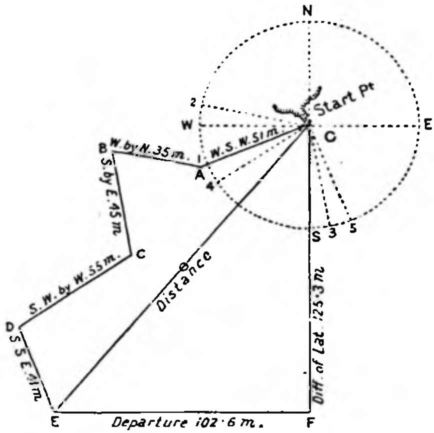
With this difference of latitude and departure made good, find the direct course and distance made good, as in plane sailing; or again turn to the Traverse Tables as shown in the sequel.

Example I.—Suppose a ship from Start Point, in lat. 50° 13′ N., sail the following true courses and distances:—W.S.W. 51 miles, W. by N. 35 miles, S. by E. 45 miles, S.W. by W. 55 miles, and S.S.E. 41 miles: required her direct true course and distance sailed, and her latitude in.

By Construction

With the chord of 60° describe the circle N E S W, to represent the compass; draw the diameters N S and E W at right angles, the one representing the meridian, and the other the parallel the ship sailed from: take the courses from the line of rhumbs, and lay them off from the meridian in their respective quarters, and number them in order, 1, 2, 3, 4, etc.; thus, from S. to 1 lay off 6 points for the first course W.S.W.; from N. to 2 lay off 7 points for the second course W. by N.; from S. to 3 lay off one point

for the third course S. by E.; from S. to 4 lay off 5 points for the fourth course S.W. by W.; from S. to 5 lay off 2 points for the fifth course S.S.E.; and from the centre of the circle draw rhumb lines to each of these points, which may be produced to any length that is necessary. Upon the first rhumb C 1, lay off the first distance 51 miles, from C to A; then will A represent the ship's place at the end of the first course; through A draw A B



parallel to the second course C 2 (Prob. IV. Geom.), and make it equal to the second distance 35 miles; through B draw B C parallel to C 3, and equal to 45 miles; through C draw C D parallel to C 4, and equal to 55 miles; and through D draw D E parallel to C 5, and equal to 41 miles. Through E draw the line E F parallel to the east and west line W E, meeting N S produced to F, and join C E. Then will C F be the difference of latitude made good, measuring 125′; E F is the departure 102′, C E the distance 162 miles, and the angle E C F the course $39\frac{1}{4}°$ or $3\frac{1}{2}$ points.

TRAVERSE TABLE

Courses.	Dist.	Diff. Lat. N.	Diff. Lat. S.	Departure. E.	Departure. W.
S. 6 pt. W.	51		19·5		47·1
N. 7 „ W.	35	6·8			34·3
S. 1 „ E.	45		44·1	8·8	
S. 5 „ W.	55		30·6		45·7
S. 2 „ E.	41		37·9	15·7	
		6·8	132·1	24·5	127·1
			6·8		24·5
	Diff.	Lat.	125·3	Dep.	102·6

To find the Lat. in

Lat. left	. . 50° 13′ N. .	
Diff. lat. 125 =	2 5 S.	
Latitude in . .	48 8 N.	

Obs.—For practice you can verify, by plane sailing, the results taken from Traverse Table, and set down in the above columns of difference of latitude and departure.

The Course and Distance made good may be calculated as already shown in plane sailing, and note that the course takes its name from the greater difference of latitude and departure columns.

BY CALCULATION

For the Course			*For the Distance*		
$\frac{\text{Dep.}}{\text{D. lat.}} = \text{tan. co.}$			$\frac{\text{Dist.}}{\text{D. lat.}} = \text{sec. co.},$		
			$\therefore$ dist. $=$ D. lat. $\times$ sec. co.		
Log. dep. $+ 10 -$ log. D. lat. $=$ L. tan. co.			Log. dist. $=$ log. D. lat. $+$ L. sec. co. $- 10$		
Dep. 102·6m.	log. (+ 10)	12·011147	Diff. lat. 125·3m.	log.	2·097951
Diff. lat. 125·3m.	log.	2·097951	Course 39° 19′	sec.	10·111452
Course S. 39° 19′ W.	tan.	9·913196	Distance 162m.	log.	2·209403

by which the direct course made good is S. 39° 19′ W. or S.W. ½ S. ; and the distance 162 miles.

But the usual method is to determine the Course and Distance by Inspection.

By Inspection.—Seek in the Traverse Table till the difference of latitude 125·3, and departure 102·6, are found side by side, each in its proper column ; the nearest to these will be 125·1 and 101·3, which give the course S. 39° W. at the top ; and the distance 161 in distance column.

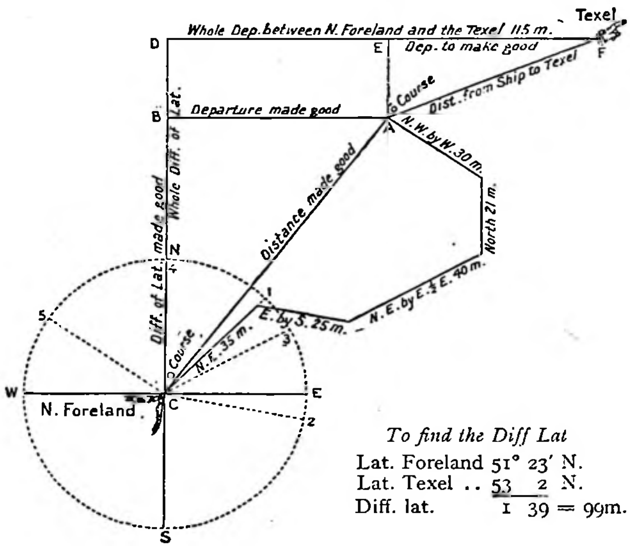
Ans. Course S. 39° 19′ W., distance 161 miles, lat. in 48° 8′ N.

Example II.—A ship from the North Foreland, in lat. 51° 23′ N., and bound to the Texel, which lies in lat. 53° 2′ N., and 115 miles to the eastward, sails the following true courses and distances :—N.E. 35 miles, E. by S. 25 miles, N.E. by E. ½ E. 40 miles, North 21 miles, and N.W. by W. 30 miles ; required her true course and distance made good, the latitude she is in, and the departure made ; also the direct true course and distance from the ship to the Texel.

BY CONSTRUCTION

With the chord of 60° describe a circle, through the centre of which draw the north and south, or meridian line, N S, and at right angles to it the east and west line W E ; lay off from the meridian, upon the circumference of the circle, the courses in their proper quarters, number them in order, and draw a rhumb line from the centre to each of the points ; then, on the first rhumb line, which is N.E. or 4 points, lay off the distance 35 miles from the centre, and through the end of this first distance draw a line parallel to the second rhumb, E. by S. 7 points, on which lay off the second distance 25 miles ; proceed thus till all the courses and distances are laid off, and through the end of the last distance draw the line A B parallel to the line W E, meeting S N produced to B ; then will A represent the ship's place ; C B will measure the difference of latitude made good 76·4 ; B A the departure 59′·6 ; C A the distance run 97 miles ; and the angle B C A the course steered, N. 38° E.

To find what course the ship must steer, and what distance she must run before she can arrive at the Texel, lay off from C to D 99 miles, the difference of latitude between the Foreland and the Texel, and through D draw D F parallel to W E, and equal to the departure 115; then will F



To find the Diff Lat

Lat. Foreland 51° 23′ N.
Lat. Texel .. 53 2 N.
Diff. lat. 1 39 = 99m.

represent the situation of the Texel: through A draw A E parallel to C D, and join A F; then A E will be the northing, or difference of latitude 23′, and E F the easting, or departure 55′·4, the ship has to make good; A F the distance the ship has to sail 60m.; and the angle E A F, measuring 67½°, the course from the ship to the Texel.

TRAVERSE TABLE

Courses	Distance	Diff. Lat.		Departure	
		N.	S.	E.	W.
N. 4 pts. E.	35	24·7		24·7	
S. 7 „ E.	25		4·9	24·5	
N. 5½ „ E.	40	18·9		35·3	
North	21	21·0			
N. 5 pts. W.	30	16·7			24·9
		81·3	4·9	84·5	24·9
		4·9		24·9	
		76·4	Diff. Lat.	59·6	Dep.

The Course and Distance made good, if desirable to calculate it by plane sailing, will be, course N. 37° 58′ E. and distance 96·9 miles.

By Inspection.—The difference of latitude 76·4, and departure 59·6, being sought in Traverse Table in their respective columns, give the course N. 38° E., and distance 97 miles.

To find the direct Course and Distance from the Ship to the Texel

Latitude of N. Foreland	51° 23′ N.	Whole departure	115m. E.
Diff. lat. made good, 76m. =	1 16 N.	Departure made good	59·6 E.
Latitude in	52 39 N.	Departure to make good	55·4 E.
Latitude of Texel	53 2 N.		
Diff. lat. to make good	0 23 N.		

By calculation, plane sailing, with difference of latitude 23′ N. and departure 55′·4 E. the direct course from the ship's position to the Texel is N. 67° 27′ E. and distance 59·98 miles.

By Inspection.—The difference of latitude 23′, and departure 55′·4, being sought in the Traverse Table, the nearest numbers answering to them will be over the course N. 6 pts. E., and opposite the distance 60 miles.

Ans. True course N. 37° 58′ E.; distance 96·9 miles; lat. in 52° 39′ N.; departure 59·6 miles. True course to Texel N. 67° 27′ E.; distance 59·98 miles.

Examples for Practice

NOTE.—The courses in these Examples are all supposed to be true courses, except in Example 8.

1. A ship from the Lizard, in lat. 49° 58′ N., sails as follows:—S. by W. 42 miles, W.S.W. 36 miles, West 18 miles, E.S.E. 22 miles, South 34 miles, and N.E. 21 miles. Required the latitude arrived at, and the course and distance made good.

Ans. Lat. in 48° 35′ N.; the course made good S. 16° 27′ W. or S. by W. ½ W. nearly; and the distance 86·13 miles.

2. A ship from lat. 9° 26′ N. sails N.E. 20 miles, North 33 miles, N.N.W. 15 miles, East 25 miles, N.E. by N. 42 miles, and S.W. ½ W. 28 miles. Required her course and distance made good, and the latitude in.

Ans. Course N. 24° 12′ E., or N.N.E. ½ E. nearly; distance 85·63 miles; and lat. in 10° 44′ N.

3. A ship from the Cape of Good Hope, in lat. 34° 22′ S., sails S.W. ½ S. 27 miles, S.E. by E. 45 miles, S.W. by S. 48 miles, West 23 miles, and S.S.W. ½ W. 18 miles. Required her course and distance made good, and her latitude in.

Ans. Course S. 24° 45′ W., or S.S.W. ½ W. nearly; distance 112 miles; and lat. in 36° 4′ S.

4. A ship from lat. 1° 12′ S. sails E. by N. ½ N. 56 miles, N. ¼ E. 80 miles, S. by E. ½ E. 96 miles, N. ½ E. 68 miles, E.S.E. 40 miles, N.N.W. ½ W. 86 miles, and E. by S. 65 miles. Required her course, distance, and latitude in.

Ans. Course N. 51° 47′ E., or N.E. ½ E. nearly; distance 193·8 miles, and lat. in 0° 48′ N.

5. A ship from lat. 46° 18′ N. sails N. 25° W. 50 miles, N. 74° E. 64 miles, S. 52° W. 36 miles, N. 35° E. 40 miles, N. 69° W. 75 miles, and S. 47° E. 48 miles. Required her course, distance, and latitude in.

Ans. Course North; distance 67·7 miles; and lat. in 47° 26′ N.

6. A ship from lat. 51° 30′ N., running at the rate of 8 knots an hour, sails W.S.W. 3 hours, N.W. 2½ hours, West 4 hours, S.W. by S. 2¼ hours, and N.W. ½ W. 2 hours. Required her course, distance, and latitude in.

Ans. Course West; distance 90·7 miles; and lat. in 51° 30′ N.

7. A ship from a port in lat. 38° 42′ N., bound to another port situated in lat. 36° 32′ N., and 137 miles to the eastward, sails the following courses: S. by W. ½ W. 55 miles, S.W. ½ S. 37 miles, South 60 miles, E.S.E. 40 miles, S.E. ¾ S. 32 miles, and N.E. by E. ½ E. 58 miles. Required her course and distance made good, and the latitude in; also the course and distance to her intended port.

Ans. The course made good is S. 23° 38′ E., and the distance 169 miles; the lat. in 36° 7′ N.; the course to the intended port, N. 70° 8′ E., and the distance 73·56 miles.

8. The course (correct magnetic) from Beachy Head to Selsea Bill is N. 67° W. and distance 40 miles; from Selsea Bill to St. Catherine's Point N. 86° W. 21 miles; from St. Catherine's Point to Portland Lights N. 69° W. 44 miles; from Portland Lights to the Start N. 85° W. 49 miles. Required the correct magnetic course and distance from Beachy Head to the Start.

Ans. The course is N. 75° 51′ W. or W.N.W. ¾ W., and distance 152·2 miles.

9. Suppose a ship to sail upon the following courses and distances:—S.E. by S. 29 miles; N.N.E. 10 miles; E.S.E. 50 miles; E.N.E. 50 miles; S.S.E. 10 miles; N.E. by N. 29 miles; West 25 miles; S.S.E. 10 miles; W.S.W. ½ W. 42 miles; North 110 miles; E. ¾ N. 62 miles; North 7 miles; West 62 miles; North 10 miles; West 8 miles; South 10 miles; West 62 miles; South 7 miles; E. ¾ S. 62 miles; South 110 miles; W.N.W. ½ W. 42 miles; N.N.E. 10 miles; and West 25 miles. Required her course and distance made good.

Ans. The ship has returned to the place she sailed from.

NOTE.—This example is taken from "Robertson's Elements of Navigation," the figure forming the outline of an anchor.

10. A ship in lat. 1° 0′ S. is bound to a port in lat. 1° 10′ N., distant 220 miles in the north-west quarter, but meeting with contrary winds, runs the following courses:—N.E. by N. 63 miles, N.W. $\frac{1}{4}$ W. 85 miles, North 96 miles, and N.N.W. 87 miles. Required the ship's place; and the course and distance to her port.

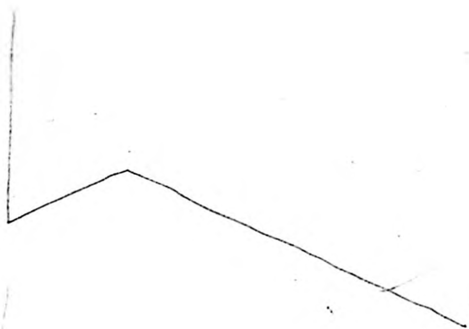
Ans. The lat. in is 3° 46′ N.; the course to her port S. 36° 42′ W. or S.W. $\frac{3}{4}$ S., and distance 194·4 miles.

11. Two ships (A) and (B) part company in lat. 1° 44′ S., and meet again at the end of two days, having run as follows:

(A) N.N.E. 96 miles, W.S.W. 96 miles, E.S.E. 96 miles, and N.N.W. 96 miles.

(B) N.N.W. 96 miles, E.S.E. 96 miles, W.S.W. 96 miles, and N.N.E. 96 miles. Required the latitude arrived at, with the direct course and distance of each ship.

Ans. There being no departure made good, and the difference of latitude being N., therefore the course made good is North, and the distance is 104 miles; also, the ships meet on the equator.



PARALLEL SAILING

In Plane Sailing it was observed that the meridians are considered as being all parallel to each other, and the length of a degree on the meridian and parallel everywhere equal, on the supposition of the earth, or at least any small part of it, being a plane: but as the earth is a sphere, or globe, and the meridians meet at the poles, it is evident that the distance between any two meridians must vary in every latitude, their greatest distance apart being at the equator, on which the difference of longitude is measured.

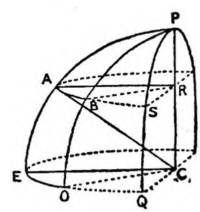
PARALLEL SAILING is the method of finding the distance between two places *in the same latitude*, when their difference of longitude is known; or, otherwise, of finding the difference of longitude corresponding to the meridian distance or *departure* made good, when a ship sails due east or west.

DIFFERENCE OF LONGITUDE between any two places is the arc of the equator contained between the meridians passing through the two places.

The MERIDIAN DISTANCE between two places is the arc of their parallel of latitude contained between the meridians of the places. It is also the *departure* made, or the *distance* run, on the parallel.

The Equator is the great circle on which longitude is reckoned; it is also the origin whence latitude is measured, *polarwise*, through successive parallels of latitude. When a ship sails 60 nautical miles on the equator she changes her position by 60′ (or 1°) of longitude; but since the parallels of latitude are small circles which successively decrease in circumference as they are nearer to the poles, it follows that the farther the parallel is from the equator, the smaller must be the number of nautical miles measuring the circumference, while, at the same time, that circumference ranges through 360° of longitude, as does the equator. As a specific instance in point, 360° of longitude are represented on the equator by 21,600 nautical miles; but the same number of degrees of longitude, on the parallel of 60° N. or S., are represented by only 10,800 nautical miles; thus the circumference of the earth on the parallel of 60° is only half what it is on the equator, and consequently half a nautical mile on that parallel is the equivalent of 1′ of longitude. It is by the methods of parallel sailing that the relation between the meridian distance and the difference of longitude between two places on the same parallel is determined.

Parallel sailing is particularly useful in making small or low islands, in which case it is usual to run into the latitude, and then steer due east or west.



The principles upon which parallel sailing depend may be thus illustrated. Let P A E C represent a section of one-fourth part of the earth, C being the centre, and P the pole; then P A E will be part of a meridian; P C the polar, and E C the equatorial radius; also let P B O represent part of another meridian, A and B two places in same parallel, being equally distant from the equator E O Q; then will A B be the meridian distance between A and B, and E O their difference of longitude; join A C; then the arcs E A or O B or the angle E C A will be the latitude of A or B; and A P or B P their co-latitude. Now E O Q C is the plane of the equator, and A B S R is a plane parallel to the equator; there-

fore A R is parallel to E C, and B R parallel to O C, and the angles E C O and A R B are equal; and as circles and similar arcs of circles are in direct ratio to their radii, therefore—

$$\frac{\text{arc } A\,B}{\text{arc } E\,O} = \frac{A\,R}{E\,C} = \frac{A\,R}{A\,C} = \text{sin. } A\,C\,R = \text{cos. } A\,C\,E$$

But arc A B is the departure or meridian distance; arc E O is the difference of longitude; and angle A C E is the latitude; therefore—

$$\text{cos. } A\,C\,E = \text{cos. lat.}$$

And (1) $\frac{\text{mer. dist.}}{\text{diff. long.}} = \text{cos. lat}$; hence, mer. dist. $=$ diff. long. $\times$ cos. lat.

(2) $\text{diff. long.} = \frac{\text{mer. dist.}}{\text{cos. lat.}} = \text{mer. dist.} \times \text{sec. lat.}$

by one or other of which formulæ all cases in parallel sailing may be computed.

By Construction.—If a triangle, as A B C (*see* Fig. in Example), be so constructed that the hypotenuse A C may represent the difference of longitude in miles, the base A B the meridian distance, and the angle C A B (between the hypotenuse and base) equal to the latitude; then any two of these parts being given, the other may be found by the ordinary solution of a right-angled triangle.

Given the Difference of Longitude between two Places, both on the same Parallel of Latitude, to find their Distance apart

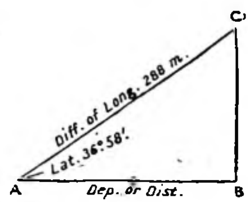
Example.—A ship in lat. 36° 58′ N., and long. 20° 25′ W., is bound to St. Mary's, one of the Western Islands, in the same latitude, and in long. 25° 13′ W.; what distance must she run to arrive at the island?

To find the Difference of Longitude

Longitude of ship	20° 25′ W.
Longitude of St. Mary	25 13 W.
Diff. long.	4 48 = 288′

BY CONSTRUCTION

Draw the line A B of any length, and make the angle C A B equal to latitude 36° 58′ (Prob. XII. Geom.); from A to C lay off the difference of longitude 288, and from C draw C B perpendicular to A B (Prob. III. Geom.); then will A B measure 230, the departure or distance required.



BY CALCULATION

To find the Departure or Distance

$$\frac{\text{Dep.}}{\text{D. Long.}} = \text{cos. lat.}, \therefore \text{dep.} = \text{D. long.} \times \text{cos. lat.}$$

$$\text{Log. dep.} = \text{log. D. long.} + \text{L cos. lat.} - 10$$

Difference of longitude 288′ ......log.		2·459392
Latitude 36° 58′ ....cos.		9·902539
Departure or distance 230·1 miles..log.		2·361931

By Inspection.—Enter Traverse Table with the lat. 37° *as a course*, and the difference of longitude 288 in the distance column, then the distance (or departure) 230 will be found in the difference of latitude column.

Ans. Distance 230·1 miles.

NOTE.—By the above Formula the meridian distance in miles and decimal of a mile, answering to a degree of longitude on any parallel of latitude, may be found.

Example.—Find the number of miles contained in a degree of longitude in lat. 48°.

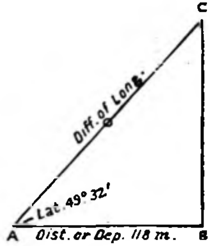
Naut. miles in a degree of long. at the equator (60′)..log.	1·778151
Latitude 48° ....cos.	9·825511
Naut. miles in a degree of long. in lat. 48°; 40·15..log.	1·603662

Given the Distance between two Places, both on the same Parallel of Latitude, to find their Difference of Longitude

Example.—A ship from lat. 49° 32′ N., and long. 10° 16′ W. sails due west 118 miles. Required her present longitude.

BY CONSTRUCTION

Draw the line A B, which make equal to the given distance 118, and make the angle C A B equal to the lat. 49½° (Prob. XII. Geom.); from B erect the perpendicular B C, cutting the line A C in C (Prob. II. Geom.); then will the line A C measure 182, the difference of longitude required.



BY CALCULATION

To find the D. Long.

$$\frac{\text{D. long.}}{\text{Dist.}} = \text{sec. lat.}$$

$$\therefore \text{D. long.} = \text{dist.} \times \text{sec. lat.}$$

$$\text{Log. D. long.} = \text{log. dist.} + \text{L sec. lat.} - 10$$

Distance 118 miles	log.	2·071882
Latitude 49° 32′	sec.	10·187752
D. long. 181·8′	log.	2·259634

To find the Longitude in

Longitude left	10°	16′	W.
D. long. 182′ =	3	2	W.
Longitude in	13	18	W.

By Inspection.—Look for the lat. 50° in the Traverse Table, as a course, and for the distance 118 in the difference of latitude column, opposite to which in the distance column will be found 184; but as the latitude is nearly half way between 49° and 50°, look with 49° as a course, and the distance 118 in the difference of latitude column, opposite to which, in the distance column, will be found 180; then half the sum of 180 and 184 will be 182, the difference of longitude.

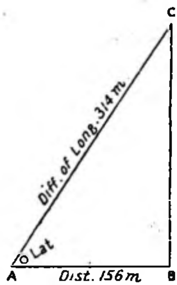
Ans. Long. in 13° 18′ W.

Given the Difference of Longitude and Distance between two Places on the same Parallel of Latitude, to find the Latitude of that Parallel

Example.—A ship sails due east 156 miles, and then finds she has altered her longitude 314 miles. Required the latitude of the parallel on which she has sailed.

BY CONSTRUCTION

Draw the line A B, and make it equal to the distance 156; from B erect the perpendicular B C, and with an extent in the compasses equal to the difference of longitude 314, set one foot in A, and with the other describe an arc cutting B C in C, and draw the line A C; then the angle C A B will measure 60¼° (Prob. XIII. Geom.), the latitude required.



BY CALCULATION

To find the Parallel of Latitude sailed on.

$$\frac{\text{Dep.}}{\text{D. long.}} = \text{cos. lat.}$$

$$\text{Log. dep.} + 10 - \text{log. D. long.} = \text{L cos. lat.}$$

Distance or departure 156 miles	log. (+ 10)	12·193125
D. long. 314′	log.	2·496930
Latitude 60° 13′	cos.	9·696195

By Inspection.—Seek in the Traverse Table till the difference of longitude and distance, viz., 314 and 156, are found opposite to each other in the distance and difference of latitude columns, which will give the lat. 60° at the bottom of the page.

Examples for Practice

1. A ship having taken her departure from North Cape, New Zealand, in lat. 34° 24′ S., and long. 173° 10′ E., being bound to Port Jackson, sails due west until she arrives in long. 163° 35′ E. Required her distance run.

Ans. Distance 474·4 miles.

2. A ship from Buchanness, in lat. 57° 29′ N., and long. 1° 47′ W., sails due east 125 miles. Required her present longitude.

Ans. Long. in 2° 6′ E.

3. A ship in lat. 32° 22′ N., and long. 52° 20′ W., sails west 365 miles. Required her distance from the Island of Bermuda, in the same latitude, and in long. 64° 43′ W.

Ans. Distance of ship from Bermuda 262·6 miles.

4. A ship in lat. 60° N., and long. 22° 30′ W., sails west 200 miles. Required her present longitude.

Ans. Long. in 29° 10′ W.

5. If a ship take her departure from Cape St. Antonio (at the entrance to the River Plate), which lies in lat. 36° 19′ S., and long. 56° 42′ W., how far must she sail due east to arrive at the meridian of the Cape of Good Hope, in long. 18° 24′ E. ?

Ans. 3,631 miles.

6. In what parallel of latitude is the departure or meridian distance one-third the difference of longitude ?

Ans. In lat. 70° 32′.

7. A ship from long. 81° 36′ W., sails west 310 miles, and then finds by observation she is in long. 91° 50′ W. On what parallel of latitude has she sailed ?

Ans. In lat. 59° 41′.

8. Suppose a ship from lat. 35° 30′ N., and long. 6° 15′ W., sails west 250 miles, north 525 miles, and then east 250 miles. Required her present latitude and longitude.

Ans. Lat. 44° 15′ N., and long. 5° 33′ W.

9. A ship from lat. 49° 32′ N., and long. 21° 56′ W., sails N.W. by N. 20 miles, S.W. 40 miles, N.E. by E. 60 miles, S.E. 55 miles, W. by S. 41 miles, and E.N.E. 66 miles. Required her present latitude and longitude.

Ans. Lat. 49° 32′ N., and long. 20° 8′ W.

10. At what rate per hour is Greenwich Observatory, in lat. 51° 28′ 38″, carried around the earth's axis ?

Ans. 560·54 geographical miles per hour.

11. Two ships in lat. 34° 20′ N. are 307 miles apart : if they both sail due north, how many miles will they be apart when they arrive in lat. 46° 34′ N. ?

Ans. 255·6 miles.

12. Two ships in lat. 50° 18′ N. are distant apart 256 miles : they sail due south until their distance apart is 356 miles. In what latitude have they arrived, and how many miles have they sailed on their south course ?

Ans. Lat. arrived in 27° 20½′ N. ; distance sailed 1,377½ miles.

MIDDLE LATITUDE SAILING

When a ship sails due north or south she keeps on the same meridian, and therefore does not change her longitude, and her distance run is the difference of latitude : consequently her place is easily determined by the latitude left, and difference of latitude. Again, when a ship sails due east or west, her difference of longitude is found by the latitude in, and the departure or meridian distance, as already explained in Parallel Sailing ; but when she sails upon any other course she changes both her latitude and longitude. Now, the difference of longitude cannot be inferred either from the departure, considered as a meridian distance in the latitude left, or that come to ; for in the greater latitude it would give the difference of longitude too much, and in the less latitude too little : for example, in lat. 50°, a departure of 100 miles is the equivalent of 155′·6 difference of longitude, while in lat. 48°, the same departure makes only 149′·5 diff. long. ; the departure is therefore accounted a meridian distance in the *mean* of the two latitudes, and then the difference of longitude is found as in Parallel Sailing ; hence this method, which is compounded of plane and parallel sailing, is called MIDDLE LATITUDE SAILING.

The middle latitude is half the sum of the two latitudes when they are of the *same* name ; this is arithmetically correct, and an assumption sufficiently *accurate for short distances ;* but when precision is required a small correction must be applied to the middle latitude, since the *true* middle latitude is always a little *nearer* to the pole than the mean of the latitude left and latitude arrived at.

Workman's Table* (see Norie's Tables) for the correction of the middle latitude is to be entered at the *top* with the *difference* of the two latitudes, and at the *side* with the *middle latitude ;* under the former, and opposite the latter, is the correction to be *added* to the middle latitude to obtain the true *middle latitude.*

When the difference of latitude is under 2°, the correction may be neglected ; when it is between 2° and 3°, add a correction of 1′.

This method of sailing, although not strictly accurate, especially in high latitudes, approaches sufficiently near to the truth for a day's run ; it is accurate in low latitudes, and generally when the ship makes a course greater than 45°.

Middle latitude should be used only when the latitudes are of the same name. If of different names, and the distance small, the departure may be assumed equal to the difference of longitude, since the meridians are sensibly parallel near the equator. But if the distance is great, the two portions of the track on opposite sides of the equator should be calculated separately.

By referring to the figures connected with the examples worked out, it will be seen that in middle latitude sailing the figure illustrative of the method consists of two triangles having a common side, one part of which properly appertains to plane sailing, and the other part to parallel sailing, with the departure as the *base* or connecting link between them.

* This Table was published by Workman in 1805 under the sanction of Dr. Maskelyne, then Astronomer Royal. The Table is not as well known as it ought to be, and, as it removes the only objection to the method of finding the difference of longitude by middle latitude sailing, it is introduced here to the notice of the navigator.

In middle latitude sailing the *departure is taken to be the sum of all the various meridian distances (or departures) corresponding to the indefinitely small parts of the rhumb line between the latitude left and latitude arrived at,* and the assumption is that this—

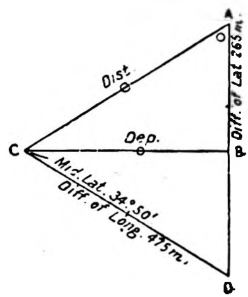
departure = the meridian distance in the middle latitude.

Given the Latitudes and Longitudes of two Places, to find the True Course and Distance from the one Place to the other

Required the true course and distance from Cape St. Vincent, in lat 37° 3′ N., and long. 9° 1′ W., to Funchal, Madeira, in lat. 32° 38′ N., and long. 16° 56′ W.

Lat. Cape St. Vincent	37° 3′ N.	—37° 3′ N.	Long. C. St. Vincent	9° 1′ W.
Lat. Funchal	32 38 N.	—32 38 N.	Long. Funchal	16 56 W.
Diff. lat.	4 25 S.	2) 69 41 Sum	D. long.	7 55 W.
	60	Mid. lat. 34 50		60
In miles	265			475′

BY CONSTRUCTION



Draw the line A D to represent the meridian of Cape St. Vincent; make the angle A D C equal to the co-mid. lat. 55° (Prob. XII. Geom.), and from D to C lay off the difference of longitude 475; from C draw the line B C perpendicular to A D (Prob. III. Geom.); make B A equal to the difference of latitude 265, and draw the line A C: then will B C represent the departure 390, the angle B A C the course 55$\frac{3}{4}$°, or 5 points nearly; and A C the distance 471 miles.

BY CALCULATION

To find the Course

$$\frac{\text{Dep.}}{\text{D. long.}} = \text{cos. mid. lat.}$$

$\therefore$ Dep. = d. long. × cos. mid. lat.

Also— $$\frac{\text{Dep.}}{\text{D. lat.}} = \text{tan. co.}$$

Therefore—

$$\frac{\text{D. long} \times \text{cos. mid. lat.}}{\text{Diff. lat.}} = \text{tan. co.}$$

L tan. course = log. d. long.

+ L cos. mid. lat. — log. d. lat.

D. long. 475′	log.	2·676694
Mid. lat. 34° 50′	cos.	9·914246
		12·590940
D. lat. 265 m.	log.	2·423246
Course S. 55° 48′ W.	tan.	10·167694

To find the Distance

$$\frac{\text{Dist.}}{\text{D. lat.}} = \text{sec. co.}$$

$\therefore$ Dist. = d. lat. × sec. co.

Log. dist. = log. d. lat.

+ L sec. co. — 10.

D. lat. 265 m.	log.	2·423246
Course 55° 48′	sec.	10·250199
Distance 471·5 m.	log.	2·673445

Because the difference of latitude is S. and the difference of longitude is W., hence the true course from Cape St. Vincent to Funchal is S. 55° 48′ W., or S.W. by W. nearly, and the distance 471·5 miles.

Also, in this case the middle latitude by Workman's Table is 34° 53′; hence the course is S. 55° 47′ W., and the distance 471·2 miles, as found by Mercator's Sailing.

By Inspection.—In Traverse Table look for the middle latitude 35° as a course, and for 475 the difference of longitude in a distance column, immediately opposite to which, in the difference of latitude column, will be found 389·1.

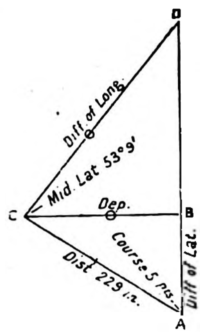
Then 265 (the difference of latitude) and 389·1 (the departure) being found nearly opposite each other in their respective columns, will give the course nearly 56°, and the distance 469 miles.

Given the Latitude left, the True Course, and the Distance, to find the Latitude and Longitude in

Example.—A ship from lat. 52° 6′ N., and long 35° 6′ W., sails N.W. by W. 229 miles. Required her present latitude and longitude.

BY CONSTRUCTION

Draw the line A D, and make the angle D A C equal to the course 5 points; lay off, from A to C, the distance 229, and draw the line C B perpendicular to the line A D; then will the departure C B measure 190, and the difference of latitude A B 127; hence the latitude in is 54° 13′, and the middle latitude 53° 9′. Now make the angle B C D equal to the middle latitude 53°, then will C D be the difference of longitude, measuring 317 miles.



BY CALCULATION

To find the Difference of Latitude

$$\frac{\text{D. lat.}}{\text{Dist.}} = \text{cos. co.}$$

$$\therefore \text{D. lat.} = \text{dist.} \times \text{cos. co.}$$

$$\text{Log. diff. lat.} = \text{log. dist.} + \text{L cos. co.} - 10$$

Distance 229	log.	2·359835
Course 5 pts.	cos.	9·744739
D. lat. 127·2 miles	log.	2·104574

To find the Latitude in

D. lat. 127 miles, or	2° 7′ N.
Latitude left	52 6 N.
Latitude in	54 13 N.
Sum of latitudes	2)106 19
Middle latitude	53 9

To find the Difference of Longitude

$$\frac{\text{D. long.}}{\text{Dep.}} = \text{sec. mid. lat.}$$

$$\therefore \text{D. long.} = \text{dep.} \times \text{sec. mid. lat.}$$

And— $$\frac{\text{Dep.}}{\text{Dist.}} = \text{sin. co.}$$

$$\text{Dep.} = \text{dist.} \times \text{sin. co.}$$

Therefore—

$$\text{D. long.} = \text{dist.} \times \text{sin. co.} \times \text{sec. mid. lat.}$$

$$\text{Log. d. long.} = \text{log. dist.} + \text{L sin. co.} + \text{L sec. mid. lat.} - 20.$$

Distance 229	log.	2·359835
Course 5 pts.	sin.	9·919846
Mid. lat. 53° 9′	sec.	10·222050
D. long. 317′·5	log.	2·501731

To find the Longitude in

Longitude left	35° 6′ W.
D. long. 317½ or	5 17½ W.
Longitude in	40 23½ W.

By Workman's Table the true mid. lat. is 53° 10½′, making diff. long. 317·8.

By Inspection.—By Traverse Table look for the course 5 points at the bottom of the page, over which, and opposite the distance 229 in its column, will be the difference of latitude 127·2, and departure 190·4, in their respective columns. Then—

Look for the middle latitude 53° as a course, and the departure 190·4 in the difference of latitude column, opposite the nearest to which, in the distance column, will be found 316, the difference of longitude.

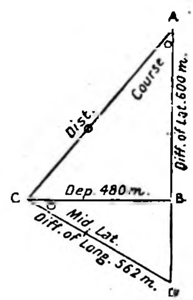
Ans. Lat. in, 54° 13′ N. Long. in, 40° 23½′ W.

*Given one Latitude, the Departure, and Difference of Longitude, to find the other Latitude, the True Course, and Distance**

Example.—A ship from lat. 36° 32′ N. sails between the south and west until she has made 480 miles of departure, and 562′ of difference of longitude. Required her present latitude, the true course, and distance run.

BY CONSTRUCTION

Having drawn the line A D, make B C perpendicular to it, and equal to the departure 480; draw C D equal to the difference of longitude 562 meeting A D in D; then the middle latitude B C D will measure 31½°; hence the latitude in is 26½°, and the difference of latitude 600: now make A B equal to 600, and join A C, which will measure the distance 787; and the course C A B will be 37½°.



* This case cannot be solved by Mercator's Sailing.

BY CALCULATION

To find the Middle Latitude

$$\frac{\text{Dep.}}{\text{D. long.}} = \text{cos. mid. lat.}$$

$\therefore$ Log. dep. + 10 — log. d. long.
= L cos. mid. lat.

Departure 480	log.	(+10)	12·681241
D. long. 562		log.	2·749736
Mid. lat. 31° 20′		cos.	9·931505

For Latitude in, and D. Lat.

Middle latitude	31° 20′
	2
Double mid. lat.	62 40
Latitude left	36 32 N.
Latitude in	26 8 N.
D. lat.	10 24 = 624m.

To find the Course

$$\frac{\text{Dep.}}{\text{D. lat.}} = \text{tan. co.}$$

$\therefore$ Log. dep. + 10 — log. d. lat.
= L tan. co.

Departure 480	log.	(+10)	12·681241
D. lat. 624		log.	2·795185
Course S. 37° 34′ W.		tan.	9·886056

To find the Distance

$$\frac{\text{Dist.}}{\text{Dep.}} = \text{cosec. co.}$$

$\therefore$ dist. = dep. × cosec. co.

Log. dist. = log. dep.
+ L cosec. co. — 10.

Departure 480	log.	2·681241
Course 37° 34′	cosec.	10·214895
Distance 787·3 miles	log.	2·896136

Ans. Lat. in, 26° 8′ N. Course S. 37° 34′ W. Dist. 787·3 miles.

The ordinary method of middle latitude sailing is but an *approximation*; the departure actually made is not *exactly* equal to the arc of the middle parallel, and the principles of parallel sailing require *that the departure should be reckoned in the parallel to which it truly belongs*. The parallel to which it truly belongs is clearly that parallel which will give the difference of longitude actually made; and hence—

$$\text{d. long.} = \text{dep.} \times \text{sec. mid. lat.}$$

should contain the *secant* of the *true* middle latitude—*i.e.*, the middle latitude in which the departure should be reckoned—instead of the *secant* of the *mean* middle latitude. Workman's Table (see Norie's Tables) converts the *mean* middle latitude into *true* middle latitude, in the sense required. But in most cases the results must be determined by computation, as the methods by *inspection* and the Traverse Tables do not admit of strict accuracy, since these tables only run to whole degrees, and one place of decimals, and consequently not much is gained by interpolation.

MERCATOR'S SAILING

MERCATOR'S SAILING is the art of finding on a plane surface the track of a ship upon any assigned course of the compass which shall be true in latitude, longitude, and distance sailed. This method is derived from the projection of Mercator's Chart, in which the degrees of longitude are everywhere equal, the degrees of latitude increase towards the poles, and the parallels, meridians, and rhumb-lines are represented by straight lines. (*See* Description of Mercator's Chart.)

Notwithstanding the inaccuracy of PLANE CHARTS, in which the degrees of longitude and latitude are everywhere equal, mariners were content to use these, and to do much computation on the basis of spherical trigonometry, until Gerard Mercator about the year 1556 published a chart in which the meridians are all parallel to each other, but in order to compensate for the expansion of the degrees of longitude he increased the distance between the parallels ; hence a chart thus constructed has obtained the name of MERCATOR'S CHART. It does not, however, appear that Mercator understood the true principles of this projection ; at least he never divulged the method on which he proceeded.

In the year 1599, Edward Wright, of Caius College, Cambridge, published the true principles of Mercator's Chart, in a work entitled "The Correction of certain Errors in Navigation," wherein he showed, by a Table of *Meridional Parts*, the length of the enlarged meridians in miles of the equator to every minute of latitude.

From the principles of Mercator's projection there exists the following relation—

$$\text{diff. long.} = \text{dep.} \times \text{sec. lat.} ;$$

or, in words, *the meridians being parallel, arcs of parallels of latitude are shown as equal to corresponding arcs of the equator, each being expanded in the ratio of the secant of its latitude.*

The sum of the lengths of all the small portions of the meridians thus increased, reckoned from the equator, and expressed in minutes of the equator, is tabulated in the Table of *Meridional Parts*. By such a Table a *Mercator's Chart* is constructed, and the various problems of *Mercator's sailing* are solved on the basis of right-angled plane trigonometry.

Let A B C (*see* the Fig., p 300) be a triangle, in which A is the course, A C the distance, A B the true (or proper) difference of latitude, and B C the departure ; then, corresponding to A B, the Table of Meridional Parts, or *increased latitudes*, gives A D as the meridional difference of latitude (mer. diff. lat.) ; and completing the right-angled triangle A D E, the difference of longitude is represented by D E.

The principles of plane sailing appertain to, and may be deduced from,

the triangle A B C; while from the triangle A D E is deduced the characteristic principle of Mercator's sailing.

To find the length of the expanded meridian between any two parallels of latitude, or, as it is called, the *meridional difference of latitude*, the same rules are to be observed as in finding the true (or proper) difference of latitude; that is, if the latitudes are of the same name, take the difference of their corresponding *meridional parts*, but if the latitudes are of contrary names, take the sum of those parts for the meridional difference of latitude.

When the course is nearly east or west, that is, when there is a large difference of longitude but only a small difference of latitude, Mercator's sailing is not so suitable as middle latitude sailing.

The same examples are introduced as in middle latitude sailing, for comparison of the two methods.

It is recommended, when finding the compass course, to convert the true course into a "New Pattern Compass" course by the Table of "Compass Equivalents," and then add all Westerly variation or deviation and subtract all Easterly variation or deviation. After applying the variation and deviation, convert it into its proper quadrant.

Example.—True course S. 70° E., variation 20° W., and deviation 15° E. Find the compass course.

$$\begin{array}{rl} \text{S. } 70^\circ \text{ E.} = & 110^\circ \\ \text{Var. W.} + & \underline{20} \\ & 130 \\ \text{Dev. E.} - & \underline{15} \\ \text{Compass course} & 115^\circ \text{ or S. } 65^\circ \text{ E.} \end{array}$$

Given the Latitudes and Longitudes of Two Places, to find the True Course and Distance from the one Place to the Other

1. *For the true Difference of Latitude.*—Latitudes of the same name, take their difference; latitudes of different names, take their sum. The result will be the true difference of latitude; then multiply the degrees by 60, and add in the miles.

2. *For the Meridional Difference of Latitude.*—From the Tables take out the meridional parts corresponding to the two latitudes; take their difference for latitudes of the same name; take their sum for latitudes of different names. The result will be the meridional difference of latitude.

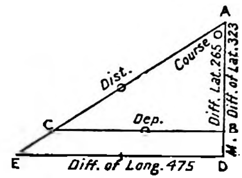
3. *For the Difference of Longitude.*—Longitudes of same name, take their difference; longitudes of different names, take their sum, and if this sum exceeds 180° subtract it from 360°. The result will be the difference of longitude; then multiply the degrees by 60, and add in the miles.

Required the true course and distance from Cape St. Vincent, in lat. 37° 3′ N. and long. 9° 1′ W., to Funchal, Madeira, in lat. 32° 38′ N. and long. 16° 56′ W.

Lat. C. St. Vincent	37° 3′ N.	Mer. parts	2396·39	Long. C. St. Vin.	9° 1′ W.
Lat. Funchal	32 38 N.	Mer. parts	2073·35	Long. Funchal	16 56 W.
Diff. lat. ...	4 25 S.	Mer. diff. lat.	323·04	Diff. long. ..	7 55 W.
	60				60
In miles	265				475′

BY CONSTRUCTION

Draw the line A D to represent the meridian of Cape St. Vincent, upon which lay off the meridional difference of latitude 323·04; from D erect the perpendicular D E (Prob. II. or XII. Geom.); make it equal to the difference of longitude 475, and draw the line A E; from A to B lay off the true difference of latitude 265, and through B draw B C parallel to D E; then will the angle E A D be the course, measuring 56°, or 5 points nearly, and A C the distance, 471 miles.



BY CALCULATION

To find the Course

$$\frac{\text{Diff. long.}}{\text{Mer. diff. lat.}} = \text{tan. co.}$$

Log. diff. long. + 10 — log. mer. diff. lat. = L tan. co.

Diff. long. 475′ log. (+10)	12·676694
Mer. diff. lat. 323′·04 log.	2·509256
Course S. 55° 46′ 52″ W. L. tan.	10·167438

To find the Distance

$$\frac{\text{Dist.}}{\text{Diff. lat.}} = \text{sec. co.} \therefore \text{dist.} = \text{diff. lat.} \times \text{sec. co.}$$

Log. dist. = log. diff. lat. + L sec. co. — 10

True diff. lat. 265 m. log.	2·423246
Course 55° 46′ 52″ L. sec.	10·249989
Distance 471·23 miles log.	2·673235

Because the difference of latitude is S. and the difference of longitude is W., hence the true course from Cape St. Vincent to Funchal is S. 55° 47′ W. or S.W. by W. nearly, and the distance 471 miles.

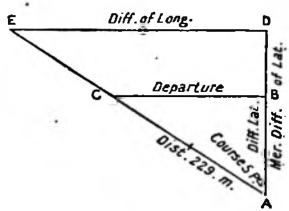
By Inspection.—Seek in the Traverse Table for the meridional difference of latitude 323, and the difference of longitude 475, till they are found together in the latitude and departure columns: the nearest to these are 320·4 and 475 in the page with 56° at the bottom, wh ch is the course; over that course, and opposite the true difference of latitude 265 in its column, is found in the distance column, 474 miles.

Given the Latitude left, the True Course and the Distance, to find the Latitude and Longitude In

A ship from lat. 52° 6′ N., and long. 35° 6′ W., sails N.W. by W. 229 miles. Required her present latitude and longitude.

BY CONSTRUCTION

Draw the line A D, and make the angle D A E equal to the course 5 points; from A to C lay off the distance 229, and from C draw C B perpendicular to A D; then will A B measure the difference of latitude 127; hence the latitude come to is 54° 13′, and the meridional difference of latitude 211·86; make A D equal to 211·86, and draw D E parallel to B C; then will the difference of longitude D E measure 317 miles.



BY CALCULATION

For the Difference of Latitude

$$\frac{\text{Diff. lat.}}{\text{Dist.}} = \text{cos. co.}$$

∴ Diff. lat. = dist. × cos. co.
Log. diff. lat. = log. dist. + L cos. co. — 10

Distance 229 m.	log.	2·359835
Course 5 pts. .	L. cos.	9·744739
Diff. lat. 127·2 miles	log.	2·104574

To find the Latitude In

Lat. left	52° 6′ N.	Mer. pts.	3674·95
Diff. lat.	2 7 N.		
Lat. in	54 13 N.	Mer. pts.	3886·81
		Mer. diff. lat.	211·86

For the Difference of Longitude

$$\frac{\text{Diff. long.}}{\text{Mer. diff. lat.}} = \text{tan. co.}$$

∴ Diff. long. = mer. diff. lat. × tan. co.
Log. diff. long. = log. mer. diff. lat. + L tan. co. — 10

Mer. diff. lat. 211·86 m.	log.	2·326049
Course 5 pts.	L. tan.	10·175107
Diff. long. 317′·07	log.	2·501156

To find the Longitude In

Longitude left	35° 6′ W.
Diff. long. 317′ or	5 17 W.
Longitude in	40 23 W.

By Inspection.—In Traverse Table over the course 5 points, and opposite the distance 229, is the difference of latitude 127·2: hence the latitude arrived at is 54° 13′; then, with the same course and half the meridional difference of latitude 106, in a latitude column, will be found 158·8 in a departure column, which, multiplied by 2, gives the difference of longitude 317·6 miles.

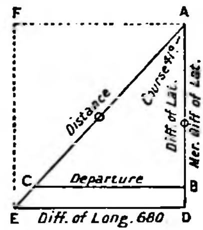
Ans. Lat. in 54° 13′ N.; Long. in 40° 23′ W.

Given the Latitude left, the True Course, and Difference of Longitude, to find the Distance and Latitude In *

A ship from lat. 34° 29′ N. sails S. 41° W. till her difference of longitude is 680 miles. Required her present latitude and distance sailed.

BY CONSTRUCTION

Draw A D, and make A F perpendicular to it, and equal to the difference of longitude 680; draw F E parallel to A D, and A E making an angle with A D equal to the course 41°, meeting F E in E, and E D parallel to F A; then A D will be the meridional difference of latitude equal to 782·3; which, subtracted from the meridional parts of the latitude left, gives the meridional parts of the latitude in; hence the lat. in is 23° 6′, and the difference of latitude 683. Make A B equal to 683, and draw B C parallel to E D; then A C will be the distance, measuring 905



BY CALCULATION

To find the Mer. Diff. Lat.

$$\frac{\text{Mer. diff. lat.}}{\text{Diff. long.}} = \text{cot. co.}$$

$\therefore$ Mer. diff. lat. = diff. long. $\times$ cot. co.

Log. mer. diff. lat. = log. diff. long. + L cot. co. — 10.

Diff. long. 680	log.	2·832509
Course 41°	L. cot.	10·060837
Mer. diff. lat. 782·3	log.	2·893346

To find the Lat. in.

Lat. left 34° 29′ N.	Mer. pts.	2206·56 N.
	Mer. diff. lat.	782·3 S.
Lat. in 23 5·25	Mer. pts.	1424·26
11 23·75		
60		
Diff. lat. 683·75 m.		

To find the Distance

$$\frac{\text{Dist.}}{\text{Diff. lat.}} = \text{sec. co.} \therefore \text{dist.} = \text{diff. lat.} \times \text{sec. co.}$$

Log. dist. = log. diff. lat. + L. sec. co. — 10

Diff. lat. 683·75	log.	2·834898
Course 41°	L. sec.	10·122220
Distance 905·98 m.	log.	2·957118

Examples for Practice in Middle Latitude and Mercator's Sailing

1. Required the true course and distance from the Cape of Good Hope, in lat. 34° 22′ S., and long. 18° 24′ E., to the Island of St. Helena, in lat. 15° 55′ S., and long. 5° 45′ W.

Ans. By middle latitude sailing the true course is N. 49° 51′ W., and distance 1717 miles.

* This example cannot be solved by middle latitude sailing.

By middle latitude sailing, using Workman's Table (in Norie's Tables), true course is N. 49° 39½′ W., and distance 1710 miles.

By Mercator's sailing the true course is N. 49° 39′ 40″ W., and distance 1710·17 miles.

2. Required on Mercator's principle the true bearing and distance of Pernambuco, in lat. 8° 4′ S., and long. 34° 53′ W., from Cape Verd, in lat. 14° 45′ N., and long. 17° 32′ W.

Ans. The true bearing is S. 37° 1′ 6″ W., and distance 1714·6 miles.

3. Required on Mercator's principle the true course and distance from Cape Sierra Leone, in lat. 9° 30′ N., and long. 13° 18′ W., to Cape St. Roque, in lat. 5° 28′ S., and long. 35° 17′ W. If the error of the compass is 10° E., required the compass course.

Ans. The true course is S. 55° 39′ 34″ W., and distance 1591·9 miles. Compass course S. 45° 40′ W.

4. Required by Mercator's sailing the true course and distance from Cape Palmas, in lat. 4° 24′ N., and long. 7° 46′ W., to St. Paul de Loando, in lat. 8° 48′ S., and long. 13° 8′ E. Required also the compass course, the variation being 20° W., and the deviation 17° W.

Ans. The true course is S. 57° 38′ 51″ E., and distance 1480 miles. Compass course S. 20° 38′ 51″ E.

5. Required the true course and distance from lat. 26° 38′ S. and long. 15° 08′ E. to lat. 44° 26′ N. and long. 63° 33′ W. on Mercator's principle; also the compass course, assuming the variation to be 12° W. and deviation of the compass 4° W.

Ans. True course, N. 45° 29′ 22″ W.; compass course, N. 29° 29′ 22″ W.; distance 6082·4 miles.

6. Required the course and distance from lat. 20° 20′ S. and long. 167° W. to lat. 37° 40′ S. and long. 170° 30′ E. on Mercator's principle; also the compass course, assuming the variation to be 10° E., and deviation of the compass 6° W.

Ans. True course, S. 48° 27′ 2″ W.; compass course, S. 44° 27′ 2″ W.; distance 1568 miles.

7. Required the true course and distance from lat. 44° 26′ N. and long. 63° 33′ W. to lat. 26° 38′ S. and long. 15° 08′ E. on Mercator's principle; also the compass course, assuming the variation to be 9° W. and deviation of the compass 7° E.

Ans. True course, S. 45° 29′ 22″ E.; compass course, S. 43° 29′ 22″ E.; distance 6082·4 miles.

8. Suppose a ship from lat. 9° 10′ N., and long. 19° 32′ W., sails in the south-east quarter till she has made 415 miles of departure, and is by observation in lat. 2° 19′ S.; required her true course steered, distance run, and longitude in.

Ans. By Mercator's sailing her true course steered is S. 31° 4′ E., distance run 8043, and long. in 12° 35′ 36″ W.

9. A ship from lat. 46° 35′ N., and long. 176° 42′ W. sails true N.W. by W. ½ W. till she arrives in lat. 51° 18′ N. Required the distance run, and longitude in.

Ans. By middle latitude sailing her distance run is 600·3 miles, and long. in 169° 52′ E.; or, using Workman's Table, 169° 51′ E.

By Mercator's sailing her distance run is 600·3 miles, and long. in 169° 50′ E.

10. Required the course and distance from lat. 20° 20′ S., and long. 20° 30′ W. to lat. 37° 40′ S. and long. 30° 20′ E. on Mercator's principle; also the compass course, supposing the variation to be 20° W. and the deviation of the compass 10° W.

Ans. True course, S. 68° 34′ 48″ E.; compass course, S. 38° 34′ 48″ E.; distance 2847·7 miles.

11. Find the true course and distance from the Cape of G. Hope in lat. 34° 22′ S., long. 18° 24′ E.; to the Falkland Is. in lat. 52° 21′ S., long. 59° 18′ W. Find also the compass course, the variation being 30° W., deviation 18° E.

Ans. By middle latitude sailing, using Workman's Table, true course S. 72° 9′ 6″ W., and distance 3520·4 miles, the same as by Mercator's sailing Compass course S. 84° 9′ 6″ W.

COMPOUND COURSES

To find the Difference of Longitude made good upon Compound Courses, by Middle Latitude and Mercator's Sailing

In the preceding cases, both of middle latitude and Mercator's sailing, the ship has been supposed to sail on a direct course; but when she makes a compound course, the several courses, after having been corrected for leeway, and the deviation and variation of the compass, are to be reduced to a single course, as in traverse sailing, and then the difference of longitude may be found either by middle latitude or Mercator's sailing, as shown below.

Suppose a ship from lat. 52° 36′ N., and long. 21° 45′ W., has made the following *true* courses and distances, viz., N.E. 36 miles; N. by W. 14 miles; N.E. by E. ½ E. 58 miles; N. by E. 42 miles; and E.N.E. 29 miles. Required her present latitude and longitude.

BY TRAVERSE TABLE

Courses.	Distance.	Diff. of Latitude.		Departure.	
		N.	S.	E.	W.
N. 4 pt. E.	36	25·5		25·5	
N. 1 ,, W.	14	13·7			2·7
N. 5½ ,, E.	58	27·3		51·2	
N. 1 ,, E.	42	41·2		8·2	
N. 6 ,, E.	29	11·1		26·8	
		118·8	diff. lat.	111·7	2·7
				2·7	
				109·0 dep.	

BY INSPECTION, USING TRAVERSE TABLE AND MERIDIONAL PARTS

Diff. lat.	118·8 N.	Course	N. $42\frac{1}{2}°$ E.
Dep.	109·0 E.	Distance	161 miles.

Latitude left	52° 36′ N. ..	52° 36′ N.	Mer. parts	3724·06
Diff. lat. 119 = ...	1 59 N.			
Latitude in	54 35 N. ..	54° 35′ N.	Mer. parts	3924·61
		2)107 11	Mer. diff. lat. ..	200·55
	Mid. lat. ..	53 $35\frac{1}{2}$		

BY MIDDLE SAILING

Then—

Mid. lat. (*as a course*) .... $53\frac{1}{2}°$ }
Departure (*in lat. column*) ..... 109 } give diff. long. (in *dist.* column) 183′

Longitude left	21° 45′ W.
Diff. long. 183 = ...	3 3 E.
Longitude in	18 42 W.

BY MERCATOR'S SAILING

Course N. $42\frac{1}{2}°$ E. (*as a course*) }
Mer. diff. lat. 200·55 (*in lat. column*) } give diff. long. (in *depart.* column) 183′·85

Longitude left	21° 45′ W.
Diff. long. 184 = ...	3 4 E.
Longitude in	18 41 W.

The above method is that generally practised at sea in estimating the difference of longitude made good in a day's run, being considered sufficiently exact for the distance sailed by a ship in that time.

Example.—A ship from Table Bay (C. Good Hope), in lat. 33° 54′ S., long. 18° 24′ E., makes good the following true courses, viz.: (1) S.W. by W. 47 miles; (2) S.W. ½ S. 80 miles; (3) W. ¾ S. 87 miles; (4) S.W. by S. 140 miles; (5) S.W. by S. ¼ S. 118 miles; (6) S.E. 124 miles; (7) W. by N. 130 miles; and (8) W.S.W. 140 miles. Required the latitude and longitude arrived at; and also the true course and distance thence to Port Stanley (Falkland Is.) in lat. 51° 41′ S., long. 57° 51′ W.

Ans. Diff. lat. 429·9 and dep. 484 give course S. 48° 23¼′ W. and dist. 647·4 miles. Lat. in is 41° 4′ S.; long. in (by mid. lat., using Workman's Table) is 8° 13′ E., and the same by Mercator's sailing.

Thence the course is S. 76° 50′ W. and dist. 2797 miles, by both sailings, using Workman's Table in mid. lat. sailing; neglecting this Table, the course by mid. lat. is S. 76° 53½′ W., and dist. 2809 miles.

CURRENT SAILING

A current is a running body of water flowing in some definite direction through the midst of the general waters of the ocean and its various seas; the effect of the progressive motion of the water is to cause all floating bodies to move more or less in the direction *towards* which the stream of current is directed; hence the *setting of a current* is that point of the compass *towards which the water runs;* and its *drift* is the *rate at which it runs* per hour, or in any other given time.

A not unusual method of ascertaining the set and drift of an unknown current is to take a boat, in calm weather, a small distance from the ship, being provided with a half-minute glass, a log, a heavy weight, or kedge, and a small boat-compass; then let down the weight by a rope fastened to the boat's stem, to the depth of about 100 fathoms, by which the boat will remain nearly as steady as at anchor; then the log being hove, its bearing will be the setting of the current, and the number of knots run out in half a minute will be its drift per hour. This method is, however, very uncertain, owing to the effect of *submarine currents.*

Current Sailing is the method of determining the true course and distance of a ship, when the ship's motion is affected by and combined with that of a current.

The current being known, it remains to apply its effect on a ship's way, which will depend on the direction and velocity of both, with regard to each other. If a ship sail in the direction of the current, it is evident that the velocity of the current must be added to that of the vessel; if her course be directly against the current, their difference will be the ship's true velocity.

Example.—Ship's course N.E., making by log 9 knots per hour; current sets N.E. 2 miles per hour; hence, ship makes good 11 miles per hour = (9 + 2).

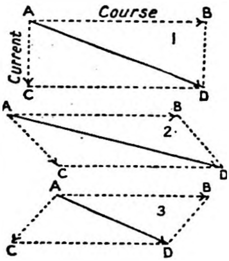
Example.—Ship's course E., making by log 8 knots per hour; current sets W. 3 miles per hour; hence, ship makes good 5 miles per hour = (8 — 3).

If the current runs stronger than the ship's rate, the ship, unable to make headway, is drifted to leeward by the current.

If a ship's course be oblique to the current, her direction by the compass will be compounded with that of the current; that is, she will proceed in the diagonal of the parallelogram formed according to the two lines of direction, and will describe or pass over that diagonal in the same time in which she would have described either of the sides by the separate velocities.

For, in diagrams 1, 2 and 3, let A B C D be a parallelogram, the diagonal of which is A D, then, by the *composition of forces*, if the wind alone would drive the ship from A to B in the same time that the current alone would drive it from A to C, then as the wind neither helps nor hinders the ship from coming towards the line C D, the current will bring it there in the

same time as if the wind did not act; and as the current neither helps nor hinders the ship from coming towards the line B D, the wind will bring it there in the same time as if the current did not act. Therefore, the ship must, at the end of that time, be found in both those lines, that is, in their meeting at D: consequently, the ship must have passed from A to D in the diagonal line A D. Hence the ship's true distance will be the third side of a triangle, whereof the other sides are the distance by the log and the drift of the current, and the true course will be the angle between that third side and the meridian.



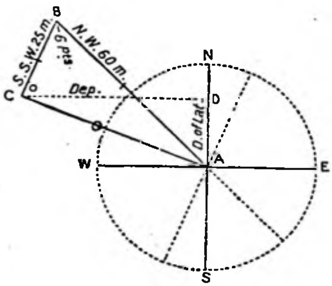
But the usual method of solving the problem of the combined effect of the *set* and *drift* of the *current* with the *course* and *distance* made by the *ship* is *by inspection*, through the *traverse table*, as will be shown in the sequel.

Given the Course steered, and Distance run by Log, also the Set and Rate of a Current, to find the Course and Distance made good

Example I.—A ship sails N.W. 60 miles, in a current that sets S.S.W. 25 miles in the same time. Required her course and distance made good.

NOTE.—The course and set must be both magnetic, or both true.

BY CONSTRUCTION



Having drawn the compass, set off 4 points from the north towards the west, and draw the N.W. line A B, which make equal to 60 miles, the distance run by the log; through B draw B C parallel to the S.S.W. and N.N.E. line, and equal to 25 miles, the set and drift of the current: now A C being joined will be the true distance, measuring 55·48 miles, and the angle N A C the true course N. 69½° W.

BY CALCULATION

In the plane triangle A B C are given the side A B 60′, the side B C 25′, and the included angle A B C (the difference between the direction of B A which is S.E. and the direction of B C which is S.S.W.) 6 points, or 67° 30′. To find the angle B A C and the side A C—

Side A B or c ..	60′		180° 00′
Side B C or a ..	25	∠ B	67 30
Sum or $c + a$ ..	85	Sum of ∠s A and C	112 30 = C + A
Diff. or $c - a$ ..	35	Half sum	56 15 = ½ (C + A)

For the Course

L tan. $\frac{1}{2}$ (C—A)
= log. $(c-a)$ + L. tan. $\frac{1}{2}$ (C + A)— log. $(c+a)$.

$(c-a)$	35	log.	1·544068
$\frac{1}{2}$ (C+A)	56° 15′	tan.	10·175107
			11·719175
$(c+a)$	85′....	log.	1·929419
$\frac{1}{2}$ (C — A)	31° 38$\frac{1}{2}$′	tan.	9·789756
$\frac{1}{2}$ (C + A)	56 15		
Diff. = ∠ BAC	24 36$\frac{1}{2}$		
∠ N A B	45		
Sum = ∠ NAC	69 36$\frac{1}{2}$ = Course.		

For the Distance

$$\frac{b}{a} = \frac{\sin. B}{\sin. A}, \therefore b = a \times \frac{\sin. B}{\sin. A}$$

log. b = log. a + L sin. B — L sin. A

a 25	log.	1·397940
∠ B 67° 30′	sin.	9·965615
		11·363555
∠ A 24° 36$\frac{1}{2}$′	sin.	9·619524
b or side A C 55′·47	log.	1·744031

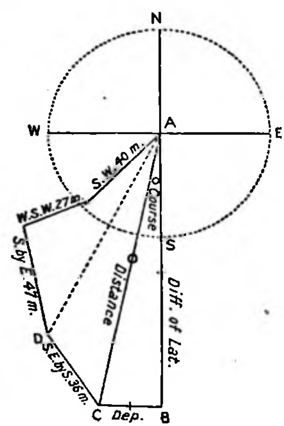
Hence the course made good, N A C, is N. 69° 36$\frac{1}{2}$′ W. or W.N.W. $\frac{1}{4}$ W. nearly, and the distance A C 55$\frac{1}{2}$ miles.

BY INSPECTION

But the most usual, and the readiest way of allowing for the effects of a current, is to consider the setting and drift as a course and distance, and enter it accordingly in a Traverse Table; then the whole difference of latitude and departure will give the true course and distance. By this method the preceding example is thus worked—

Course N.W.	60 m. gives D. Lat.	42·4 N.	....Dep.	42·4 W.
Current S.S.W.	25 m. „ „	23·1 S.	.... „	9·6
		19·3 N.	....	52·0 W.

Then, the difference of latitude A D 19·3, and the departure C D 52, give the course D A C = N. 69° 38′ W., and the distance A C = 55·46, by plane sailing; or, preferentially (because more simply and sufficiently accurate), by Traverse Table course = N. 69$\frac{1}{2}$° W., and distance = 55·5 miles.



Example II.—Suppose a ship in 24 hours sails as follows—S.W. 40 miles, W.S.W. 27 miles, and S. by E. 47 miles, being throughout the time in a current setting S.E. by S., at the rate of 1$\frac{1}{2}$ miles per hour. Required her direct course, and distance made good.

BY CONSTRUCTION

Draw the compass, and lay off the several courses and distances, as in traverse sailing; then will D represent the place of the ship by the log. From D draw D C parallel to the S.E. by S. line, and equal to 36 miles, for the set and drift of the current in 24 hours; then will C be the ship's true place, the angle B A C the true course, measuring 11° 50′, A C the distance 117, A B the difference of latitude 114·6, and B C the departure 24 miles.

BY CALCULATION

This process would be too tedious; the solution would be performed

BY INSPECTION

Working the question by inspection, the set and drift of the current is taken into account in the Traverse Table after the courses and distances per log, and in the same manner.

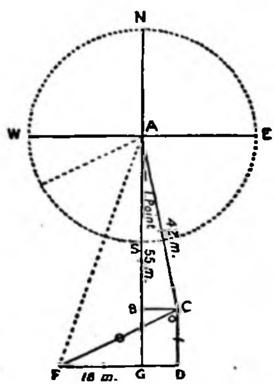
Courses	Distance	Difference of Lat.		Departure	
		N.	S.	E.	W.
S.W.	40		28·3		28·3
W.S.W.	27		10·3		24·9
S. by E.	47		46·1	9·2	
S.E. by S. (Current)	36		29·9	20·0	
		Diff. lat.	114·6	29·2	53·2
					29·2
				Dep.	24·0

Then, by *inspection*, in Traverse Table difference of latitude 114·4, and departure 24·3 (the nearest), give course made good S. 12° W., and, distance 117 miles.

Or by plane sailing, difference of latitude A B 114·6, and departure B C 24·0, give the true course C A B = S. 11° 49¾′ W., or about S. by W., and the distance A C = 117·1 miles.

Given the Course and Distance by Dead Reckoning, or by Log, and the True Position of the Ship (or the Position by Observation), to find the Set and Drift of the Current

Example I.—A ship sailing in a current has by dead reckoning run S. by E. 42 miles; but by observations finds she has made 55 miles of southing, and 18 miles of westing. Required the set and drift of the current.



BY CONSTRUCTION

Having drawn the compass, set off 1 point from the south towards the east, and draw the S. by E. line A C, which make equal to 42 miles; through C draw the line C B parallel to the east and west, then will A B be the difference of latitude 41·19, and B C the departure 8·19, made by the log.

From A to G lay off 55, the difference of latitude, by observation, and through G draw a line parallel to the east and west; from G towards F lay off 18, the true departure, and draw the line C F. From C

draw a line parallel to A G, meeting the line F G produced in D; then the angle F C D will be the set of the current, measuring S. 62° W.; and the side C F the drift, 29·6 miles.

BY CALCULATION

In the right-angled triangle A B C are given the course B A C = 1 point, and the distance A C 42 miles. To find the difference of latitude A B 41·19, and the departure B C 8·19—

Subtract A B 41·19 S., the difference of latitude by the log, from A G 55 S., the difference of latitude by observation; and the remainder B G 13·81 will be what the ship is to the southward of her reckoning.

To the departure B C (equal to G D) 8·19 E., add the departure F G 18 W., and the sum F D 26·19 will be what the ship is to the westward of her account.

The difference of latitude C D (equal to B G) 13·81, and the departure D F 26·19, give the angle or course S. 62° 12′ W., and the side C F or distance 29·59 for the set and drift of the current.

BY INSPECTION (*General Rule*)

1. Under the difference of latitude and departure by dead reckoning write the difference of latitude and departure by observation, each with its proper name.

2. Find the *difference* of the two differences of latitude, and the two departures, as follows—

a. Take the difference of the two differences of latitude if both have the same name; if of different names take their sum. This sum or difference will be N. when the difference of latitude by observation is *north* of that by dead reckoning; if otherwise S.

b. Take the difference of the two departures if both have the same name; if of different names take their sum. This sum or difference will be E. when the departure (or longitude) by observation is *east* of that by dead reckoning; if otherwise W. If the difference of longitude is taken you must turn it into departure.

3. Enter the Traverse Table with these differences, as difference of latitude and departure, each in its proper column, and take out the corresponding course and distance, which will be the set and drift of the current.

Thus, *Ex*. p. 309; S. by E. 42 m. = 41·2 S. and 8·2 E. in Trav. Tab.

By D.R., diff. lat.	41′·2 S.	....	Dep.	8′·2 E.
By Obs.,	,, 55 ·0 S.	....	,,	18 ·0 W.
Diff. of diff. lats.	13 ·8 S.		Sum of deps.	26 ·2 W.

Then, difference of latitude 13′·8 S. and departure 26′·2 W. give course S. 62½° W., and distance 29·5 m., which correspond to the *set* (or direction) and *drift* (or rate) of the current in the interval; S. and W. because observation is S. and W. of the dead reckoning.

Example II.—A ship from latitude 39° 50′ N., longitude 149° 20′ E. sails N. 46° E. 203 miles, by dead reckoning, and then by observation is in latitude 42° 26′ N., longitude 153° 8′ E. Required the set and drift of the current.

Course N. 46° E. and dist. 203 m. = 141′ N., and 146′ E.
Hence, by D.R. lat. is 42° 11′ N.; and long. is 152° 33′·5 E.

Lat. left	39° 50′ N.	Long. left	149° 20′ E.
Lat. in	42 26	Long. in	153 8
True diff. lat.	2 36 = 156′ N.	True diff. long.	3 48 = 228′ E.
	1 18		
Mid. lat.	41 · 8		

Mid. lat. 41° as course, and 228 in dist. col., give 172′ dep. (in D. lat. col.).

By D.R., diff. lat.	141′ N.	Dep.	146′ E.
By Obs., „	156	„	172
Diff. of diff. lats.	15 N.	Diff. of deps.	26 E.

Then, difference of latitude 15′ N. and departure 26′ E. give N. 60° E. as *Set*, and 30 m. as *drift* of current in the given time.

Given the Course to (or Bearing of) a Port, and the Set and Drift of the Current, to shape a Course to counteract the Effect of the Current, and so as to keep the Port on the same Bearing

General Method *by Inspection and the Traverse Table.*—(1.) Find the angle between the direction of the ship's course towards her port and the set of the current; thus, if they are in *adjacent* quadrants of the compass take their *sum;* but their *difference* if they are in the *same* or *opposite* quadrants. Enter the Traverse Tables with this *angle* (or its supplement, when more than 8 points or 90°) as a *course* and the *rate* of the current in *distance* column; take out the corresponding difference of latitude, and departure.

(2.) Then enter the Traverse Tables with the ship's *run* in the *distance* column, and the above departure in departure column: take out the corresponding difference of latitude, and the course. This course is the *angle* by which the ship's course towards her port must be corrected (*i.e.*, augmented or diminished) according to whether the current carries the ship towards, or away from her port; and the course thus corrected is the required *course to make good*, allowance being made for deviation and variation of the compass.

(3.) Lastly, the sum or difference of the two differences of latitude already found, according to whether the set of the current favours, or is adverse to the ship's progress, will be the distance that the ship makes good towards her port.

Note.—It may sometimes be convenient, when entering the Traverse Table, to multiply the ship's rate and also the drift of the current by 5 or 10.

If a ship be making leeway as well as being set out of her course by the current, find the true or magnetic course after allowing for the current, then allow the leeway to windward of that course before finding the compass course.

Example 1.—Given the bearing of the port S.W.; the current setting S.S.E. 2 miles per hour; ship's rate of sailing 11 knots per hour. Shape

the course so as to keep the port on the same bearing, and find the ship's rate of approach to her port in 10 hours, the current remaining the same.

Angle between S.W. and S.S.E. = 6 points.
In 10 hours ship makes 110 miles, and current sets 20 miles.
6 points and 20 m. = 7·7 D. lat., and 18·5 dep.
(Dist.) 110 m. and dep. 18·5 = 108·4 D. lat., and course (angle) $9\frac{1}{2}°$.

Then, since the set of current is S.S.E., and the bearing of the port is S.W., the angle must be allowed to the *right*; $45° + 9\frac{1}{2}° =$ S. $54\frac{1}{2}°$ W., the required course to preserve the bearing.

And, 108·4 + 7·7 = 116·1 miles, the ship's rate of approach towards her port in 10 hours, the drift of current being partly with the ship.

Example 2.—Suppose the current to set N.N.W., and all the other *data* to remain as in *Ex.* 1, above.

Then the resolved elements would still be the same as those already given; but for the course we should have—

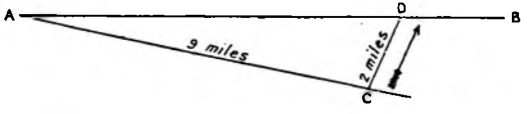
$45° - 9\frac{1}{2}° =$ S. $35\frac{1}{2}°$ W., since the set of the current is N.N.W., and the angle must be allowed to the *left* to preserve the S.W. bearing of the port.

Also, 108·4 — 7·7 = 100·7 miles, is the ship's rate of approach towards her port in 10 hours, since the drift of current is against the ship.

Example 3.—The port bears East, and the current sets N.N.E. at the rate of 2 miles per hour, the ship making 9 knots per hour. Shape a course so as to keep the port on the same bearing, and find the hourly rate of the ship towards her port.

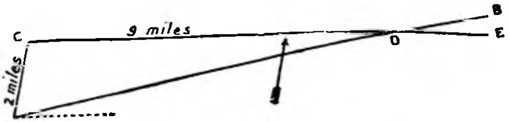
BY CONSTRUCTION

Method 1. From any point C lay off the set and drift of the current towards D, in this case N.N.E. 2 miles; on D lay off the bearing of the port from A to B, in this case East; from C towards the line of bearing at



A lay off the ship's rate, in this case 9 miles; then D A C is the angle (12°) to the *right* of the bearing of the port, on which to put the ship; hence the course is S. 78° E.; and the ship's hourly rate A D is $9\frac{1}{2}$ knots.

Method 2. Set off from A to B the bearing of the port, in this case East; from A (the ship's position) lay off A C as the set and drift of the current, in



this case N.N.E. 2 miles; from C in the direction towards the line of bearing A B lay off C D 9 miles (the ship's rate); extend the line C D to E, then

B D E is the angle (12°) to the right of the bearing of the port, on which to put the ship, in this case S. 78° E.

The *dotted* line extending from A (which is parallel to the line C D), also shows the direction on which to place the ship's head to counteract the effect of the current.

BY CALCULATION

$$\frac{\sin. x}{\sin. 6 \text{ pts.}} = \frac{2'}{9'}, \sin. x = \frac{2' \sin. 6 \text{ pts.}}{9'},$$

whence $x = 11° 51'$, the change of course to be applied to East in a direction *opposite* to the trend of the current, making course S. 78° 9′ E.

$$\frac{d}{9'} = \frac{\sin. 100° 39'}{\sin. 6 \text{ pts.}}, d = \frac{9' \sin. 100° 39'}{\sin. 6 \text{ pts.}},$$

whence $d = 9{\cdot}57$ m., the hourly distance made good towards the port.

By Inspection.—N.N.E. to East is 6 points; then 6 points and 2 miles give D. lat. 0·8 and dep. 1·9.

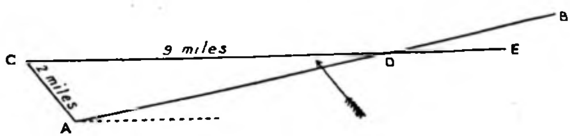
(Dist.) 9 m. and dep. 1·9 = 8·8 D. lat. and course 12° (or angle).

Then 12° to *right* of East is S. 78° E. = course to shape; and 0·8 + 8·8 = 9·6, the ship's hourly rate of approach to her port, the drift of current being partly with the ship.

Example 4.—Suppose the port to bear N. 75° E., the current setting N. 40° W. 2 miles per hour, and the ship's rate of sailing to be 9 knots per hour. Shape a course such as to keep the port on the same bearing, and give the hourly rate of the ship towards her port.

BY CONSTRUCTION

Lay off the bearing of the port from A to B, in this case N. 75° E.; from A towards C lay off the set and drift of the current, N. 40° W. 2 miles; from



C towards the line A B set off 9 miles (the ship's rate), such that it shall touch the line of the port's bearing at D; join C and D, and prolong the line to E; then will B D E be the angle (11½°) which the ship must steer to the *right* of A B, to make good the course to her port, in this case N. 86½° E.; the distance A D (8 miles) is ship's rate per hour, as the set of the current is *against* the ship.

The *dotted* line extending from A, and parallel to the line C D, also shows the direction of the altered course.

BY CALCULATION

The angle between the direction of the bearing and that of the current ($75° + 40°$) = $115°$; hence—

$$\frac{\sin. x}{\sin. 115°} = \frac{2'}{9'}, \sin. x = \frac{2' \sin 115°}{9'},$$

whence $x = 11° 37'$, the change of bearing to be applied to N. 75° E., in a direction *opposite* to that towards which the current is setting the ship, and course is therefore N. 86° 37′ E.

$$\frac{d}{9} = \frac{\sin. 53° 23'}{\sin. 115°}, \ d = \frac{9' \sin. 53° 23'}{\sin. 115°},$$

whence $d = 7{\cdot}97$ m., the ship's rate per hour towards her port.

By Inspection.—N. 75° E. and N. 40° W. make an angle 115°, the supplement of which is 65°.

65° and 2 m. = ·85 D. lat. and 1′·81 Dep.

(Dist.) 9 m. and dep. 1·81 = 8·81 D. lat. and course $11\frac{1}{2}°$, the angle of change of bearing.

Then $75° + 11\frac{1}{2}°$ = N. $86\frac{1}{2}°$ E., the required course to steer.

And 8′·81 — ′·85 = 7′·96, the ship's rate of approach towards her port, since the drift of the current is *against* the ship.

NOTE.—A ship's course and distance will invariably be affected by what is called the *set* (direction) and *drift* (rate) of any current in which she may be sailing. A tidal current, generally called the *stream* of tide, is only experienced when near land, and this, in some places, runs with great strength; allowance has to be made for this on the course set, as otherwise the ship may soon be ashore. For the present it is sufficient to state that the stream of tide affects the ship's course in the same manner as a general current.

In different parts of the ocean there are great *oceanic* currents setting in various directions, and in some places, as in the heart of the Gulf Stream, round the Cape of Good Hope, and in other places, with great velocity; you must note this set and drift in order to appreciate, through this problem, their effect on the ship's course.

Charts generally show the direction of the current by means of an arrow, and if there is a numeral near it, you have also the *mean* drift; in a Day's Work allowance must always be made for the effect of the current.

N.B.—If the chart be on a small scale it is advisable to lay off the current for 3 or 4 hours, also laying off the distance sailed in the same time, in order to find the course to steer, as the length of the lines is increased and a better result is obtained.

Examples for Practice

1. A ship sails by her log N.W. by N. 72 miles, in a current that sets W.N.W. 36 miles in the same time. Required her course and distance, corrected for the effect of the current.

Ans. The course made good is N. 44° 51′ W., or N.W. nearly, and distance 104 miles.

2. A ship sails E. by N. 7·5 knots an hour, in a current setting S.W. 4 knots an hour. What will be her course and distance made good in 24 hours?

Ans. The course will be S. 73° 12′ E., or E.S.E. ½ E. nearly, and the distance 113·5 miles.

3. A ship sailing at the rate of 9 knots an hour, and wanting to double a cape bearing from her N.W. by W., finds she is in a current setting S.S.W. 3½ miles an hour. What course must she steer to counteract the effect of the current?

Ans. Course to steer for the cape is N. 33° 50′ W., or N.W. by N. nearly.

4. A ship sailing by her log 9 miles an hour is bound to a port which lies N.W. by N. from her, distant 56 miles, and finds she is in a current setting N.E. ½ N. 3 miles an hour. What course must she steer in the current, and distance make good; and how long will it take her to arrive at her port?

Ans. The course to be steered is N. 51° 21′ W., or N.W. ¾ W. nearly; the distance to run is 53·61 miles; and the time it will take nearly 6 hours.

5. A ship bound from Bombay to England, being on the edge of the Bank of Agulhas on April 21st, at noon, was by observation in lat. 35° 3′ S., and in long. by chronometer, 26° 52′ E.; on the 22nd the latitude, by observation at noon, was 35° 13′ S., and the longitude by chronometer, 25° 5′ E., having sailed by her reckoning N. 81° W. 39 miles. Required the set and drift of the current.

Ans. The current set S. 71° 49′ W., or W. 18° 11′ S., and its drift in 24 hours was 51·57 miles, being at the rate of 2·15 per hour.

6. A ship bound to a port bearing S.E. is in a current setting N. by W. 1½ miles per hour. Find the courses on which she must be laid to make good the course to the port, when her rate is respectively 6, 7, 8, and 9 knots per hour; and give the ship's hourly rate of approach to the port, with each rate of sailing.

Ans. (6) Course S. 37° 1′ E. and 4·69 knots; (7) course S. 38° 10′ E. and 5·7 knots; (8) course S. 39° 2′ E. and 6·71 knots; (9) course S. 39° 41′ E. and 7·72 knots.

NOTE.—Current sailing in practice is done on the chart in a correct and expeditious manner and the calculations given above are for the benefit of students who wish to learn the methods by calculation.

OBLIQUE SAILING AND TAKING THE DEPARTURE.

In the navigation of the ship this is an important problem. In previous Editions of Norie's "Epitome" the subject is discussed under the head of OBLIQUE SAILING, which is stated to be the application of oblique-angled plane triangles to various cases at sea, as in coasting along shore, approaching or leaving the land, surveying coasts or harbours, etc.; but much may be done by *inspection*, as will be shown presently.

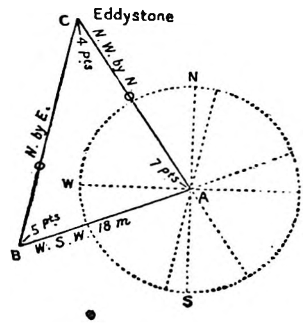
In this kind of sailing, *to set an object* means to observe what rhumb, or point of the compass, is directed to it; and the *bearing* of an object is the rhumb on which it is seen; also the bearing of one place from another is reckoned by the name of the rhumb passing through those two places: hence the bearing of two places from each other will be upon opposite points of the compass; thus, if one place bears E.N.E. from another, the latter will bear W.S.W. from the former, being in the same line, but in opposite directions.

A great variety of examples might be given in this sailing; hence, in this place, it is sufficient to select those only that appear to be useful in practice, and advantageous to the student who wishes to understand the problem on the basis of the solution of plane triangles.

Example I.—Sailing down the English Channel, I observed the Eddystone to bear N.W. by N.; and after sailing W.S.W. 18 miles, I found it bore from me N. by E. Required the distance of the ship from the Eddystone at both stations.

BY CONSTRUCTION

Describe the circle N.W.S.E., to represent the compass, and draw the diameters W.E. and N.S. at right angles to each other; draw the N.W. by N., W.S.W., and N. by E. rhumb lines, and on the W.S.W. line lay off 18 from A to B, taken from any scale of equal parts; through B draw B C parallel to the N. by E. line, meeting the N.W. by N. line A C in C; then will A represent the place of the ship at her first station, B her place at the second station, and C the place of the Eddystone; A C will be the ship's distance from the Eddystone at the first station, measuring 21 miles, and B C the distance at the second station, measuring 25 miles.



BY CALCULATION

In the plane triangle A B C are given the side A B 18 miles, the angle C A B equal to 7 points (being the angle contained between N.W. by N. and W.S.W.); the angle A B C equal to 5 points (being the angle contained between N. by E. and E.N.E. the opposite to the W.S.W. rhumb); and the angle B C A equal to 4 points (the angle between S. by W. and S.E. by S.); to find the sides A C and B C.

To find the Side A C *or b.*

$$\frac{b}{c} = \frac{\sin. B}{\sin. C} \therefore b = \frac{c \sin. B}{\sin. C}$$

$$\log. b = \log. c + L \sin. B - L \sin. C$$

c or side A B 18 m.	Log.	1·255273
∠ B 5 points	Sin.	9·919846
		11·175119
∠ C 4 points	Sin.	9·849485
b or side A C 21·17 m.	Log.	1·325634

To find the Side B C *or a.*

$$\frac{a}{c} = \frac{\sin. A}{\sin. C} \therefore a = \frac{c \sin. A}{\sin. C}$$

$$\log a = \log. c + L \sin. A - L \sin. C$$

c or side A B 18 m.	Log.	1·255273
∠ A 7 points	Sin	9·991574
		11·246847
∠ C 4 points	Sin	9·849485
a or side B C 24·97m.	Log	1·397362

Hence the distance of the Eddystone from the ship's first station is 21·17 miles, and from the second station 24·97, or 25 miles nearly.

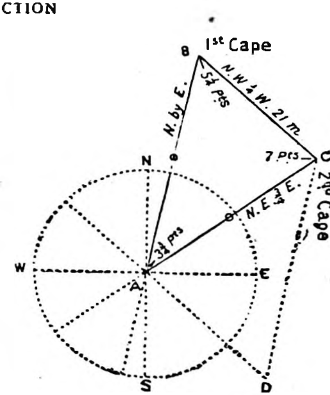
See another form under Example II.

Example II.—Coasting along shore, I observed two capes: the first bore N. by E., and at the same time the second bore N.E. ¾ E.; now, by the Chart, these capes bear from each other N.W. ¼ W., and S.E. ¼ E., distant 21 miles. Required my distance from both places at that time.

NOTE.—The bearings must all be considered to be true.

BY CONSTRUCTION

Having drawn the compass N.W.S E., the centre of which is to represent the ship's place, draw the N. by E. and N.E. ¾ E. rhumb lines A B and A C, being the bearings of the capes from the ship; draw likewise the N.W. ¼ W. and S.E. ¼ E. line, the bearing of the capes from each other, on which from A to D lay off 21 miles, the distance between the capes; through D draw D C parallel to the N. by E. line, and through C draw C B parallel to the N.W. ¼ W. and S.E. ¼ E. line; then B will represent the place of the first cape, C the second cape, A B the distance of the first cape from the ship, measuring 31 miles, and A C the distance of the second cape, measuring 27 miles.



BY CALCULATION

In the plane triangle A B C are given the angle B A C $3\frac{3}{4}$ points (the angle between N. by E. and N.E. $\frac{3}{4}$ E.); the angle A B C $5\frac{1}{4}$ points (the angle between S. by W. and S.E. $\frac{1}{4}$ E.); and the angle A B C 7 points (the angle between N.W. $\frac{1}{4}$ W. and S.W. $\frac{3}{4}$ W.); and the side B C 21 miles; to find the sides A B and A C.

To find the Side A B *or c.*

$$\frac{c}{a} = \frac{\text{sin. C}}{\text{sin. A}} \therefore c = \frac{a \text{ sin. C}}{\text{sin. A}}$$

log. c = log. a + L. sin. C — L sin. A

a or side B C 21 m.	Log.	1·322219
∠ C 7 pts.	Sin.	9·991574
		11·313793
∠ A $3\frac{3}{4}$ pts.	Sin.	9·827084
b or side A C 30·67 m.	Log.	1·486709

To find the Side A C *or b.*

$$\frac{b}{a} = \frac{\text{sin. B}}{\text{sin. A}} \therefore b = \frac{a \text{ sin. B}}{\text{sin. A}}$$

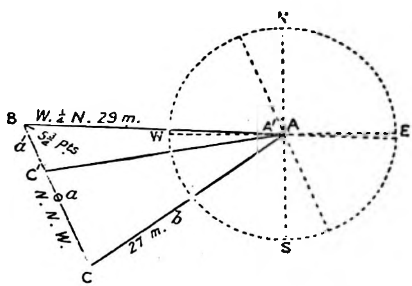
log. b = log. a + L sin. B — L sin. A

a or side B C 21 m.	Log.	1·322219
∠ B $5\frac{1}{4}$ pts.	Sin.	9·933350
		11·255569
∠ A $3\frac{3}{4}$ pts.	Sin.	9·827084
b or side A C 26·82m.	Log.	1·428485

Hence the distance of the ship from the first cape is 30·67 miles, and from the second cape 26·82, or 27 miles nearly. *See* another form under Example I.

Example III.—Being close in with a point of land at A, I ran 27 miles on a direct course to the westward, and then found the point of land at B to bear N.N.W.; now the bearing of B from A (by Chart) is W. $\frac{1}{4}$ N., and the distance 29 miles. Required the course steered, and the distance of the ship from B.

BY CONSTRUCTION



Describe a circle, and divide it into 4 equal parts by the diameters N.S. and W.E., the extremities of which will represent the cardinal points of the compass; and the centre of the circle the place the ship sailed from (A); draw the W. $\frac{1}{4}$ N. line A B equal to 29 miles, then will B represent the place of the point B; through B draw B C parallel to the N.N.W. line, and with the distance run, 27 miles in the compasses, set one foot in A, and with the other describe an arc cutting B C in C, and draw the line A C; then will C represent the ship's place, B C the distance of the ship from the point B, measuring 19 miles, and the angle S A C the course steered from the south, measuring $53\frac{1}{2}$°.

BY CALCULATION

In the plane triangle A B C are given the side A B, equal to 29 miles; the side A C 27 miles; and the angle A B C $5\frac{3}{4}$ points (the angle between E. $\frac{1}{4}$ S. and S.S.E.); to find the angle B A C, and the side B C.

The angle being given opposite the less side, this is the ambiguous case as shown in figure.

To find the Angle A

$$\frac{\text{sin. A}}{\text{sin. B}} = \frac{a}{b} \therefore \text{sin. A} = \frac{a}{b} \times \text{sin. B}$$

$$\text{L sin. A} = \text{log. } a + \text{L sin. B} - \text{log. } b$$

c or side A B 29	log.	1·462398
∠ B $5\frac{3}{4}$ points	sin.	9·956163
		11·418561
b or side A C 27	log.	1·431364
∠C 76° 9′	sin.	9·987197
180		
Or ∠ C′ 103 51		

∠ B ($5\frac{3}{4}$ pts.)	64° 41′	..	64° 41′
∠ C	76 9	..	103 51
Sum	140 50	..	168 32
	180		180
∠ A	39 10	or A′	11 28

To find the Side B C *or a.*

$$\frac{a}{b} = \frac{\text{sin. A}}{\text{sin. B}} \therefore a = \frac{b \text{ sin. A}}{\text{sin. B}}$$

$$\text{log. } a = \text{log. } b + \text{L sin. A} - \text{L sin. B}$$

To find B C′ *or a′.*

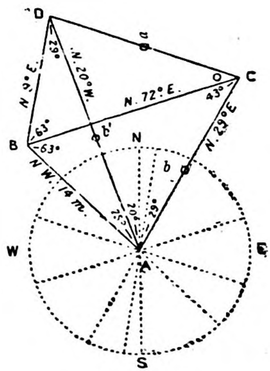
$$\frac{a'}{b} = \frac{\text{sin. A}'}{\text{sin. B}} \therefore a' = \frac{b \text{ sin. A}'}{\text{sin. B}}$$

b or side A C 27	log.	1·431364	....		1·431364
∠ A 39° 10′	sin.	9·800427	∠ A 11° 28′	sin.	9·298412
		11·231791			10·729776
∠ B $5\frac{3}{4}$ pts.	sin.	9·956163	....		9·956163
a or side B C 18·86	log.	1·275628	*a′* or side B C′ 5·938	log.	0·773613

Now the bearing of A B, which is W. ¼ N. or W. 2° 49′ N., subtracted from the angle B A C 39° 10′, makes the bearing of A C to be W. 36° 21′ S.; hence the course steered is S. 53° 39′ W. or S.W. ¾ W. nearly, and the distance of the ship from the point B is 18·86, or 19 miles nearly.

Otherwise, if the lay of the land between A and B permitted, the course might be as follows: W. 2° 49′ N. subtracted from B A C 11° 28′ makes the bearing of A C to be W. 8° 39′ S.; hence the course steered would be S. 81° 21′ W., and the distance from the point B 5·94, or 6 miles nearly.

Example IV.—Coasting along, at noon, a point C bore N. 29° E., and another point D bore N. 20° W.; running N.W., at the rate of 7 knots an hour, at 2 p.m. the point C bore N. 72° E., and the point D bore N. 9° E. Required the bearing and distance of the point D from C.



BY CONSTRUCTION

Draw the compass as before, and let the centre A represent the first station, from which draw the first bearing A C, N. 29° E., and the second bearing A D, N. 20° W.; also draw the N.W. line A B equal to 14 miles, the distance run in 2 hours; then will B represent the second station; through B draw B C parallel to N. 72° E., and B D parallel to N. 9° E., meeting the lines A C and A D in C and D; join D C; then will the line C D be their distance, measuring 18 miles nearly, and the bearing of D from C will be N. 70½° W., or W.N.W. ½ W.

BY CALCULATION

In the plane triangle A B C are given the side A B 14 miles; the angle A C B equal to 43° (the angle between S. 72° W. and S. 29° W.); and the angle A B C equal to 63° (the angle between N. 72° E., or E. 18° N. and S. 45° E.); to find the side A C.

To find the Side A C *or b.*

$$\frac{b}{c} = \frac{\text{sin. B}}{\text{sin. C}} \therefore b = \frac{c \text{ sin. B}}{\text{sin. C}}$$

$$\text{log. } b = \text{log. } c + \text{L sin. B} - \text{L sin. C}$$

c or side A B 14	log.	1·146128
∠ B 63°	sin.	9·949881
		11·096009
∠ C 43°	sin.	9·833783
b or side A C 18·29	log.	1·262226

In the plane triangle A B D are given the side A B 14 miles; the angle A D B equal to 29° (the angle between S. 20° E. and S. 9° W.); and the angle A B D equal to 126° (the angle between N. 9° E. or E. 81° N. and S. 45° E.); to find the side A D.

To find the Side A D *or b′.*

$$\frac{b'}{c} = \frac{\text{sin. B}}{\text{sin. D}} \therefore \text{A D} = \frac{c \text{ sin. B}}{\text{sin. D}}$$

$$\text{log. A D} = \text{log. } c + \text{L sin. B} - \text{L sin. D}$$

Side c 14 m.	log.	1·146128
∠ B 126°	sin.	9·907958
		11·054086
∠ D 29°	sin.	9·685571
Side A D 23·36	log.	1·368515

In the plane triangle A C D are given the side A C 18·29; the side A D 23·36; and the included angle C A D equal to 49° (the angle between N. 29° E. and N. 20° W.); to find the angle A C D, and the side C D.

Side A D or c	23·36		180°
Side A C or d	18·29	∠ C A D	49
Sum or $c + d$	41·65	Sum of ∠ˢ D & C	131 = C+D
Diff. or $c - d$	5·07	½ sum of ∠ˢ	65°30′ = ½ (C+D)

For the Angle A C D.

$$\text{L tan. } \tfrac{1}{2}(\text{C} - \text{D}) = \begin{cases}\text{log. } (c - d) + \text{L tan. } \tfrac{1}{2} \\ (\text{C} + \text{D}) - \text{log. } (c + d)\end{cases}$$

$c - d$ 5·07	Log.	0·705008
½ (C+D) 65° 30′	Tan.	10·341296
		11·046304
$c + d$ 41·65	Log.	1·619615
½ (C—D) 14° 57′	Tan.	9·426689
½ (C+D) 65 30		
Sum = ∠ACD 80 27		

For Side C D *or a.*

$$\frac{a}{c} = \frac{\text{sin. A}}{\text{sin. C}} \therefore \text{C D} = \frac{c \text{ sin. A}}{\text{sin. C}}$$

$$\text{log. C D} = \text{log. } c + \text{L sin. A} - \text{L sin. C}$$

Side c 23·36	Log.	1·368473
∠ A 49°	Sin.	9·877780
		11·246253
∠ C 80° 27′	Sin.	9·993939
Side C D 17·88	Log.	1·252314

Sum because ∠ A C D is opposite the greater side.

Now the angle A C D 80° 27′ added to 29°, the bearing of A C from the south, gives the bearing of C D = S. 109° 27′ W., which, subtracted from 180°, leaves the bearing N. 70° 33′ W. or W.N.W. ¼ W. nearly: hence the bearing of D from C is W.N.W. ¼ W., and the distance 18 miles nearly.

TAKING A DEPARTURE BY A SINGLE BEARING AND DISTANCE

In this case the point of land or other object is set by compass, and its distance is estimated by the eye.

This is the general method of taking a departure, and it is sufficiently accurate when the ship is leaving the land, bound on an oversea voyage. But since distances are almost always over-estimated, probably to the extent of a fifth or more of the whole, this method should never be relied upon when coasting.

TAKING A DEPARTURE (AND DETERMINING THE DISTANCE) BY TWO BEARINGS OF THE SAME OBJECT

In this case the ship's course, supposed to continue unaltered between the two times of observation, lies more or less across the line of direction of the object.

Take the bearing of the object, and note the number of points contained between it and the ship's head (or course): after the bearing of the object has altered not less than two or three points, again take the bearing of the object, and note the number of points contained between it and the ship's head. Each of these is a "difference" between the course and bearing at the instant of observation.

(A) *To find the distance when the second bearing was observed*

Enter Table* (p. 322) with the *first* difference at the *side*, and the *second* difference at the *top*, and take out the number corresponding thereto, as a multiplier: reference to the Table sufficiently explains the mode of entry. Multiply the number got from the Table by the number of miles run in the interval of time between taking the two observations, and the product is the distance (in miles) of the ship from the object at the time the *second* bearing was taken.

Norie's Tables now contain a new table, "Distance off by two Bearings and Distance run between them." It is entered with the angle between the ship's course and the first bearing in degrees at the top, and the angle between the first and second bearings in degrees at the side. Under the former, and opposite the latter, take out the number corresponding to any two arguments, and this number multiplied by the distance the ship has sailed between the first and second bearings will be the distance off at the time the second bearing was taken.

* CONSTRUCTION OF THE TABLE: To the L *sine* of the difference between the course and first bearing, add the L *co-secant* of the difference between the first and second bearings. Their sum is the logarithm of a natural number, which is the multiplier in the Table.

TABLE FOR FINDING THE DISTANCE OF AN OBJECT BY TWO BEARINGS AND THE DISTANCE RUN BETWEEN THEM.

Difference between the Course and Second Bearing in Points of the Compass.

4	¼	½	¾	5	¼	½	¾	6	¼	½	¾	7	¼	½	¾	8	¼	½	¾	9	¼	½	¾	10	¼	½	¾	11	¼	½	¾	12	¼	½	¾	13	Pts.
1·00	0·89	0·81	0·74	0·69	0·64	0·60	0·57	0·54	0·52	0·49	0·48	0·46	0·45	0·43	0·42	0·41	0·41	0·40	0·39	0·39	0·39	0·38	0·38	0·38	0·38	0·38	0·39	0·39	0·39	0·40	0·41	0·41	0·42	0·43	0·45	0·46	2
	1·12	1·00	0·91	0·83	0·77	0·72	0·67	0·64	0·60	0·58	0·55	0·53	0·51	0·50	0·48	0·47	0·46	0·45	0·45	0·44	0·44	0·43	0·43	0·43	0·43	0·43	0·43	0·43	0·44	0·44	0·45	0·45	0·46	0·47	0·48	0·50	2¼
		1·23	1·10	1·00	0·92	0·85	0·79	0·74	0·70	0·67	0·64	0·61	0·59	0·57	0·55	0·53	0·52	0·51	0·50	0·49	0·49	0·48	0·48	0·47	0·47	0·47	0·47	0·47	0·48	0·48	0·49	0·49	0·50	0·51	0·52	0·53	2½
			1·34	1·20	1·09	1·00	0·93	0·86	0·81	0·77	0·73	0·69	0·67	0·64	0·62	0·60	0·58	0·57	0·56	0·55	0·54	0·53	0·52	0·52	0·52	0·51	0·51	0·51	0·52	0·52	0·52	0·53	0·54	0·55	0·56	0·57	2¾
				1·45	1·30	1·17	1·08	1·00	0·93	0·88	0·83	0·79	0·75	0·72	0·69	0·67	0·65	0·63	0·61	0·60	0·59	0·58	0·57	0·57	0·56	0·56	0·56	0·56	0·56	0·56	0·56	0·57	0·57	0·58	0·59	0·60	3
					1·56	1·39	1·26	1·16	1·07	1·00	0·94	0·89	0·84	0·80	0·77	0·74	0·72	0·69	0·68	0·66	0·64	0·63	0·62	0·61	0·61	0·60	0·60	0·60	0·60	0·60	0·60	0·60	0·61	0·61	0·62	0·63	¼
						1·66	1·48	1·35	1·23	1·14	1·07	1·00	0·94	0·90	0·86	0·82	0·79	0·76	0·74	0·72	0·70	0·69	0·67	0·66	0·65	0·65	0·64	0·64	0·64	0·64	0·64	0·64	0·64	0·65	0·65	0·66	½
							1·76	1·57	1·42	1·31	1·21	1·13	1·06	1·00	0·95	0·91	0·87	0·84	0·81	0·78	0·76	0·74	0·73	0·71	0·70	0·69	0·68	0·68	0·67	0·67	0·67	0·67	0·67	0·68	0·68	0·69	¾
								1·85	1·65	1·50	1·37	1·27	1·19	1·11	1·05	1·00	0·95	0·92	0·88	0·85	0·82	0·80	0·78	0·76	0·75	0·74	0·73	0·72	0·71	0·71	0·71	0·71	0·71	0·71	0·71	0·72	4
									1·94	1·73	1·57	1·44	1·33	1·24	1·17	1·10	1·05	1·00	0·96	0·92	0·89	0·86	0·84	0·82	0·80	0·79	0·77	0·76	0·76	0·75	0·74	0·74	0·74	0·74	0·74	0·75	¼
										2·02	1·81	1·64	1·50	1·39	1·30	1·22	1·15	1·09	1·04	1·00	0·96	0·93	0·90	0·88	0·86	0·84	0·82	0·81	0·80	0·79	0·78	0·78	0·78	0·78	0·78	0·78	½
											2·10	1·88	1·70	1·56	1·45	1·35	1·27	1·20	1·14	1·08	1·04	1·00	0·97	0·94	0·91	0·89	0·87	0·85	0·84	0·83	0·82	0·81	0·81	0·80	0·80	0·80	¾
												2·17	1·94	1·77	1·62	1·50	1·40	1·31	1·24	1·18	1·12	1·08	1·04	1·00	0·97	0·94	0·92	0·90	0·88	0·87	0·86	0·85	0·84	0·83	0·83	0·83	5
													2·24	2·01	1·82	1·67	1·54	1·44	1·35	1·28	1·21	1·16	1·11	1·07	1·03	1·00	0·97	0·95	0·93	0·91	0·90	0·88	0·87	0·87	0·86	0·86	¼
														2·30	2·06	1·87	1·71	1·58	1·48	1·39	1·31	1·25	1·19	1·14	1·10	1·06	1·03	1·00	0·97	0·95	0·94	0·92	0·91	0·90	0·89	0·89	½
															2·36	2·11	1·92	1·76	1·63	1·52	1·43	1·35	1·28	1·22	1·17	1·12	1·09	1·05	1·02	1·00	0·98	0·96	0·94	0·93	0·92	0·91	¾
																2·11	2·16	1·96	1·80	1·66	1·55	1·46	1·38	1·31	1·25	1·19	1·15	1·11	1·08	1·05	1·02	1·00	0·98	0·97	0·95	0·94	6
																	2·46	2·20	2·00	1·83	1·69	1·58	1·48	1·40	1·33	1·27	1·22	1·17	1·13	1·10	1·07	1·04	1·02	1·00	0·98	0·97	¼
																		2·50	2·24	2·03	1·86	1·72	1·61	1·51	1·42	1·35	1·29	1·24	1·19	1·15	1·11	1·08	1·06	1·03	1·02	1·00	½
																			2·53	2·27	2·06	1·89	1·75	1·63	1·53	1·44	1·37	1·31	1·25	1·21	1·17	1·13	1·10	1·07	1·05	1·03	¾
																				2·56	2·29	2·08	1·91	1·76	1·65	1·55	1·46	1·39	1·32	1·27	1·22	1·18	1·14	1·11	1·09	1·06	7
																					2·58	2·31	2·10	1·92	1·78	1·66	1·57	1·47	1·40	1·34	1·28	1·23	1·19	1·15	1·12	1·09	¼
																						2·60	2·33	2·11	1·93	1·79	1·67	1·57	1·48	1·41	1·34	1·29	1·24	1·20	1·16	1·13	½
																							2·61	2·34	2·12	1·94	1·80	1·68	1·57	1·49	1·41	1·35	1·29	1·24	1·20	1·16	¾
																								2·61	2·34	2·12	1·94	1·80	1·68	1·58	1·49	1·41	1·35	1·29	1·24	1·20	8
																									2·61	2·34	2·12	1·94	1·80	1·68	1·57	1·49	1·41	1·35	1·29	1·24	¼
																										2·60	2·33	2·11	1·94	1·79	1·67	1·57	1·48	1·41	1·34	1·29	½
																											2·58	2·31	2·10	1·92	1·78	1·66	1·56	1·47	1·40	1·34	¾
																												2·56	2·29	2·08	1·91	1·76	1·65	1·55	1·46	1·39	9
																													2·53	2·27	2·06	1·89	1·75	1·63	1·53	1·44	¼
																														2·50	2·24	2·03	1·86	1·72	1·61	1·51	½
																															2·46	2·20	2·00	1·83	1·69	1·58	¾
																																2·41	2·16	1·96	1·79	1·66	10
																																	2·36	2·11	1·92	1·76	¼
																																		2·30	2·06	1·87	½
																																			2·24	2·01	¾
																																				2·17	11

Difference between the Course and First Bearing in Points of the Compass.

This table gives the factor which the distance run by the ship between the bearings is to be multiplied by in order to find the distance from the object at the time of taking the second bearing.

Example.—The Eddystone bore N.W. by N.; after running W.S.W. 18 miles, it bore N. by E. Find the distance of the ship from the Eddystone at the time when the *last* bearing was taken.

Difference between N.W. by N. and W.S.W. is 7 points; difference between W.S.W. and N. by E. is 11 points. Opposite 7 at the side, and under 11 at the top, stands 1·39, which multiplied by 18 (miles) gives 25 mil s, the distance of the ship from the Eddystone when the *last* bearing was taken (*see* Example 1, p. 316).

(B) *To find the distance when the* FIRST *bearing was observed*

Take the supplement of each difference, that is subtract each from 16 points. Then, enter Table p. 322 with the supplement of the *second* difference at the *side*, and the supplement of the *first* difference at the *top*; take out the number corresponding thereto, and multiply it by the distance run; the product is the distance (in miles) of the ship from the object at the time the *first* bearing was taken.

Example.—Find the distance of the ship from the Eddystone at the time the first bearing was taken, the elements being as in the previous Example.

First difference being 7 points, the supplement is 9 points; the second difference being 11 points, the supplement is 5 points. The distance run is as before, 18 miles. Then, 9 at the top, and 5 at the side, gives 1·18, which multiplied by 18 gives 21·24 miles as the distance of ship from the Eddystone at the time when the *first* bearing was taken (*see* Example 1, p. 316).

N.B.—In iron ships and steamers the bearings are affected to the extent of the deviation due to the *direction of the ship's head* at the time of observing the bearings; and thus the deviation must be applied as well as the variation of the compass, for *true* readings; only the deviation for *magnetic* readings.

General Rule for computing the distance of the ship from the object at the time of taking the second bearing.—Note the number of points (or degrees) contained between the first bearing of the object and the course of the ship; when the bearing of the object has altered not less than 2 or 3 points, note the distance run. Then,

Add together the *logarithm* of the distance run, the L *sine* of the angle between the first bearing and the course, and the L *co-secant* of the angle between the two bearings; the sum (rejecting tens in the index) will be the *logarithm* of the distance from the object when the *last* bearing was taken.

Example 1.—Cabrera Island Light bore N.N.W. at 2 a.m.; and at 4 a.m. it bore N.E. by N.; in the interval of the observations the course was west, and the distance run 12 miles. Find the distance of the Lighthouse when the *second* bearing was taken.

Angle between the bearings (N.N.W. and N.E. by N.) = 5 points.

Angle between first bearing and course (N N.W and west) = 6 points.

Then, dist. run 12 m.	Log.	1·0792
6 points	Sin.	9·9656
5 points	Co-sec.	10·0802
Dist. of Cabrera Is. Light 13·3 m.	Log.	1·1250

Example 2.—At noon, Ascension Island bore S. 80° E.; and at 5 p.m. it bore S. 46° E.; in the interval of the observations the course was N.N.W., and the distance run 19 miles; find the distance of the island when the *second* bearing was taken.

Angle between the bearings (S. 80° E., and S. 46° E.) = 34°.

Angle between first bearing and course (S. 80° E., and N. 22° 30′ W.) = 122° 30′.

Then, dist. run 19 m.	Log.	1·2788
122° 30′	Sin.	9·9260
34°	Co-sec.	10·2524
Dist. of Ascension Is. 28·7 m.	Log.	1·4572

Rule *by Inspection and the Traverse Table.*—When the angle between the first bearing and the course is more or less than 8 points, the method by *Inspection* from the Traverse Table is as follows:

Enter Traverse Table with the angle between the first bearing and the course, as a course; and the distance run (in distance column); take out the departure.

N.B.—If the angle is more than 8 points (or 90°) take its supplement; *i.e.* subtract it from 16 points (or 180°), and enter the Traverse Table with this difference as a course.

Then, enter Traverse Table again, with the difference of bearings as a course, and the departure (just found) in departure column; the *distance* of the object at the time of taking the second bearing will be found in the distance column.

Example 1 (p. 323).—	Angle between 1st bearing and course 6 points. Dist. 12 m., in dist. col.	in Trav. Tab. give	Dep. 11·1 in dep. col.
	Diff. of bearings, 5 points. Dep. 11·1 in dep. col.	in Trav. Tab. give	Dist. 13·5 m. in dist. col.

By Table p. 322: Difference between course and first bearing is 6 points; difference between course and second bearing is 11 points; hence 1·11 × 12 = 13·32 miles.

Example 2 (p. 324).—	Angle between 1st bearing and course, 122½°. (Take 57½°, the supplement.) Dist. 19 m., in dist. col.	in Trav. Tab. give	Dep. 16 in dep. col.
	Diff. of bearings, 34° Dep. 16, in dep. col.	in Trav. Tab. give	Dist. 28·5 m. in dist. col.

If the angle between the first bearing and the course be exactly 8 points (or 90°) the calculation is shorter than by the preceding Rule p. 323; thus—

RULE.—To the *logarithm* of the distance run add the L *co-secant* of the difference of the two bearings of the object; the sum (less index 10) will be the *logarithm* of the distance of the object at the second bearing.

Example.—Cape Horn bore north, and after running west 20 miles, it bore N.E. $\frac{1}{2}$ N.; find the distance when the second observation was taken.

Angle between the bearings (N. and N.E. $\frac{1}{2}$ N.) = $3\frac{1}{2}$ points.
Angle between first bearing and course (N. and W.) = 8 points.

Then, distance run 20 m.	Log.	1·3010
$3\frac{1}{2}$ points	Co-sec.	10·1976
Dist. of cape at second bearing 31·52 m.	Log.	1·4986

RULE *by Inspection and Traverse Table.*—When the angle between the first bearing and the course is 8 points, the distance at either bearing is readily determined *by Inspection* from the Traverse Table; thus:

Enter Traverse Table with the difference of bearings as a course, and the distance run in Departure column; then the distance column gives the *distance* of the object at the time of taking the *last* bearing; and the latitude column gives the *distance* of the object at the *first* bearing. Taking the example above as an instance, we have—

Diff. of bearings, $3\frac{1}{2}$ points } in Trav. } Dist. 31·5 m.
Dist. run 20 m., in Dep. col. } Tab.give } D. lat. 24·3 m.

Hence, by Inspection, the distance of the cape was 31·5 miles from the position where the second bearing was taken; and the ship was 24·3 miles from the cape when the first bearing was observed.

By Table p. 322: Difference between course and first bearing is 8 points; difference between course and second bearing is $11\frac{1}{2}$ points; opposite 8 at the side, and $11\frac{1}{2}$ at the top, is 1·58, which multiplied by 20 gives 31·6 miles as distance from cape at time of taking the second bearing.

Also, supplement of 8 is 8; and supplement of $11\frac{1}{2}$ is $4\frac{1}{2}$; opposite $4\frac{1}{2}$ at the side, and under 8 at the top is 1·22; then 1·22 multiplied by 20 = 24·4 miles, the distance of ship from the cape at first observation.

Source of Errors.—In determining the position by two bearings of the same object, errors will chiefly arise from observation of the bearings. The most favourable application of the method will be when the triangle is equilateral.

(C) *Position by the Isosceles Triangle "Doubling the Bearing"*

In connection with two bearings of the same object, this is a very important problem in navigation, when determining the distance of a ship from the land. If the ship continues on the same course until the angle between the two bearings is equal to the angle between the course and first bearing, the triangle is isosceles, and the distance of the land, point, or lighthouse, from the *position of the observer when the second bearing was taken* is *equal to the distance run between the two bearings.*

PRACTICAL RULE.—On taking the bearing note the difference between the bearing and the course, then, when the bearing has changed the same

amount as this difference, the ship is distant from the point the exact distance she has gone in the interval.

Example.—Course W.N.W., with a point of land bearing N.W. by N. (*the difference is 3 points, hence the next bearing to be taken is north*); when the same point bore north, ship had run on the W.N.W. course 6½ miles; therefore the ship was 6½ miles from the point when the second bearing was observed.

Example.—Between two islands the course is north; a point of land bore N.E. (*the difference is 4 points, hence the next bearing to be taken is east*); and when the same point bore east, the ship, still on the same course, had run 2½ miles; therefore the ship was 2½ miles from the point at the time of taking the second bearing. (This is the 4-point bearing, with which most navigators are familiar.)

Example.—The ship's course being S. 85° E., a point of land bore S. 55° E. (*the difference is 30°, hence the next bearing to be taken is S. 25° E.*); when the same point bore S. 25° E. the ship had run on her course 8 miles; therefore the distance from the point at the last observation was 8 miles.

There are other methods of *taking a departure and determining the distance from the land, as,* for instance, *by the Chart,* etc.; these will be fully explained in connection with the *Use of the Chart.*

Distance by the Altitude of a High Light, etc., seen on the Sea Horizon

The distance of the visible horizon from an observer is equal to the true depression of the eye. With the eye 10 feet above the sea the visible horizon is distant about 3½ nautical miles; with the eye 20 feet above the sea the horizon is distant a little over 5 miles. When an object breaks the continuity of the horizon the distance of the object is less than the depression. But if a peak or light is seen beyond the range of the sea-horizon the distance between the eye and the object is equal to the sum of the depressions corresponding to the two heights. The solution of the problem depends on the uniform curvature of the sea, by means of which all terrestrial objects disappear at certain distances from the observer.

The distance of visibility, in miles, is the square root of the height, in feet; an accidental relation which gives a result approximately correct.

$$\text{More accurately, the distance (in miles)} = \frac{8}{7}\sqrt{\text{Height (in feet)}}.$$

But distances may be got by inspection from Table for Finding the Distance of Terrestrial objects at Sea in Norie's Nautical Tables, in which the elevation in feet is given in one column, and the distance of visibility is expressed in nautical miles in the other column; if the position from which the object is seen be elevated, *add together the distances corresponding to the height of the observer and the height of the object;* the sum is the greatest distance of the object's visibility from the observer.

Example.—Height of eye at masthead 90 ft.	....	10·9 miles.
Height of light (per chart) 240 ft.	....	17·8
Distance of light from observer	....	28·7 miles.

Terrestrial refraction will interfere at times with the accuracy of the result.

$\sqrt{\text{ht of eye}} + \sqrt{\text{Ht of Lt House}}$

Examples for Practice

1. Running down Channel, and wanting to take my departure from the Lizard, at 2 h. p.m. I observed it bear from me N. by W.; and after sailing W. by N. $\frac{1}{4}$ N., at the rate of 8 knots per hour, at 3h. 30m. p.m. it bore from me N.N.E. $\frac{1}{2}$ E. Required my distance from the Lizard at the time of taking the second bearing.

Ans. The distance 17·1 miles.

2. Entering a river by night, I observed two lights; that from the lighthouse bore from me N. by E. $\frac{1}{4}$ E., and that from the light vessel W. $\frac{1}{4}$ S., the former bearing from the latter N.E. $\frac{3}{4}$ N., distant 18 miles. Required my distance from each of the lights.

Ans. Distance from lighthouse 14·54 miles, and from the light vessel 7·198 miles.

3. Being off the Burlings (on the coast of Portugal), I ran 34 miles on a direct course between the south and west, and then observed Cape Rocca bearing from me S. by E. $\frac{1}{2}$ E.; Cape Rocca bears from the Burlings S. by W. $\frac{3}{4}$ W., distant 43 miles. Required the course steered, and my distance from Cape Rocca.

Ans. The ship's course was S. 32° W. or S.S.W. $\frac{3}{4}$ W. nearly, and the distance from the cape was 12·17 miles.

4. Sailing between two small islands, I observed the first bear from me S.W. by W. $\frac{3}{4}$ W., and the second E.S.E.; after running S. by W. $\frac{1}{2}$ W. 15 miles, the first bore from me N.W. by W., and the second E. $\frac{1}{2}$ N. Required the bearings, and distance between the islands.

Ans. The first island bore from the second N. 84° 28′ W., the second from the first S. 84° 28′ E., and their distance was 42·49 miles.

5. Two ships, A and B, sail from the same port C; A sails N.E. by N. 84 miles, and B sails S.E. 76 miles. Required their bearings and distance from each other.

Ans. Bearing of A from B is N. 3° 16½′ W., of B from A is S. 3° 16½′ E., and their distance 123·8 miles.

6. Being off the coast of South America, in lat. 47° 4′ 30″ S., and long. 65° 26′ W., I found the (true) bearing of a cape to be W. 20° S., and after running S. 12° 30′ W. 32 miles the cape bore N. 34° W. Required the latitude and longitude of the cape.

Ans. The lat. of the cape is 47° 12′ 42″ S., and long. 65° 59′ W.

7. Wanting to know the distance of a ship at anchor from the shore, I chose two stations, A and B, that were distant from each other 2·5 miles. From the station at A, I took with a sextant the angle subtended by the station at B and the ship, and found it to be 64° 15′; then from the station at B, I found the angle between the station A and the ship 73° 55′ Required the distance of the ship from both stations.

Ans. The distance of the ship from the station at A was 3·602, and from the station at B 3·376 miles.

8. Sailing along a coast I observed two objects, a church and a mill, in one, the church being the nearer object; and at the same time I measured the angle subtended at the ship by the church and a tower on the coast, and found it to be 25° 36′; now, by a chart, the distance from the church to the tower was 1·5 miles, from the church to the mill 0·75 of a mile, and from the mill to the tower 1·9 miles. Required the distance of the ship from the church and the tower.

Ans. The distance of the ship from the church was 3·459 miles; from the tower 3·246 miles.

THE VARIOUS CASES OF PLANE SAILING ARE SHOWN IN THE FOLLOWING TABLE.

Case.	Given.	Required.	Solution.
1	Diff. lat. and dep.	Course .... Distance	..Tan. course $= \frac{\text{dep.}}{\text{diff. lat.}}$ ..Dist. $= \frac{\text{dep.}}{\text{sin. course}}$
2	Course and distance	Diff. lat. .... Departure	..Diff. lat. = dist. × cos. course ..Dep. = dist. × sin. course
3	Course and departure	Distance ...... Diff. lat.	..Dist. $= \frac{\text{dep.}}{\text{sin. course}}$ ..Diff. lat. $= \frac{\text{dep.}}{\text{tan. course}}$
4	Distance and diff. lat.	Course .... Departure	..Cos. course $= \frac{\text{diff. lat.}}{\text{dist.}}$ ..Dep. = dist. × sin. course
5	Course and diff. lat.	Distance ...... Departure	..Dist. $= \frac{\text{diff. lat.}}{\text{cos. course}}$ ..Dep. = diff. lat. × tan. course
6	Distance and departure	Course .... Diff. lat.	..Sin. course $= \frac{\text{dep.}}{\text{dist.}}$ ..Diff. lat. = dist. × cos. course

THE VARIOUS CASES OF PARALLEL SAILING ARE SHOWN IN THE FOLLOWING TABLE.

Case.	Given.	Required.	Solution.
1	Diff. long.	Departure	..Dep. = diff. long. × cos. lat.
2	Departure	Diff. long.	..Diff. long. = dep. × sec. lat.
3	Diff. long. and dep.	Latitude	..Cos. lat. $= \frac{\text{dep.}}{\text{diff. long.}}$

THE VARIOUS CASES OF MIDDLE LATITUDE SAILING ARE SHOWN IN THE FOLLOWING TABLE.

Case.	Given.	Required.	Solution.
1	Both lat. and long.	Departure	Dep. .... = Diff. long. × cos. middle lat.
		Course	Tan. course .. = $\frac{\text{Cos mid lat.} \times \text{d. long.}}{\text{d. lat.}}$
		Distance	Dist. .... = Sec. course × diff. lat.
2	Both lat. and departure	Course	Tan. course .. = $\frac{\text{Dep.}}{\text{Diff. lat.}}$
		Distance	Distance .... = $\frac{\text{Dep.}}{\text{Sin. course}}$
		Diff. long.	Diff. long. .... = $\frac{\text{Dep.}}{\text{Cos. mid. lat.}}$
3	One lat. course and distance	Diff. lat.	Diff. lat. .... = Dist. × cos. course
		Departure	Departure .... = Dist. × sin. course
		Diff. long.	Diff. long. .... = Dep. × sec. mid. lat.
4	Both lat. and course	Departure	Departure .. = Diff. lat. × tan. course
		Distance	Distance .... = $\frac{\text{Diff. lat.}}{\text{Cos. course}}$
		Diff. long.	Diff. long. .... = Dep. × sec. mid. lat.
5	Both lat. and distance	Courses	Cos. course .. = $\frac{\text{Diff. lat.}}{\text{Distance}}$
		Departure	Departure .. = Dist. × sin. course
		Diff. long.	Diff. long. .. = Dep. × sec. mid. lat.
6	One lat. course and departure	Diff. lat.	Diff. lat. .... = $\frac{\text{Dep.}}{\text{Tan. course}}$
		Distance	Distance .... = $\frac{\text{Dep.}}{\text{Sin. course}}$
		Diff. long.	Diff. long. .. = Dep. × sec. mid. lat.
7	One lat. distance and departure	Course	Sin. course .. = $\frac{\text{Dep.}}{\text{Dist.}}$
		Diff. lat.	Diff. lat. .... = Dist. × cos. course.
		Diff. long.	Diff. long. .. = Dep. × sec. mid. lat.

THE VARIOUS CASES OF MERCATOR SAILING ARE SHOWN IN THE FOLLOWING TABLE.

Case.	Given.	Required.	Solution.
1	Both lat. and long.	Course	Tan. course .... = $\frac{\text{Diff. long.}}{\text{Mer. pts.}}$
		Distance	Distance ...... = Sec. course × diff. lat.
		Departure	Departure ...... = Diff. lat. × tan. course
2	Both lat. and departure	Course	Tan. course .... = $\frac{\text{Departure}}{\text{Diff. lat.}}$
		Distance	Distance ...... = Diff. lat. × sec. course
		Diff. long.	Diff. long. ...... = Mer. pts. × tan. course
3	One lat. course and distance	Departure	Departure ...... = Dist. × sin. course
		Diff. of lat.	Diff. of lat. .... = Dist. × cos. course
		Diff. of long.	Diff. of long. .... = Mer. pts. × tan. course
4	Both lat. and course	Distance	Distance ...... = $\frac{\text{Diff. lat.}}{\text{Cos. course}}$
		Departure	Departure ...... = Diff. lat. × tan. course
		Diff. of long.	Diff. of long. .... = Mer. pts. × tan. course
5	Both lat. and distance	Course	Cos. course .... = $\frac{\text{Diff. lat.}}{\text{Distance}}$
		Departure	Departure ...... = Distance × sin. course
		Diff. of long.	Diff. of long. .... = Mer. pts. × tan. course
6	One lat. course and departure	Diff. of lat.	Diff. lat. ...... = $\frac{\text{Dep.}}{\text{Tan. course}}$
		Distance	Distance ...... = $\frac{\text{Dep.}}{\text{Sin. course}}$
		Diff. of long.	Diff. long. ...... = Mer. pts. × tan. course
7	One lat. distance and departure	Course	Sin. course ...... = $\frac{\text{Dep.}}{\text{Distance}}$
		Diff. of lat.	Diff. lat. ...... = Distance × cos. course
		Diff. of long.	Diff. long. ...... = Mer. pts. × tan. course

CORRECTION OF COURSES

DEFINITION

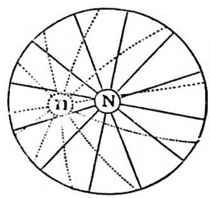
Variation of the compass.—When off Gravesend, or off the Isle of Wight, if you look at the Pole Star (as showing the *true* north very nearly),* and note its position by a correct compass you will find it to bear nearly N. by E. $\frac{1}{2}$ E. : or, if you look at the sun *at noon*, when it is on the meridian and at its greatest altitude, and consequently *true* south of you, it will be found to bear by compass S. 17° W. How is this ?—

Every bar magnet or magnetised needle has two poles, one at each end ; each pole also differs in quality, or property, from the other ; this may be tested by presenting a magnet end on, first to the N. end of the compass needle and then to the S. end of the same needle, when it will be found that there is repulsion of one end, and attraction of the other end of the needle ; on the principle that—

Like poles repel, and *unlike poles attract, one another.*

Now the earth being a great magnet, with two magnetic poles, it acts on this law, and attracts to its northern part the end of the needle that has magnetism *unlike* to that of the northern hemisphere ; hence, when speaking of the compass, the end of the magnet that turns to the north is the north-*seeking* end.

In neither hemisphere, however, do the magnetic poles occupy the same place as the true poles of the earth. The *magnetic* pole of the northern hemisphere is about 1200 miles south of the earth's *true* N. pole, and the *magnetic* pole of the southern hemisphere is about 990 miles north of the earth's *true* S. pole. But the magnetic needle points to the magnetic, *not* to the true poles.



In the annexed fig. is a part of the N. hemisphere on the stereographic projection, with the terrestrial meridians radiating in straight lines from the true north pole (N.) ; and the position of the magnetic pole is shown at *n* ; the magnetic meridians (dotted lines) trending towards the magnetic pole *n* cut the terrestrial meridians at an angle, which is the Variation of the Compass.

The direction of the magnetic needle at any place is the magnetic meridian, hence *the angle that the magnetic meridian makes with the true meridian* is the Variation of the Compass ; the *angular value* (large or small) is *the measure of the variation*, and if the direction of the needle

* The Pole star (Polaris in the Little Bear, which is easily found by means of the *pointers* in the Great Bear or Charles' Wain) is 1¼° off the true celestial pole, and therefore *true* north only twice in 24 hours ; but it is always so nearly true north that for practical purposes at sea, when in the northern hemisphere, it may be taken to show true north.

Note

trends to the *right* of the true meridian, we say the variation is *east;* if it trends to the *left*, we say it is *west;* and if it trends in the same direction *as* the true meridian, we say there is *no variation.*

There are two meridians, trending from one magnetic pole to the other, where the variation is 0° (*nil*); roughly speaking, one of these meridians crosses the Atlantic in a diagonal direction and thence through N. America; the other crosses Australia, Western Asia, and Eastern Europe; between them, over one part of the globe, as in the North Atlantic, in the greater part of the South Atlantic, and in the Indian Ocean, the *variation* is *westerly;* and again between them, over the remaining part of the globe, as in the whole of the Pacific Ocean, the *variation* is *easterly.*

NOTE.—The accompanying "Chart of the World, showing the lines of equal magnetic variation" illustrate these latter remarks. The plain lines indicate W. variation; the dotted lines, E. variation; and the thick lines *no* variation.

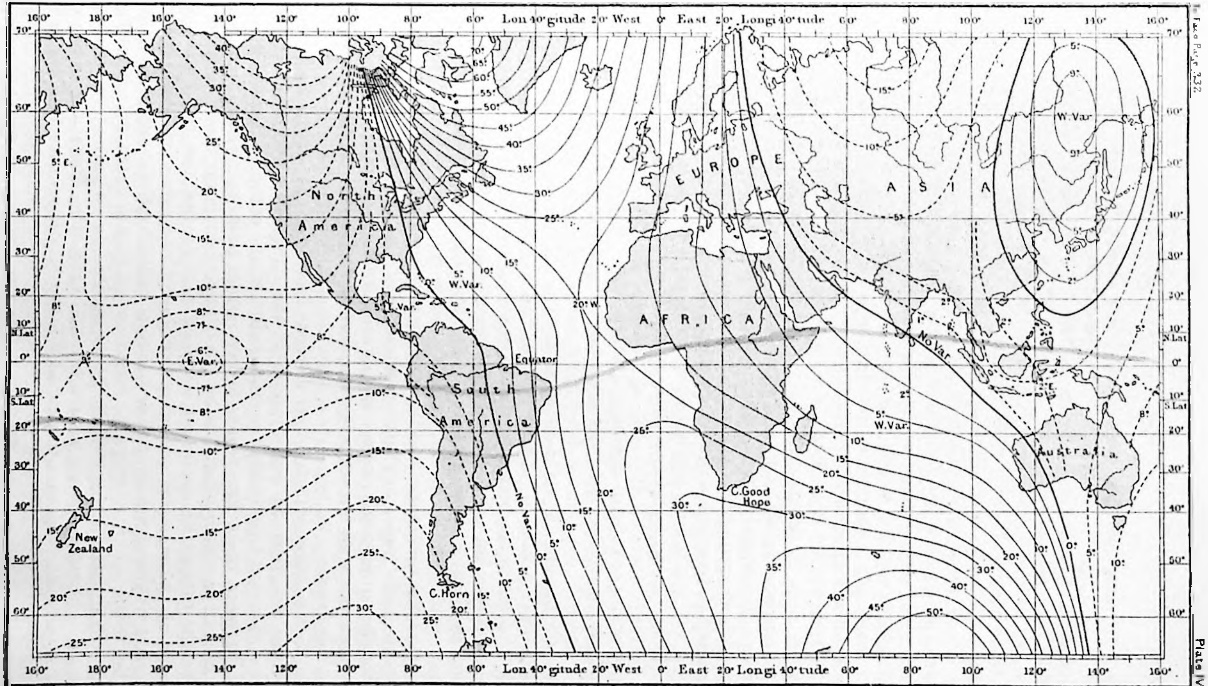
As illustrative of secular magnetic change, it may be noted that the *line of no variation* passed through London in the year 1657; the variation had previously been easterly; since that date it has been westerly, and attained its western *maximum* (24½°) in 1816; since the latter date it has been decreasing at the rate of 7′ annually, and is now 16¾° W. The magnetic needle will again point true north about the year 1976, the cycle of change being about 320 years.

DEVIATION OF THE COMPASS.—Ships called composite (partly wood and partly iron), and such as are built wholly of iron, are strongly magnetic—due to the magnetic direction in which they have been built, and the amount of hammering and twisting to which the iron has been subjected while in that position. The effect on the compass when in its place on board is to cause the magnetic needle to *deviate* from the magnetic meridian; not that this is an error, strictly speaking, for the compass is only acting in obedience to a law of magnetism, but for the practical purposes of navigation it is an error that might lead to serious consequences. Now, exactly as the magnetic needle, unaffected by local surroundings, points to the magnetic pole, forming an angle with the true meridian, so in like manner the same *needle, when under the influence of an iron ship's magnetism, forms an angle with the magnetic meridian;* and this is called the Deviation of the Compass.

Unlike variation, which, for any given place, is of the same amount and in the same direction on every point of the compass, deviation attains its greatest value on two nearly opposite points of the compass; and also, somewhere between these two points are two other points on which there is little or no deviation; *nearly* half round the compass, from one point of no deviation to the other, the *deviation* is called *easterly*, because the *needle* lies *to* the *right of* the *magnetic meridian*; between the same two points, on the other part of the compass, the *deviation* is called *westerly*, because the *needle* then lies *to* the *left of* the *magnetic meridian.* A glance at the Deviation Table, p. 339, will illustrate this. N.E. by N. and S. by W. are the points of *no* deviation; between them, round by eastward, the deviation is westerly, and it attains its greatest amount at S.E. by E.; from the same two points, but round by westward, the deviation is easterly, and attains its greatest amount at W. by S. and W.S.W. Thus the deviation differs, not only in amount but in name, for different directions of a ship's head. Every iron ship's compass has deviation peculiar to itself, the direction and amount

CHART OF THE WORLD, showing LINES of EQUAL MAGNETIC VARIATION.

Easterly Variation is shown by Broken Lines; Westerly Var. by Continuous fine Lines. No Var. by thick black Lines.



Imray, Laurie, Norie & Wilson, Ltd. London.

of which for its various points must be ascertained by what is called *swinging ship for the errors of the compass.*

When a ship is "*close hauled*," that is, when she is sailing as near to the direction of the wind as she can be brought, that part of the wind which acts upon the hull and rigging, together with a considerable part of the force exerted on the sails, tend to drive her immediately from the direction of the wind, or, as it is termed, to *leeward*. But since the bow of a ship exposes less surface to the water than the side, the resistance will be less in the fore and aft direction than in the athwartship direction; the velocity, therefore. in the direction of her head will, in most cases, be greater than the velocity in the direction of her side, and the ship's real course will, on the basis of the composition of forces, be between the two directions: the effect produced is a continual drifting of the vessel from the wind with the head still in the same direction as if no drifting took place, and consequently the compass showing the same course; but if the ship drifts in this manner, her keel will make a streak or *wake* in the water in a direction opposite to the point towards which she is moving. The angle contained between the line of the ship's *apparent* course and the line she really describes through the water, is termed her leeway—which may be also expressed as the *angle which the ship's keel makes with her actual path through the water.*

The amount of leeway to be allowed will depend upon a variety of circumstances, as the build and trim of the ship; the force of the wind; the quantity of sail carried; the rate through the water; the sails being properly set to receive the action of the wind, etc.: hence no general rules can be laid down for estimating it.

With the wind aft or on the quarter there can be no leeway. A ship close hauled, wind moderate and sea smooth, makes no leeway. In heavy weather with small sail on, there is often considerable leeway. For a ship when she lies-to, observe the points to which her head comes up and then falls off, and take the middle point for the apparent course on which to allow the leeway.

The only method that ought to be relied on, in practice, of ascertaining the amount of leeway, since it is a correction of the *apparent* course, is that of actually measuring the angle before mentioned. This can be done as follows: draw a small semicircle on the taffrail, with its diameter inwards and at right angles to the ship's keel; divide the semi-circumference into 16 points, and their halves; then observe the angle contained between the semidiameter which points right aft, and that which points in the direction of the wake, and it will be the angle of leeway required. But the most accurate method of determining the leeway is to have a semicircle drawn on the taffrail, as before described, with a low crutch or swivel in its centre. Then, after heaving the log, the line is to be slipped into the crutch just before it is drawn in, and the points and quarters contained between the direction of the log-line and the fore and aft line of the semicircle will be the quantity of leeway.

When the ship makes leeway the direction of her head by compass is only the apparent, not the real course; and the amount of leeway is always allowed to the *apparent* course in a direction *from* the wind; hence, looking towards the ship's bow—

RULE.—Wind on *starboard* side, allow leeway *to* the *left* hand.
Wind on *port* side, allow leeway *to* the *right* hand.

Example.—Ship's head by compass N.N.W., leeway $1\frac{1}{2}$ points, with wind N.E., gives the actual direction of her course N.W. $\frac{1}{2}$ N.

Example.—Ship's head by compass W.S.W., leeway $\frac{3}{4}$ point, with wind south, gives the actual direction of her course W.S.W. $\frac{3}{4}$ W.

Example.—Ship's head by compass south, leeway $2\frac{1}{4}$ points, with wind W.S.W., gives the actual direction of her course S.S.E. $\frac{1}{4}$ E.

These examples will make the method of allowing for leeway clear; but remember that leeway is not an error of the compass, only a correction to be applied to the course under special circumstances, and is independent of direction of ship's head as to amount.

Correct the following courses for leeway:—

Apparent Courses steered.	Leeway.	Winds.	Corrected Compass Courses.
East	$\frac{3}{4}$ pt.	S.S.E.	E. $\frac{3}{4}$ N.
N.N.E.	1 ,,	N.W.	N.E. by N.
N.W.	2 ,,	N.N.E.	W.N.W.
S.E.	$1\frac{1}{2}$,,	S.S.W.	S.E. by E. $\frac{1}{2}$ E.
W.S.W.	$2\frac{1}{2}$,,	South	W. $\frac{1}{2}$ N.
South	$2\frac{1}{4}$,,	E.S.E.	S.S.W. $\frac{1}{4}$ W.
E. $\frac{1}{2}$ S.	$1\frac{3}{4}$,,	N.N.E.	S.E. by E. $\frac{3}{4}$ E.
W. $\frac{1}{2}$ N.	$1\frac{1}{4}$,,	N.N.W.	W. $\frac{3}{4}$ S.

COURSES.—There are three different kinds of courses, each of which has its special corrections, to be applied in a special way, when converting one course into another.

I. The COMPASS COURSE is the angle that the ship's fore and aft line makes with the axis of the compass needle, which should coincide with the N. and S. line of the compass card.

This course will be affected by the deviation due to the direction of the ship's head, and the variation shown by the chart as appertaining to the locality where the ship is being navigated. If there is deviation, its application reduces the compass course to the magnetic course; and the further application of the variation to the latter gives the true course.

II. The MAGNETIC COURSE is the angle that the ship's track makes with the magnetic meridian.

This course is affected only by the variation, the application of which converts it into a true course. When there is no deviation, the compass course will be the magnetic course, but it must not be rashly concluded that because a ship is all wood there is no deviation. There are sure to be iron fittings which may affect the compass.

III. The TRUE COURSE is the angle that the ship's track makes with the terrestrial or true meridian.

This is the *true course made good* and is derived from the *course steered.* Since the latter does not necessarily, and at all times, coincide with the *correct* compass course, it follows that the true course made good must be obtained through the application of the corrections for leeway, deviation, and variation—any or all of them, as required; the true course is the

basis of the ship's position by *dead reckoning* from day to day, or at any intermediate time.

PRELIMINARY AND CAUTIONARY OBSERVATION ON THE CORRECTION OF COURSES.—Let it be clearly understood, and without the necessity for further repetition, that since the compass is the representation of the visible horizon which bounds the observer in every direction at sea, he must never lose sight of the conception that the position of the ship, and hence his own as navigator, is the centre of the compass card. This is most important, for, in speaking of the corrections of the various courses, these corrections, now it may be to the *right*, and now to the *left*, bear reference entirely to that central position, with the navigator looking in the direction of the point towards which the ship's head is directed.

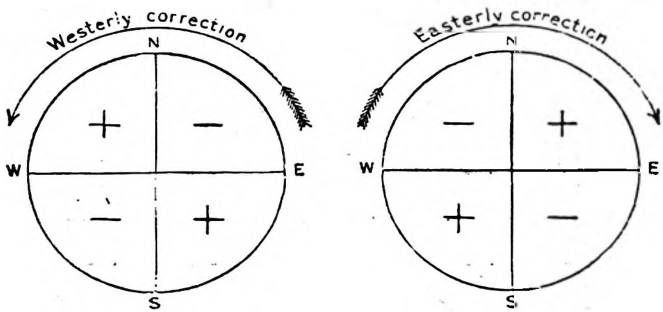
Hence the significance of the terms *right* and *left*, as compared with the movements of the hands of a watch, is this—*right* is with watch-hands, *left* is against watch-hands.

The Magnetic Course and the Variation being given, to find the True Course

Variation of the compass is the angle that the magnetic meridian makes with the true meridian. For any given place it affects every compass course and bearing alike, to the same amount and in the same direction ; thus, off the Isle of Wight, where the variation is 16° W., the compass needle trends 16° westward of the terrestrial meridian, so that the Pole Star bears by compass N. 16° E., which is the compass bearing corresponding to *true* north. In that vicinity every compass course and bearing is affected to the extent of 16° W.

RULE.—Variation *westerly*, allow it to the *left of* the magnetic course.
Variation *easterly*, allow it to the *right of* the magnetic course.

This rule you can often carry out by looking at a compass card, and making the correction mentally, but not always ; you should aim, however, at getting an accurate result without reference to the compass card.



To correct a Compass Course steered, for the Deviation

Take the deviation for the given course from the Deviation Table (page 339) ; then, to get the *correct* magnetic course—

RULE.—Deviation *westerly*, allow it to the *left* of the compass course. Deviation *easterly*, allow it to the *right* of the compass course.

The modern practice is to use the algebraic sum of the variation and deviation, which is then called the compass error. For example, variation 15° W., deviation 10° E., compass error will be 5° W., which apply as before, E. to the right, W. to the left.

The diagrams should assist materially.

The westerly correction it will be observed is + from N. to W., — from W. to S., + from S. to E., and — from E. to N.

The easterly correction is the opposite.

When the variation, deviation, or compass error is additive, if the sum exceeds 90° take it from 180°, and the remainder will be S. if previously N., but N. if previously S.; also, when the variation is subtractive but exceeds the course, subtract the course from the variation, and name the remainder E. if the course was W., but W. if the course was E.

Modern practice tends to abolish points and quadrantal degrees in the compass and to treat it as a circle of 360°, north being 0° or 360°, E. 90°, S. 180°, and West 270°.

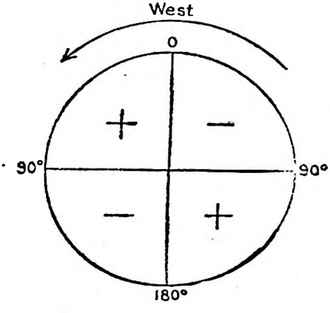
By turning quadrantal compass courses into their equivalent value reckoned from north right round to 360° by table of compass equivalents all easterly errors become plus and all westerly errors minus, and all leeway when on the port tack is plus and on the starboard tack minus. After correcting the course it is easily turned into its proper quadrant by the same table. This is by far the best method.

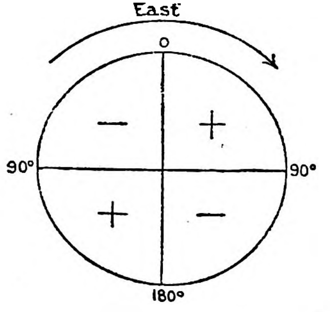
The excess of 360° would be shown as from 0° towards 90° and the defect as from 360° towards 270°. For example, compass course 350°, error 20° E., 350° + 20° = 370° = 10°, that is = N. 10° E. Again, compass course 5°, error 30° W., 360° + 5° = 365° — 30° = 335° or N. 25° W.

Examples in the Correction of Courses

As shown above, a course may be affected by three things—variation, deviation, leeway.

An algebraical sum can be made of the variation and deviation; the leeway is applied separately.





Compass Co.	N. 45° E.	Var. 16° W. Dev. 16° E. = Error 0.
Error	0	
True Co.	N. 45° E.	

Compass Co.	N. 23° 30′ E.	Var. 17° W. Dev. 5° W. = Error 22° W.
Error	— 22 00 W.	By diagram westerly error in N.E. quadrant
True Co.	N. 1 30 E.	is subtractive.

Compass Co.	N. 67° 30′ W.	Var. 10° E. Dev. 8° E. = Error 18° E.
Error	— 18 00 E.	By diagram easterly error in N.W. quadrant
True Co.	N. 49 30 W.	is subtractive.

Compass Co.	S. 80° E.	Var. 10° W. Dev. 12° W. = Error 22° W.
Error	+ 22 W.	As it has exceeded 90° it must be in the N.
	S. 102 E.	and E. quadrant.
	180	
True Co.	N. 78 E.	

Compass Co.	S. 10° W.	Var. 10° W. Dev. 2° E. = Error 8° W.
Error	— 8 W.	
True Co.	S. 2 W.	

Compass Co.	S. 10° W.	Var. 20° W. Dev. 10° W. = Error 30° W.
Error	— 30 W.	Take the difference in these cases and carry
True Co.	S. 20 E.	across to the other quadrant.

Compass Co.	S. 5° E.	Var. 40° E. Dev. 5° W. Error = 35° E.
Error	— 35 E.	As before, take the difference and carry
True Co.	S. 30 W.	across to next quadrant.

Compass Co.	S. 78° W.	Var. 5° E. Dev. 10° E. Error = 15° E.
Error	+ 15 E.	
	S. 93 W.	
	180	
True Co.	N. 87 W.	

Compass Co.	N. 70° W.	Var. 20° W. Dev. 0° = Error 20° W.
Error	+ 20 W.	
True Co. S. or	N. 90 W.	

Compass Co.	N. 80° W.	Var. 15° W. Dev. 15° W. Error = 30° W.
Error	+ 30 W.	
	N. 110° W.	
	180	
True Co.	S. 70° W.	

Compass Co.	N. 10° W.	Var. 10° E. Dev. 2° E. = Error 12° E.
Error	— 12 E.	Take the difference and carry across to the
True Co.	N. 2 E.	next quadrant.

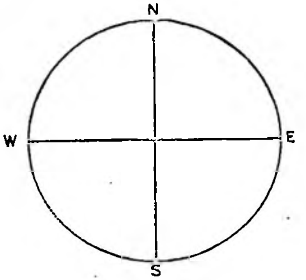
Compass Co.	N. 20° E.	Var. 10° W. Dev. 15° W. = Error 25° W.
Error	— 25 E.	Take the difference and carry across to the
True Co.	N. 5 W.	next quadrant.

Examples with Leeway

Take the courses as given on page 334 and proceed as follows—

NOTE.—Leeway is associated with heavy weather and in heavy weather a course cannot be steered to degrees —therefore, both course and leeway are generally expressed in points. Having first corrected the course for leeway proceed as before.

Compass Co. East, leeway $\frac{3}{4}$ pt., wind S.S.E. If the ship's head is E. and the wind S.S.E. it must press on starboard side and force her to the left or northward, therefore—



This fig. represents a compass.

Compass Co. E. or N. 8 pts. E.
Leeway $\frac{3}{4}$ pts.
N. $7\frac{1}{4}$ pts. E. = N. 81° 34′ E.

Wind S.S.E., therefore, allow leeway to the left.

then, if the var. is 10° W. and dev. 10° W., Error = — 20 W.

The true course would be N. 61 34 E.

Compass Co. N.N.E. = N. 2 pts. E.
Leeway 1 pt.
N. 3 pts. E. = N. 33° 45′ E.
Error + 8
True Co. = N. 41 45 E.

Wind N.W., Port tack, therefore, allow leeway to the right.

Var. 10° E. Dev. 2° W.
Error = 8° E.

Compass Co. N.W. = N. 4 pts. W.
Leeway 2 pts.
N. 6 pts. W. = N. 67° 30′ W.
Error — 18 E.
True Co. = N. 49 30 W.

Wind N.N.E., Starboard tack, therefore, allow leeway to the left.

Var. 20° E. Dev. 2° W.
Error = 18° E.

Compass Co. S.E. = S. 4 pts. E.
Leeway $1\frac{1}{2}$ pts.
S. $5\frac{1}{2}$ pts. E. = S. 61° 53′ E.
Error + 2 30 W.
True Co. = S. 64 23 E.

Wind S.S.W., Starboard tack, therefore, allow leeway to the left.

Var. 12′·30″ W. Dev. 10° E.
Error = 2° 30′ W.

Compass Co. W.S.W. = S. 6 pts. W.
Leeway $2\frac{1}{2}$ pts.
S. $8\frac{1}{2}$ pts. W.; that is, $\frac{1}{2}$ pt. in excess of W., therefore N. $7\frac{1}{2}$ pts. W. = 84° 23′.

Wind south, Port tack, therefore, allow leeway to the right.

N. 84° 23′ W.
Error + 50 30 W.
N. 134 53 W.
180 00
True Co. = S. 45 07 W.

Var. 30° 15′ W. Dev. 20° 15′ W.
Error = 50° 30′ W.

Compass Co. S. = 0
Leeway $2\frac{1}{2}$ pts.

Wind E.S.E., port tack, therefore allow leeway to the right.

S. $2\frac{1}{2}$ pts. W. = S. 25° 19′ W.
Error + 11 15 E.
True Co. = S. 36 34 W.

Var. 16° E. Dev. 4° 45′ W.
Error = 11° 15′ E.

Compass Co. E. $\frac{1}{2}$ S. = S. $7\frac{1}{2}$ pts. E.
Leeway $1\frac{3}{4}$ pts.

Wind N.N.E., port tack, therefore allow leeway to the right.

S. $5\frac{3}{4}$ pts. E. = S. 64° 41′ E.
Error + 10 30 W.
True Co. = S. 75 11 E.

Var. 9° 30′ E. Dev. 20° W.
Error = 10° 30′ W.

Compass Co. W. $\frac{1}{2}$ N. = N. $7\frac{1}{2}$ pts. W.
Leeway $1\frac{1}{4}$ pts.

Wind N.N.W., Starboard tack, therefore allow leeway to the left.

N. $8\frac{3}{4}$ pts. W., or $\frac{3}{4}$ pt. in excess of west, therefore S. $7\frac{1}{4}$ pts. W. = S. 81° 34′ W.

S. 81° 34′ W.
Error + 35 34 E.
S. 117 08 W.
180 00
True Co. = N. 62 52 W.

Var. 15° 15′ E. Dev. 20° 19′ E.
Error = 35° 34′ E.

For further examples see Day's Work.

Form of Deviation Table

In the following "Deviation Table," column 1 represents the *direction of the ship's head by compass*, when the Deviations of column 2 were ascertained; column 3 gives the magnetic courses.

1 Ship's Head (course) by Standard Compass.	2 Deviation.	3 Correct Magnetic Course made good.	1 Ship's Head (course) by Standard Compass.	2 Deviation.	3 Correct Magnetic Course made good.
	° ′	° ′		° ′	° ′
North.	5 15 E.	N. 5 15 E.	North.	5 15 E.	N. 5 15 E.
N. by W.	6 0 E.	N. 5 15 W.	N. by E.	4 0 E.	N. 15 15 E.
N.N.W.	7 0 E.	N. 15 30 W.	N.N.E.	2 20 E.	N. 24 50 E.
N.W. by N.	8 15 E.	N. 25 30 W.	N.E. by N.	0	N. 33 45 E.
N.W.	9 40 E.	N. 35 20 W.	N.E.	3 0 W.	N. 42 0 E.
N.W. by W.	11 30 E.	N. 44 45 W.	N.E. by E.	6 40 W.	N. 49 35 E.
W.N.W.	13 30 E.	N. 54 0 W.	E.N.E.	10 40 W.	N. 56 50 E.
W. by N.	15 30 E.	N. 63 15 W.	E. by N.	15 0 W.	N. 63 45 E.
West.	17 15 E.	N. 72 45 W.	East.	19 0 W.	N. 71 0 E.
W. by S.	18 0 E.	N. 83 15 W.	E. by S.	22 30 W.	N. 78 45 E.
W.S.W.	18 0 E.	S. 85 30 W.	E.S.E.	24 40 W.	N. 87 50 E.
S.W. by W.	16 40 E.	S. 72 55 W.	S.E. by E.	25 30 W.	S. 81 45 E.
S.W.	14 0 E.	S. 59 0 W.	S.E.	24 40 W.	S. 69 40 E.
S.W. by S.	10 0 E.	S. 43 45 W.	S.E. by S.	22 15 W.	S. 56 0 E.
S.S.W.	4 50 E.	S. 27 20 W.	S.S.E.	18 15 W.	S. 40 45 E.
S. by W.	1 0 W.	S. 10 15 W.	S. by E.	13 0 W.	S. 24 15 E.
South.	7 15 W.	S. 7 15 E.	South.	7 15 W.	S. 7 15 E.

EXAMPLES FOR PRACTICE.

Correct the following Compass Courses for Leeway, Variation, and Deviation of the Compass.

Where the Deviation is not given, it is to be taken from the Table on p. 339.

Compass Course.	Wind.	Leeway.	Variation.	Deviation.	Answers: Corrected Courses.
1. N.N.E.	N.W.	¼ pt.	1¾ pt. E.	20° W.	N. 25° 0′ E.
2. S.E. by E.	S. by W.	½ "	1¼ " E.	17° W.	S. 64° 49′ E.
3. S.W. by S.	W. by N.	¾ "	1¼ " E.	11° W.	S. 28° 23′ W.
4. N.W.	W.S.W.	¼ "	¾ " E.	13° W.	N. 46° 45′ W.
5. N.E.	E.S.E.	1¼ "	1½ " W.	14° E.	N. 28° 4′ E.
6. S. by E.	E. by S.	¾ "	1½ " W.	10° E.	S. 3° 41′ E.
7. W. ½ S.	N.N.W.	½ "	¾ " W.	11° E.	S. 81° 19′ W.
8. W. ¾ N.	N. by W.	¾ "	1¾ " W.	6° W.	S. 62° 19′ W.
9. W. by N. ½ N.	S.W. by S.	¾ "	19° W.	–	N. 71° 30′ W.
10. S. by W.	S.E. by E.	1 "	12° E.	12° E.	S. 46° 30′ W.
11. S. by E.	E. by S.	1½ "	16° W.	—	S. 28° 23′ E.
12. E. by N. ½ N.	S.E. ½ S.	¾ "	16° E.	14° E.	S. 85° 19′ E.
13. N. ¼ E.	E.N.E.	½ "	14° W.	—	N. 11° 53′ W.
14. N.W. ¼ N.	N.N.E.	¾ "	7° E.	15° W.	N. 58° 38′ W.
15. S.W. ¼ W.	S.S.E.	¾ "	3° W.	—	S. 67° 55′ W.
16. E. ¾ S.	S. by E.	¾ "	16½° W.	12½° W.	N. 61° 0′ E.
17. W. by S.	S. by W.	1¼ "	4½° E.	—	N. 64° 41′ W.
18. S. ¼ W.	W.S.W.	½ "	7½° W.	11½° E.	S. 1° 11′ W.
19. West.	N.N.W.	1 "	11¼° W.	22½° E.	West.
20. East.	N.N.E.	1¼ "	14° E.	14° W.	S. 75° 56′ E.
21. North.	W.N.W.	1 "	11¼° E.	22½° W.	North.
22. South.	E.S.E.	1½ "	19½° W.	—	S. 9° 53′ E.
23. E. ¾ N.	N. by E.	1¾ "	23½° W.	20° W.	N. 57° 45′ E.
24. S. ¼ E.	W.S.W.	½ "	19½° E.	27° E.	S. 38° 4′ W.
25. N.E. by E. ½ E.	N. by W.	¾ "	6½° E.	—	N. 66° 20′ E.
26. South.	E.S.E.	1½ "	22½° W.	5½° E.	S. 0° 8′ E.
27. East.	N.N.E.	1¼ "	10° W.	5° W.	N. 89° 4′ E.
28. E. ½ N.	S.S.E.	1¼ "	29½° W.	—	N. 28° 49′ E.
29. N.E. ¼ N.	E.S.E.	½ "	11¼° E.	21½° E.	N. 69° 19′ E.
30. S. by W. ½ W.	E.S.E.	¾ "	15° W.	—	S. 7° 57′ W.

There are various *graphic* methods of delineating the deviation of the compass, and for converting correct magnetic courses into compass courses, and compass courses into correct magnetic courses; that introduced here is due to Mr. J. R. Napier, F.R.S., of Glasgow. It is equally applicable whether the points on which observations have been made are or are not precisely equidistant; and only requires a moderate degree of dexterity in drawing a curved line.

Construction of the Diagram.—As originally proposed the diagram consisted of a vertical line divided into 32 equal parts, representing the 32 points of the compass; it was also subdivided into 360 equal parts, representing degrees, reckoned from 0° at North and South to 90° at East and West; thus it might be considered as the margin of a compass card, cut at the North point, and straightened out.

The diagram that accompanies this work is what has just been described, with the exception that the vertical line is lengthened by a point of the compass at top and bottom to give greater facility in drawing the curve of deviation, should an observation be *near*, but not exactly at North.

The vertical line is intersected at each point of the compass by two straight lines—one plain, the other dotted—each inclined to the vertical line at an angle of 60°.

Observations for the deviations should be made on as many points as convenient, but those on eight points (at or near N., N.E., E., S.E., S., S.W., W., and N.W.) will be ample to furnish a curve sufficiently exact for practical purposes.

Construction of the Curve of Deviation.—*Easterly* deviations are to be laid down on the *dotted* line to the *right* of the vertical line; *Westerly* deviations on the *dotted* line to the *left*.

The method of constructing the curve will be best understood by explaining the projection corresponding to the observations No. 1 (thick curve) below:—

No. 1 (Thick Curve).

Ship's Head by Standard Compass.	Deviation.
	° ′
North	6 30 W.
N.E.	17 40 E.
East	25 0 E.
S.E.	17 40 E.
South	6 30 E.
S.W.	8 40 W.
W.	25 0 W.
N.W.	26 40 W.

No. 2 (Thin Curve).

Ship's Head by Standard Compass.	Deviation.
	° ′
North	4 0 E.
N.E.	7 40 W.
East	25 30 W.
S.E.	28 20 W.
South	4 0 W.
S.W.	22 40 E.
West	25 30 E.
N.W.	13 20 E.

1. Take (with dividers) from the vertical line a distance equal to deviation $6\frac{1}{2}°$, and lay it off on the dotted line, from North towards the left—deviation being W.; at the extremity of the distance make a dot or cross.

2. Take from the vertical line, a distance equal to $17\frac{3}{4}°$, and lay it off on the dotted line, from N.E. towards the right—deviation being E.; make a dot.

3. Take from the vertical line, a distance equal to 25°, and lay it off on the dotted line, from East towards the right—deviation being E.; make a dot.

4. Take from the vertical line, a distance equal to $17\frac{3}{4}°$, and lay it off on the dotted line, from S.E. towards the right—deviation being E.; make a dot.

5. Take from the vertical line, a distance equal to $6\frac{1}{2}°$, and lay it off on the dotted line, from South towards the right—deviation being E.; make a dot

6. Take from the vertical line, a distance equal to $8\frac{3}{4}°$, and lay it off on the dotted line, from S.W. towards the left—deviation being W.; make a dot.

7. Take from the vertical line, a distance equal to 25°, and lay it off on the dotted line, from West towards the left—deviation being W.; make a dot.

8. Take from the vertical line, a distance equal to $26\frac{3}{4}°$, and lay it off on the dotted line, from N.W. towards the left—deviation being W.; make a dot.

9. Repeat at the lower end of the vertical line the first admeasurement from North.

10. With a pencil and a light hand draw a flowing curve, passing through all the dots; when satisfied that the curve is good, draw it in ink. You then have *a curve of deviations*, by means of which the deviation on any compass course or any magnetic course may be found, as well as the compass course corresponding to any *correct* magnetic course.

In a similar manner lay down the deviations corresponding to observations No. 2 (*thin curve*).

Application of the Curve of Deviation:—

I. *To find the Deviation on any Compass Course.*—On the vertical (central) line find the given course; measure (with dividers) the distance from that point to where the curve cuts the dotted line proceeding from the point; that distance measured on any part of the vertical line will give the deviation in degrees.

Thus, the deviation, by *thick curve*, on N.E. by N. is 13° E.; and on W.S.W. is $17\frac{1}{2}°$ W.

To find the Deviation on any Magnetic Course.—On the vertical line find the given magnetic course; measure with a pair of dividers along, or parallel to the plain line, until you reach the curve; the distance in the dividers measured on any part of the graduated central line will be the deviation on that magnetic course.

II. *From a given Compass Course to find the corresponding Correct Magnetic Course.*—On the vertical line find the given compass course, from

which move in a direction parallel to the *dotted* line till you arrive at the curve, and thence move in a direction parallel to the *plain* line till you get back to the vertical line. The point on the vertical line at which you arrive is the *correct* magnetic course required. (*a*)

Thus, by *thick curve*, compass course N.E. by N. gives *correct* magnetic course N. 47° E., and compass course W.S.W. gives *correct* magnetic course S. 50° W.

III. *Given a Correct Magnetic Course to find the corresponding Compass Course.*—On the vertical line find the given *correct* magnetic course, from which move in a direction parallel to the *plain* line till you arrive at the curve, and thence move in a direction parallel to the *dotted* line till you get back to the vertical line. The point on the vertical line at which you arrive is the compass course required.(*b*)

Thus, by *thick curve*, for *correct* magnetic course S.E., steer by compass S. 67° E.; and for *correct* magnetic course W. by N., steer by compass N. 51° W.

IV. *Bearings require correction for the direction of the ship's head.*—Thus, if the bearing by compass is N.E., and the ship is heading by compass S.E. on which the deviation is 28° W. (by *thin curve*), the correct magnetic bearing will be 28° to left of N.E., *i.e.*, N. 17° E.

(*a*) " From compass course magnetic course to gain
Depart by dotted and return by plain.

(*b*) " But if you seek to steer a course allotted,*
Depart by plain and return by dotted."

With the deviation as above, give the courses you would steer by the Standard Compass to make the following courses, correct magnetic—(*using thin curve*).

	E.N.E.	N. by W.	N.E.	S.W.
Ans.	S. 86° E.	N. 19° W.	N. 58° E.	S. 29° W.

Supposing you have steered the following courses by the Standard Compass, find the correct magnetic courses made from the above deviation table—(*using thin curve*).

	S.S.W.	S.E. by E.	E.N.E.	W. by N.
Ans.	S. 33° W.	S. 87° E.	N. 50° E.	N. 56° W.

You have taken the following bearings of two distant objects by your Standard Compass as above; with the ship's head at S. by E., find the bearings, correct magnetic—(*using thin curve*).

Bearings by compass S.W. ½ W. and N.W. by N. give
Ans. Correct magnetic bearings S. 37½° W. and N. 47° W., since the deviation on S. by E. is 13° W.

(*From thick curve*).—Give the courses you would steer by the Standard Compass to make the following courses, correct magnetic.

	E. by N.	W. by S.	S.E. by S.	N.N.W.
Ans.	N. 57° E.	N. 74° W.	S. 53° E.	N. 10° W.

* The course allotted is the magnetic Course from the chart.

(*From thick curve*).—Supposing you have steered the following courses by the Standard Compass, find the correct magnetic courses made.

	E. by S	W. by S.	E. by N.	N.W. by N.
Ans.	S. 54° E.	S. 58° W.	S. 75° E.	N. 55° W.

(*From thick curve*).—You have taken the following bearings of two distant objects by your Standard Compass with the ship's head at E. by N.; find the bearings, correct magnetic?

Bearings by compass N.E. and S.E. give

Ans. Correct magnetic bearings N. 70° E. and S. 20° E., since the deviation on E. by N. is 25° E.

(*From thin curve*).—Give the courses you would steer by Standard Compass to make the following courses, correct magnetic.

	E. by N.	S.S.E.	W. by N.	N.W. by N.
Ans.	S. 73° E.	S. 11° E.	S. 75° W.	N. 47° W.

(*From thin curve*).—Supposing you have steered the following courses by the Standard Compass, give the correct magnetic courses.

	S.W. by W.	N.W.	E.S.E.	S.S.E.
Ans.	S. 80° W.	N. 32° W.	N. 83° E.	S. 43° E.

(*From thin curve*).—You have taken the following bearings of two distant objects by your Standard Compass, with the ship's head at S.E. by S.; find the bearings, correct magnetic.

Bearings by compass N.E. by E. and N.W. ½ N.

Ans. Correct magnetic bearings N. 31° E. and N. 64° W., since the deviation on S.E. by S. is 25° W.

(*From thick curve*).—Give the courses you would steer by the Standard Compass to make the following courses, correct magnetic.

	E. by N.	S.S.E.	W. ½ N.	N.W. by N.
Ans.	N. 57° E.	S. 39° E.	N. 57° W.	N. 18° W.

Supposing you have steered the following courses by the Standard Compass, give the correct magnetic courses made good—(*using thick curve*).

	S.W. by W	N.W. ½ N.	S.E. by E.	N. by W.
Ans.	S. 43° W.	N. 65° W.	S. 37° E.	N. 23° W.

You have taken the following bearings of two distant objects by your Standard Compass, with the ship's head at W. by S. ½ S.; find the bearings, correct magnetic—(*using thick curve*).

Bearings by compass	N.E. by E.	and	N.W. ½ N.
	N. 56¼° E.		N. 39½° W.
Dev. on W. by S. ½ S.	= 21 W.	. .	21 W.
Bearings Corr. Mag.	N. 35¼ E.		N. 60½ W.

N.B. Bearings must always be corrected for the deviation due to the direction of the ship's head at the time the bearings were taken; in this case the deviation on W. by S. ½ S. must be used.

✓

THE DAY'S WORK

A SHIP'S RECKONING is that account by which it can be known at any time *where the ship is,* and on what course or courses she must steer to gain her port.

Shaping the Course.—Having taken the departure, the course must then be shaped, and from that moment the run by log must be noted. Having decided on the track, the true or magnetic course can be taken from the chart ; if the latter, allowance must be made for the deviation due to the direction of the ship's head to obtain the compass course. If sailing in a known current there should be deduced and allowed a proper amount for it. Should the wind be ahead, the tack must be chosen upon which the greatest distance will be made good towards the port of destination.

The LOG-BOOK is a book ruled in order to contain the daily copies of the remarks written at intervals during the day of everything connected with the progress of the ship, on her voyage, and any occurrences worthy of notice. It is strictly a journal, each page being ruled for one day ; with other important entries superadded, it is the official, and only authentic, record of the daily proceedings on board ship, and is divided as follows, under the headings of :—

H.	K.	.	Courses.	Winds.	Lee-way.	Dev	Remarks, Monday, April 10th.

The column on the left contains the 24 *Hours* from the noon of one day to the noon of the next, divided into two portions of 12 hours each.

In the second and third columns are the *K*nots and *F*athoms (or *K*nots and *T*enths) the ship is found to run per hour, set against the hours when the log was hove.

N.B.—The log is generally hove once in two hours, and the intervening rate inferred. The "patent log" gives the miles run between the intervals of observing it.

The fourth column contains the *Courses* the ship steers.

The fifth column gives the direction of the *Winds*.

The sixth column notes the *Leeway*, when any has been made.

The seventh column is for the *Deviation* on the ship's head by compass.

The eighth column, for *Remarks*, gives :—The kind of wind and weather —as moderate, fresh, a gale, squally, strong gale, foggy, cloudy, etc. ; the state of the sea—as smooth, moderate, high, or heavy, etc. ; the alteration of the sails—as tacked ship, squared the yards, etc. ; the business doing aboard, as to the employment of the crew ; when the ship was pumped, and the water in the well ; the ships sighted or spoken ; if in soundings, the depth of water and nature of the bottom, etc. ; the *Variation* of the compass ; and what other remarks the *officer of the watch* thinks it his duty to insert.

In the Royal Navy the time is reckoned as on shore, and has been kept in civil time since 1805, by order of the Admiralty. In the Merchant Service it has been the custom to begin the day at noon, and presumably this continues to be the case in the majority of ships, and with most Masters.

But there can be no doubt that the ship's log should be kept in Civil Time. 24 hours to the day,—the first 12 hours being A.M., and the second 12 hours P.M.; by this mode of reckoning, the noon of the Date by Log agrees with the astronomical day, and there can be no confusion in the correction of the elements taken from the Nautical Almanac. The civil day begins at midnight and ends at the midnight following; the astronomical day begins at noon and ends at the noon following: but the noon of any given date, as May 12, or November 20, is the same in both methods of reckoning; hence the civil day is 12 hours in advance of the astronomical day

The barbarism of reckoning by a nautical day, 12 hours in advance of the civil, and 24 hours in advance of the astronomical, day, cannot be too much deprecated, and must have frequently led to errors in the computation of the astronomical data. Besides, two modes of reckoning dates must surely be enough, without the complication of a third, and wholly useless date.

DEAD RECKONING is the diff. lat., dep., and diff. long.,—the lat. in and long. in—and the course and distance made good,—derived from the courses and distances made during the 24 hours. The latitude and longitude obtained from the log-board or log-book is written lat. by D.R. and long. by D.R.; or lat. by acc. (account) and long. by acc.—to distinguish them from what is obtained by *observation* of the sun, etc., which is written lat. by obs., and long. by obs.

NOTE.—*The error in the Distance arising from an error of a quarter of a point in the Course is about one mile in twenty.*

The Day's Work

WE will now proceed to the solution of the day's work—which is the determination of the ship's position by *dead reckoning*, the basis of which is to be found in the log-book, which has been copied from the scrap log

Summary of Rules for working a Day's Work

When correcting a course or bearing, always suppose yourself standing in the *centre* of the compass, looking in the *direction* of the course or bearing

A compass course may be affected by leeway, deviation and variation.

A magnetic course can be affected by leeway and variation, *not* by deviation.

A compass bearing may be affected by deviation (due to the direction of the ship's heading or course), and by variation.

A magnetic bearing can be affected only by variation.

When *correcting for leeway*—

Allow it to the right if on the port tack;
Allow it to the left if on the starboard tack;

that is, in a direction *from* the wind.

When *correcting* a compass course *for deviation*—

Allow it to the *right*, if Easterly;
Allow it to the *left*, if Westerly.

1 Reverse the departure bearing; express it in degrees, and apply the deviation (due to the direction of the ship's head), and the variation; the result is to be taken as a course.

2. Correct each course in succession, allowing the leeway, applying the deviation, and the variation.

3. The current being *magnetic*, treat it as a course, but only apply the variation. The drift is taken as distance.

4. The true courses being thus found are then, with their respective distances, to be entered in the following form :—

True Courses	Dist	Diff Lat		Departure.	
		N	S	E	W.

5. Enter Traverse Table with the several courses and distances, and fill up the difference of latitude and departure columns. Remember to read from the top of Traverse Table if course is less than 45°, but from the bottom if over 45°. Also, if distance exceeds 600 miles, enter the Traverse Table with *half* the distance, take out the corresponding difference of latitude, and departure, and then double them.

6. Add together the quantities in the N. column, and also in the S. column; then take the difference of the two sums, giving the remainder the name of the greater quantity (N. or S.); the result will be the difference of latitude made good.

7. Add together the quantities in the E. column, and also in the W. column; then take the difference of the two sums, giving the remainder the name of the greater quantity (E. or W.); the result will be the departure made.

8. For the course and distance made good, enter Traverse Table and seek out the page where the difference of latitude and departure made are side by side, reading headings of columns from the top if difference of latitude is greater than departure, but from the bottom if difference of latitude is the less. The course (in degrees) is taken from the top of the page if difference of latitude is greater than departure, but from the bottom if difference of latitude is the less. The distance comes from distance column alongside the given difference of latitude and departure; name

the course according to the difference of latitude and departure, and write the result thus—course made good N. or S. . . .° E. or W. ; Dist. . . . miles.

9. FOR THE LATITUDE IN.—Under the latitude left write the difference of latitude ; if both have the same name (N. or S.) take their sum for *latitude in*, of the same name as latitude left ; if one is N. and the other S. take their difference for the *latitude in*, of the same name as the greater.

10. *For the Middle Latitude.*—Add together the latitude left and latitude in, if both have the same name (N. or S.), and divide the sum by 2.

11. FOR THE DIFFERENCE OF LONGITUDE.—Enter the Traverse Table with

Mid. lat. (as a course) and dep. (in D. lat. col.)	the dist. (in dist. col.) is the required diff. long.

Or, for accuracy, by middle latitude sailing.

log. diff. long. = log. dep. + L sec. mid. lat.

Dep. . . . m.	log.	. . . .
Mid. lat. . .° . .	sec.	. . . .
Diff. long. . . .'	log	. . . .

If one latitude is N. and the other S. the departure may be taken as the difference of longitude.

12. FOR THE LONGITUDE IN.—Under the longitude left write the difference in longitude with the same name as the departure. If longitude left and difference of longitude have the same name (E. or W.) take their sum for the *longitude in*, of the same name as longitude left ; *except* this sum exceed 180°, in which case subtract it from 360°, and mark the remainder W. when longitude left is E., but E. when longitude left is W. If longitude left and difference of longitude have different names (one E. and the other W.), take their difference for the *longitude in*, of the same name as the greater.

The day's work is thus completed.

EXPLANATION OF DAY'S WORK, NO. 1

The courses, leeway, and variation being in *points* of the compass, you will retain them so, and use Traverse Table for points.

1st. Begin with the bearing of the point of land ; reverse it, and enter it as a course after correcting for variation.

2nd. Take each course in succession, correct for leeway (if any), and then for variation ; also sum up the distance run on each course. The third column being marked F, remember that 8 furlongs = 1 knot.

3rd. Take the current as the last course, and correct for variation.

4th. Then proceed as directed in the foregoing summary.

No. 1—*This is a copy of the Scrap log, divested of the bulk of the remarks*

H	K	F	Courses	Winds.	Lee-way. Pts	Remarks.
1	6	0	W. by N.	North	0	P.M. The Departure is taken from the Lizard in
2	6	4				Lat. 49° 58′ N.
3	6	4				Long. 5° 12′ W.
4	6	2				bearing by compass (magnetic) N.E. by E. distant 14 miles.
5	5	6				
6	6	4	S.W. by W.	N.W. by W.	¼	
7	6	4				
8	7	0				
9	6	6	S.W.	W.N.W.	½	
10	6	4				Variation 2 points West.
11	6	6				
12	7	0	N.N.W.	West	¾	
1	6	6				A.M.
2	6	4				
3	5	6				
4	6	0	N.N.E. ½ E.	W.N.W.	0	
5	6	4				
6	5	6				
7	5	0				
8	4	6				
9	5	6	W.S.W.	N.W.	½	Current set ship N.W. by N.
10	6	4				(magnetic) 8 miles during
11	6	0				the last 12 hours.
12	5	6				

Dep. Co.
S. 5 W.
2 W.
S. 3 W.

1st Co.
N. 7 W.
2 W.
N. 9 W.
16
S. 7 W.

2nd Co.
S. 5 W.
¼
S. 4¾ W.
2 W.
S. 2¾ W.

Cur. Co.
N. 3 W.
2 W.
N. 5 W.

Prove the rest yourself.

Corrected Courses.	Dist.	Diff. Lat. N.	Diff. Lat. S.	Departure E.	Departure W.
S. 3 pts. W.	14		11·6		7·8
S. 7 W.	31		6·0		30·4
S. 2¾ W.	20		17·2		10·3
S. 1½ W.	20		19·1		5·8
N. 3¼ W.	26	20·9			15·5
N. ½ E.	28	28·0		1·4	
S. 3½ W.	24		17·8		16·1
N. 5 W.	8	4·4			6·7
		53·3	71·7	1·4	92·6
			53·3		1·4
		Diff. Lat. 18·4		Dep. 91·2	

Lat. left	49° 58′ N.
Diff. Lat.	18 S.
Lat. in	49 40 N.
	2)99 38
Mid. Lat.	49 49

Mid. Lat. 49° and Dep. 91·2 (in Lat. col.) in Trav. Tab. give 139 in dist. col., for Diff. Long.
Mid. Lat. 50° and Dep. 91·2 (in Lat. col.) in Trav. Tab. give 142 in Dist. Col., for Diff. Long.
Take 141 for Diff. Long. corresponding to Mid. Lat. 49¾°.

Long. left	5° 12′ W.
Diff. Long. 141′ =	2 21 W.
Long. in	7 33 W.

Diff. Lat. 18·4 S. and Dep. 91·2 W. in Trav. Tab. give Course made good S. 78½° W. and Dist. 93 miles.

We have refined here, which is not generally necessary: it would have been sufficient to have taken Mid. Lat. 50° to find the D. Long.

These results would be entered in the log-book in something like the following form, beneath the day's (24 hours) transactions :—

Course. made good.	Dist. run.	Diff. Lat.	Dep.	Latitude.		Diff. Long.	Longitude.	
				By D. R.	By Obs		By D. R.	By Obs.
S. 78½° W.	93 m.	18′·4 S.	91′·2 W	49° 40′ N.		141′W	7° 33′ W.	

Where there is no deviation of the compass, there is no absolute necessity, under ordinary circumstances, to correct the courses for variation, until you have arrived at a difference of latitude and departure from the courses corrected for leeway ; it saves trouble, and the result is equally accurate.

The following is the day's work just solved ;—the courses are corrected for leeway, and with the distances, a difference of latitude and departure are found in the usual way ; then this difference of latitude and departure give a course and distance (1):—Now correct the Course (just obtained) for variation, and you get the true course (2) ;—with the true course and distance get the true difference of latitude and true departure (3) ; and the remainder of the solution is as before.

N.B.—The figures in this method (below) are as many as by the former one, but you remember we have not been at the trouble of correcting each course separately for variation.

Courses. Corrected for Leeway.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
S. 5 W.	14		7·8		11·6
N. 7 W.	31	6·0			30·4
S. 4¾ W.	20		11·9		16·1
S. 3½ W.	20		15·5		12·7
N. 1½ W.	20	25·2			6·3
N. 2¼ E.	28	25·3		12·0	
S. 5½ W.	24		10·3		21·7
N. 3 W.	8	6·7			4·4
		63·2	45·5	12·0	103·2
		45·5			12·0
	Diff. Lat.	17·7		Dep.	91·2

(1) Diff. Lat. 17·7 N. and Dep. 91·2 W. in Traverse Table give Course N. 79° W. and Dist. 93 m.

(2) Then Course corrected for 2 pts. W. var. gives True Co. S. 78½° W.

(3) True Co. S. 78½° W. or W. by S. and Dist. 93 in Traverse Table give True Diff. Lat. 18·1 S., and True Dep. 91·2 W.

Lat. left............	49°	58′	N
Diff. Lat.		18	S.
Lat. in	49	40	N.
	2)99	38	
Mid. Lat.	49	49	

Mid. Lat. 49¾° and Dep. 91·2 (in Lat. col.) give in Traverse Table Dist. 141, which is Diff. Long.

Long. left..........	5°	12′	W.
Diff. Long. 141′ = ..	2	21	W.
Long. in	7	33	W.

At sea, in the second day's work, and so on, there would be no departure other than the latitude and longitude left on the previous day.

DAY'S WORK No. 2 has leeway, deviation, and variation ; and the solution is by correcting the courses, first, for leeway (if any) ; then for deviation ; and, lastly, for variation ;—unless you correct by the *error of the compass*, that is, taking deviation and variation together.

H.	K.	T.	Courses.	Winds.	Leeway. Pts.	Dev.	Remarks.
1	6	6	S. by W. ½ W.	W. by S.	¼	5° E.	P.M. The Departure is taken from the Old Head of Kinsale in Lat. 51° 37′ N. Long. 8° 32′ W. bearing by Compass N.W. distant 12 miles; Ship heading S. by W. ½ W. with Deviation 5° E.
2	6	4					
3	6	6					
4	6	4					
5	7	0	S.W. ¾ W.	W.N.W.	¼	9° W.	
6	7	4					
7	7	6					
8	8	0					
9	8	0	W.S.W.	N.W.	½	12° W.	
10	7	6					
11	7	4					
12	7	0					
1	7	0					A.M.
2	6	6					
3	6	4					Variation 24° W.
4	7	0	N. ½ W.	W.N.W.	¾	4° W.	
5	7	6					
6	7	6					
7	8	4					
8	7	4					Allow for Current setting W. by N. (magnetic) 14 miles during the last 16 hours.
9	8	0	S.E. ½ E	S.W.	0	11° E.	
10	8	0					
11	7	6					
12	8	4					

Dep. Co.	1st Co.	2nd Co.
S. 45° E.	S. 1½ W.	S. 4¾ W
5 E.	¼	¼
S. 40 E.	S. 1¼ pt. W.	S. 4½ W.
24 W.	S. 14° W.	S. 51°W
S. 64 E.	5 E.	33 W
	S. 19 W.	S. 18 W
Or S. 45° E.	24 W.	
24—5 = 19 W.	S. 5 E.	
S. 64 E.	etc., etc.	

Prove the rest yourself, and to Current apply only the Var.

Corrected Courses	Dist.	Diff. Lat. N.	Diff. Lat. S.	Departure E.	Departure W.
S. 64° E.	12		5·3	10·8	
S. 5 E.	26		25·9	2·3	
S. 18 W.	30		28·5		9·3
S. 26 W.	50		44·9		21·9
N. 22 W.	38	35·2			14·2
S. 64 E.	32		14·0	28·8	
S. 77 W.	14		3·1		13·6
		35·2	121·7	41·9	59·0
			35·2		41·9
		Diff. Lat. 86·5		Dep. 17·1	

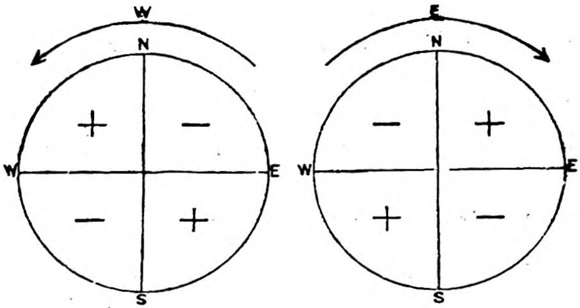
Lat. left.	51° 37′ N.	
Diff. Lat. 86·5 =	1 27 S.	
Lat. in	50 10 N.	
	2)101 47	
Mid. Lat.	50 53	

Mid. Lat. 51° and Dep. 17·1 (in Lat. col.) Traverse Table give 27 in Dist. col., for Diff. Long ; or by calculation as below.

Mid. Lat.	50° 53′	sec. 0·2000		Long. left.	8° 32′ W	
Dep.	17·1	log. 1·2330		Diff. Long.	27 W	
D. Long.	27′·1	log. 1·4330		Long. in	8 59 W	

Diff. lat. 86·5 S. and dep. 17·1 W. in Traverse Table for points give course made good S. by W., and distance 88 miles.

DAY'S WORK No. 3, with courses corrected by using plus and minus signs as shown by diagram.



BEARING REVERSED.
N.N.E.½E. = N 28° 8' E
Var. 20 W, Dev. 7 W } − 27 0 W
N 1 8 E

1ST COURSE.
N.N.W. = N 22° 30' W
Leeway = 5 38 L
N 28 6 W
Var. 20 W, Dev. 7 W } + 27 0 W
N. 55 6 W

2ND COURSE.
W.N.W. = N 67° 30' W
Leeway = 8 26 L
N 75 56 W
Var. 20 W, Dev. 18 W } + 38 0 W
N 113 56 W
180 00
S 66 4 W

3RD COURSE.
S.S.W.½W = S 28° 8' W
Leeway = 2 49 R
S 30 57 W
Var. 20 W, Dev. 9 W } − 29 00 W
S 1 57 W

H.	Courses.	K.	T.	Winds.	Leeway	Dev.	Remarks, etc.
					Pts.		
1	N.N.W.	9	8	N.E.	½	7° W.	Departure from a point in
2		9	8				Lat. 62° 11' N.
3		9	8				Long. 5° 8' E.
4		9	6				bearing by compass
5	W.N.W.	9	6	North	¾	18° W.	S.S.W. ½ W
6		8	0				distance 10 miles.
7		8	0				Ship's Head N.N.W.
8		9	2				Dev. as per log.
9	S.S.W.½W.	9	2	S.E.	¼	9° W	
10		9	8				
11		10	0				
12		10	0				
1	S. by E.	10	0	E. by S.	0	3° E.	Variation 20° W.
2		10	0				
3		10	0				
4		10	0				
5	S. ½ E.	9	6	W.S.W.	1	2° E.	A Current set S.W. by W., correct
6		8	6				magnetic 36 miles
7		8	6				from the time the
8		8	2				departure was taken
9	N.N.E.	8	2	N.W.	1½	6° E.	to the end of the
10		7	8				day.
11		7	8				
12		7	2				

Day's Work No. 3, *continued.*

4TH COURSE.

S. by E. =	S 11° 15′ E
Var. 20 W } Dev 8 S }	+17 00 W
	S 28 15 E

5TH COURSE.

S ½ E =	S 5° 38′ E
Leeway =	11 15 L
	S 16 53 E
Var. 20° W } Dev. 2 E }	+18 00 W
	S 34 53 E

6TH COURSE

N.N.E. =	N 22° 30′ E
Leeway =	14 4 R
	N 36 34 E
Var 20 W } Dev 6 E }	14 00 W
	N 22 34 E

CURRENT

S.W. by W. =	S 56° 15′ W
Var. .... —	20 00 W
	S 36 15 W

Courses.	Dist.	N.	S.	E.	W.
N. 1° E.	10	10·0		0·2	
N. 55 W.	39	22·4			31·9
S. 66 W.	36		14·6		32·9
S. 2 W.	39		39·0		1·4
S. 28 E.	40		35·3	18·8	
S. 35 E.	35		28·7	20·1	
N. 23 E.	31	28·5		12·1	
S. 36 W	36		29·1		21·2
		60·9	146·7	51·2	87·4
			60·9		51·2
		D. Lat	85·8	Dep.	36·2

Lat. left	62° 11′ N.	Diff. Lat.... 85′·8 S }	= Course S. 23° W.		
Diff. Lat. 85·8 =	1 26 S	Depart...... 36′·2 W. }	Distance 92 m.		
Lat. in	60 45 N.				
	2) 122 56			Long. left ...	5° 8′ E.
Mid. Lat.	61 27	Sec	10·3206	D. Long.......	1 16 W.
Dep.	36·2	Log	1·5587	Long. in......	3 52 E.
D. Long	60) 75·7	Log.	1·8793		
	1° 16′				

For D. Long. it is sufficient to take logs. to four places of decimals.

Examples of Day's Work for Practice

Example 1 —

H	Courses.	K	T	Winds	Leeway (Pts.)	Remarks.
1	E. ½ N.	4	8	S.S.E.	1½	The departure is taken from a point in Lat. 47° 34′ N. Long. 52° 40′ W. bearing by compass W. by N. ¾ N. distant 17 miles.
2		5	2			
3		4	2			
4		5	8			
5	N. by E. ¾ E.	6	8	East	½	
6		7	8			
7		8	8			
8		9	6			
9	S.E. ¾ S.	9	8	E.N.E.	¼	
10		8	8			
11		8	6			Variation 2¾ points W.
12		7	8			
1	W.N.W. ½ W.	8	8	S.W.	½	
2		9	8			
3		8	8			
4		7	6			
5	N.E. ¾ E.	8	8	N.N.W.	¼	
6		9	8			A current set E.N.E. ½ E., correct magnetic 17 miles, from the time the departure was taken to the end of the day.
7		9	6			
8		8	8			
9	S.E. by E. ¾ E.	7	8	N.E.	½	
10		6	2			
11		6	8			
12		7	2			

Ans. D. Lat. 81′·2 N.; Dep. 73′·2 E.; Course N. 42° E., Dist. 109 m.; Lat. in 48° 55′ N.; Long. in 50° 50′ W.

Example 2.—

H.	Courses.	K.	T	Winds.	Lee-way	Dev	Remarks
					Pts.		
1	N.W. by W.	9	8	N. by E.	1	16° W	The departure is taken from a
2		8	2				point in
3		8	8				Lat 38° 43′ S
4		8	2				Long 77° 35′ E
5	S.E. by S.	9	8	S.W. by S.	½	11° E	bearing by compass
6		9	2				S.S.E ½ E. distant 16 miles
7		10	8				Ship's head N W. by W. with
8		10	2				Dev. as per log. 16° W.
9	S. ¼ W.	10	2	E.S.E.	—	2° W.	
10		10	2				
11		10	2				
12		10	4				
1	W. ½ N.	9	8	N. by W.	¼	26° W.	Variation 25° W.
2		9	6				
3		8	8				
4		8	8				
5	N.N.E. ½ E.	7	8	N.W.	1¾	8° E.	A current set S.E. by E., cor-
6		8	6				rect magnetic 39 miles, from
7		8	8				the time the departure was
8		8	8				taken to the end of the day.
9	S. by E.	9	2	E. by S.	½	3° E.	
10		9	2				
11		9	2				
12		9	4				

Ans. D. lat. 109′ S.; dep. 47′·5 E.; course S. 23½° E., dist. 119 m.; lat. in 40° 32′ S.; long. in 78° 37′ E.

Example 3.—

H.	Courses.	K.	T.	Winds	Lee-way	Dev.	Remarks.
					Pts.		
1	N.N.E.	8	8	N.W.	1	6° E.	The departure taken from a
2		8	8				point in
3		8	6				Lat. 61° 19′ N.
4		7	8				Long. 179° 19′ E
5	N E. by E.	10	8	N. by W.	¼	16° E.	bearing by compass
6		9	8				N.W. by W. distant 18 miles.
7		9	8				Ship's head N.N.E. with
8		9	6				Deviation as per first course.
9	E. ½ S.	8	6	S. by E.	1¼	21° E.	
10		8	0				
11		7	2				
12		6	2				
1	S.S.W.	6	0	S.E.	2½	7° W.	Variation 20° E.
2		6	6				
3		6	8				
4		6	6				
5	W.byN.½N.	7	2	N. by W.	1¾	19° W.	
6		7	2				
7		7	8				
8		7	8				
9	S.S.E.	8	6	S.W.	¾	7° E.	
10		8	8				A current set S.E. by E., cor-
11		8	8				rect magnetic 38 miles, dur-
12		9	8				ing the 24 hours.

Ans. D. lat. 97′·4 S.; dep. 74′·8 E.; course S. 37½° E., dist. 123 m.; lat. in 59° 42′ N.; long. in 178° 9′ W.

Example 4.—

H.	Courses.	K.	T.	Winds.	Lee-way. Pts.	Dev.	Remarks.
					Pts.		
1	N.E. ½ E.	9	8	N.N.W.	1	Use Deviation Table, page 339.	Departure taken from a point in
2		7	6				Lat. 60° 51′ N.
3		7	8				Long. 0° 53′ W.
4		7	0				bearing by compass
5	N.N.E.	8	0	N.W.	½		S.S.W. ½ W., distant 12 miles.
6		9	2				Ship's head N.E. ½ E. by compass.
7		9	0				
8		9	0				
9	E.S.E.	8	0	South	1¼		
10		7	0				
11		7	2				
12		7	0				
1	S.E. ¼ E.	8	6	S.S.W.	¾		Variation 26° W.
2		9	0				
3		9	8				
4		9	8				
5	E. by N.	8	0	N. by E.	1¾		
6		7	0				
7		6	0				
8		6	8				
9	N. ½ W.	6	6	West	2½		
10		6	6				A current set S.W. by S., correct magnetic 36 miles, during the 24 hours.
11		6	6				
12		6	2				

Ans. D. lat. 110′ N.; dep. 94′ E.; course N. 40½° E., dist. 145 m.; lat. in 62° 41′ N.; long. in 2° 26′ E.

Example 5.—

H.	Courses.	K.	T.	Winds.	Lee-way.	Dev.	Remarks.
1							
2	N. 39° W.	6	4	S.W.	3°	3° E.	Departure taken from a point of land in
3		5	6				Lat. 40° 19′ S.
4		4	0				Long. 9° 44′ W.
5		5	4				bearing East by compass.
6	N. 22½° W.	5	4	West	3° 6	8½° E.	distant 20 miles.
7		5	0				Ship's head as per first course.
8		5	6				
9		6	4				
10	N. 22½° E.	6	4	N.W.	6°	20½° E.	
11		5	4				
12		6	2				Variation 20° W.
1	S. 48° E.	6	2	N.W.	0	6° W.	
2		6	0				
3		6	2				
4	S. 34° E.	4	6	S.W.	0	9° W.	
5		5	4				
6	N. 3° W.	5	6	W. by N.	3°	14° E.	
7		6	4				
8		7	0				A current set the ship W. by N. ½ N., correct magnetic 36 miles, during the 23 hours.
9		6	4				
10		6	0				
11		5	0				
12		5	0				

Ans. D. lat. 73′ N.; dep. 53′ W.; course N. 36° W., dist. 90 m.; lat. in 39° 6′ S.; long. in 10° 53′ W.

Day's Work, involving the finding of the Set and Drift

The second method of correcting courses is shown in the two following examples, and should be used in preference to any other method. It consists of reckoning the course from 0° at North, through East, South and West to 360° at North. In this method all easterly deviation or variation is plus, and all westerly deviation or variation is minus; all leeway on the port tack is plus, and all leeway on the starboard tack is minus. When the course has been corrected it is turned into its proper quadrant by the "Table of Compass Equivalents" in Norie's Tables.

To find the set and drift, take the difference between the position by observation and the position by dead reckoning; turn the difference of longitude into departure, using for this purpose the same middle latitude as was used when finding the difference of longitude; then, with this departure and the d. latitude, search in the Traverse Table until they are found side by side in their proper columns, when the number of degrees at the top or bottom, as the case may be, will give the set, and the miles in the distance column the drift. When correcting courses all quantities less than 30′ are dropped, and for quantities more than 30′ increase the degrees by one.

To construct the Figure for the Day's Work (see Fig. 1)

Draw a line in a North and South direction of any convenient length. Take from any scale, in a pair of dividers, the d. latitude, and if the d. latitude be South, lay it off from the top of the line; but if the d. latitude be North, lay it off from the bottom of the line and mark the point at the top or bottom A. With one leg of the dividers on A the other leg will reach a point, which mark C, then A C is the d. latitude. From C draw a line at right angles to A C to the right, if the departure is East, but to the left if the departure is West; take from the same scale, in a pair of dividers, the departure, and with one leg on C, the other leg will reach a point, which mark B; now join A B, and the triangle is complete. The angle at A is the course; A B is the distance; A C the d. latitude; and C B the departure.

To draw the Figure for D. Longitude

At B lay off an angle equal to the middle latitude, and draw B D, then B D is the difference of longitude.

Example 6.—

(1)
S. 23° E. = 157°
Leeway + 11
168°
Dev. — 18
150
Var. + 12
S. 18° E. = 162°

(2)
79°
Leeway + 6
Var. + 12
97
Dev. — 15
N. 82°E.

(3)
South = 180°
Leeway + 17
Var. + 12
209
Dev. — 7
S. 22° W. 202°

(4)
S. 45° W. = 225°
Leeway + 23
Var. + 12
Dev. + 14
N 86° W. = 274°

(5)
S 56° E. 124°
Dev. — 26
98
Var + 12
S 70° E = 110°

H.	Courses.	K.	T.	Winds.	Leeway.	Dev.	Remarks.
					Pts.		
1	S. 23° E.	8		E.N.E.	1	18° W.	The departure was taken from a position in
2		8			1		
3		7	5		1		Lat. 60° 00′ N.
4		7	5		1		
5	N. 79° E.	9		N. by E.	½	15° W.	Long. 179° 30′ E.
6		9			½		
7		9			½		
8		9			½		
9		8	5		½		
10		8	5		½		Variation 12° E.
11	South	6	6	E.S.E.	1¼	7° 15′ W.	
12		6	6		1½		
1		5	4		1½		
2		5	4		1¾		
3	S. 45° W.	4	5	S.S.E.	2	14° E.	
4		4	5		2		
5		5			2		
6	S. 56° E.	7		S. by W.	0	25° 30′ W.	
7		7			0		
8		7			0		
9		8			0		
10		9	5		0		
11		10	5		0		
12		11			0		

The position by Obs. was found to be Lat. 58° 50′ N., Long 177° 12′ W. Find the set and drift of the current.

Courses.	Dist.	N.	S.	E.	W.
S. 18° E.	31		29·5	9·6	
N. 82° E.	53	7·4		52·5	
S. 22° W.	24		22·3		9·0
N 86° W.	14	1·0			14·0
S. 70° E.	60		20·5	56·4	
		8·4	72·3	118·5	23·0
			8·4	23·0	
			63·9	95·5	

Course S. 56° E.
Dist. 115 miles.

Lat. left 60° 00′ N.
Diff. lat. 1 4 S.
Lat. in 58 56 N.
2) 118 56
Mid. lat. 59 28 N.

To find d. long.
d. long. = dep. × Sec. Mid. lat.
Dep. 95·5 log. 1·9800
Mid. Lat. 59° 28 Sec. 10·2941
6) 188′ Log. 2·2741
d. long. 3° 8′ E.

Long. left 179° 30′ E.
Diff. long. 3 8 E.
182 38 E.
360 00
Long. in 177 22 W.

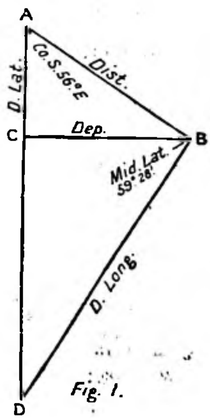


Fig. 1.

To find the set and drift by inspection is simply finding the course and distance from the dead reckoning position to the position by observation.

D. R. lat.	58°	56′ N.		D. R. long.	177°	22′ W.
Observed lat.	58	50 N.		Observed long.	177	12 W.
d. lat. =		6		d. long. =		10

With middle latitude 59° 28′ as course, and d. longitude 10′ in distance column gives 5·15 in departure column in Norie's New Tables. Then with d. latitude 6′ and departure 5′·15, it will be found that the required set is S. 41° E., drift 8′; South because the latitude by observation is South of the latitude by dead reckoning; and East because the longitude by observation is East of longitude by dead reckoning. The mean of departures given by middle latitude 59° and 60° is taken to find the departure for middle latitude 59° 28′, and this mean is 5·15, as shown above.

Example 7.—

(1) 225°
Dev. — 7
218
Var. + 18
236°

(2) South = 180°
Leeway — 6
174
Dev. — 7
167
Var. + 18
S. 5° W. = 185°

Bearing reversed.
(1) S. 45° W. = 225°
Dev. — 7
218
Var. + 18
S. 56° W. = 236°

(2) South = 180°
Leeway — 6
174
Dev. — 7
167
Var. + 18
S. 5° W. = 185°

H.	Courses.	K.	T.	Winds.	Lee-way.	Dev.	Remarks.
					Pts.		
1	South	8		W.S.W.	½	7° W.	Diego Ramirez bore by compass N.E. 30 miles. Ship's head South.
2		8	5				
3		9					
4		9					
5		8	5				
6		8					
7	N. 54° W.	7	5	S.W. by S.	¾	10° E.	
8		7	5				
9		7					
10		7					
11		7	5				Variation for the first 12 hours 18° E., for the second 12 hours 22° E.
12		7	5				
1	N. 34° W.	8		W. by S.	½	8° E.	
2		8					
3		8	5				
4		8	5				
5		9					
6		9					
7	S. 68° W.	9	5	N.E.	0	18° E.	
8		9	5				
9		10					
10		10					Current set East true 1 knot per hour for the day.
11		12					
12		12					

Example 7—continued.

(3) N. 54° W. = 306°
Leeway + 8
Dev. + 10
Var. + 18

N. 18° W. = 342°

(4) N. 34° W. = 326°
Leeway + 6
Dev. + 8
Var. + 22

N. 2° E. + 362°

(5) S. 68° W. = 248°
Dev. + 18
Var. + 22

N. 72° W. = 288°

(6) N. 90° E.

Course.	Dist.	N.	S.	E.	W.
S. 56° W.	30		16·8		24·9
S. 5° W.	51		50·8		4·4
N. 18° W.	44	41·8			13·6
N. 2° E.	51	51·0		1·8	
N. 72° W.	63	19·5			59·9
N. 90° E.	24			24·0	
		112·3	67·6	25·8	102·8
		67·6			25·8
		44·7			77·0

Lat left 56° 31′ S.
45 N.
Lat in 55 46 S.
2) 112 17
Mid lat. 56 8½

Long left 68° 43′ W.
2 18 W.
Long. in 71 01 W.

Course and dist. by inspection N. 60° W.
Dist. 89 miles.

To find d. long.—
D. long = dep. × sec. Mid. lat.
dep. 77′ log. 1·8865
Mid lat. 56 8½ sec. 10·2540
138·2 log. 2·1405
D. long. 2° 18′·2 W.

To find the course—
Tan course = log. dep. + 10 — log. d. lat.
Dep 77′ log 11·8865
D. lat. 44·7 log 1·6503
Course N. 59° 52′ W. Tan = 10·2362

To find the distance—
Dist. = d. lat. × sec. course
D. Lat. 44·7 log 1·6503
Co 59° 52′ sec. 0·2993
Dist. 89·05 m log 1·9496

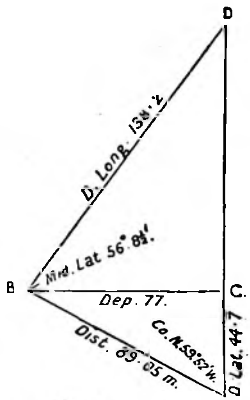


Fig. 2.

Example 8.—

H.	Courses.	K.	T.	Winds.	Lee-way.	Dev.	Remarks.
					Pts		
1	N.E. ¾ N.	8	8	E.S.E.	½	21½° E.	Point of Departure—
2		8	8				Lat. 49° 42′ S.
3		8	6				Long 178°42′ E.
4		8	8				
5	E.N.E. ½ E.	9	0	North	½	15° E.	
6		8	2				
7		9	2				
8		9	6				
9	S.W. ¾ W.	10	0	S.S.E.	¾	15° W.	
10		9	8				
11		8	8				Variation 14° 15′ E.
12		8	4				
1	S. by E. ½ E.	7	8	East	¾	13½° W.	
2		6	8				
3		5	8				
4		5	6				
5	N. ¾ E.	6	8	E. by N.	¾	18° E.	Position by observation—
6		7	8				Lat. 49° 15′ S.
7		8	8				Long. 178° 54′ W.
8		9	6				
9	N. by W. ½ W.	10	0	N.E. by E.	½	12° E.	Find the set and drift.
10		9	8				
11		8	8				
12		9	4				

Ans. D. lat. 17·2′ N.; dep. 66·9′; Course N. 75½° E., dist. 69 m.; lat. in 49° 24·8′ S.; long. in 179° 35′ W.; Set N. 70° E., drift 28 m.

Example 9.—

H.	Courses.	K.	T.	Winds.	Lee-way.	Dev.	Remarks.
					Pts.	Use Deviation Table, page 339.	
1	S.S.W.	8	8	S.E.	½		Departure taken from a point
2		8	8				of land in
3		8	8				Lat. 60° 18′ N.,
4		7	6		1		Long. 172° 4′ W.
5	S.W. by W.	7	6	S. by E.	1		bearing by compass E.N.E.,
6		7	8				distant 12 miles.
7		8	8				Ship's head S.S.W. with dev.
8		7	8				as per Table, p. 339.
9	W. by S.	7	8	S. by W.	1¾		
10		7	6				
11		7	6				
12		7	0				Variation 20° E.
1	S. by E.	7	0	E. by S.	2¼		
2		8	2				
3		8	8				
4		8	0				
5	S.S.E.	8	0	East	1½		
6		8	8				
7		9	2				
8		8	0				
9	W. ½ N.	8	2	N. by W.	¾		A current set W. by S., cor-
10		7	8				rect magnetic 30 miles, from
11		8	8				the time the departure was
12		9	2				taken to the end of the day.

Ans. D. lat. 30·9′ S.; dep. 157·9′ W.; Course S. 79° W. dist. 161 m.; lat. in 59° 47′ N.; long. in 177° 20′ W.

Example 10.—

H.	Courses.	K.	T.	Wind.	Compass Error.	Remarks.
1	N. 75° E.	17	5	N.W.	3° E.	P.M.
2		17	5			At noon the ship was in
3		18	—			Lat. 37° 20′ S.
4		19	—			Long. 177° 15′ E.
5		19	—			
6		20	—			
7		20	—			
8		20	—			
9		20	—			
10		19	—			
11		19	—			Variation 19° E.
12		19	—			
1		19	—			A.M.
2		18	5			
3		18	5			
4		18	—			
5	N. 45° E	17	5	N.W.	6° E.	
6		17	—			
7		17	—			A current set cor. mag. South
8		17	—			20 miles after 4 a.m.
9		17	5			
10		18	—			
11		18	—			
12		18	—			

Ans. D. lat. 132·0′ N. ; dep. 397·7′ E. ; course N. 72° E., dist. 418 m. ; lat. in 35° 8′ S. ; long. in 174° 32′ W.

Example 11.—

H.	Courses.	K.	T.	Wind.	Dev.	Remarks.
1	N. 80° E.	14	5	S.W.	15° W.	P.M. Point of Departure—
2		14	5			Lat. 49° 58′ N.
3		14	5			Long. 10° 12′ W.
4		14	5			
5		14	5			
6		14	5			
7		14	5			
8		15	5			
9	N. 87° W.	15	—	S.W.	17° E.	
10		15	—			
11		15	—			A.M.
12		15	—			
1		14	5			
2		14	5			
3		14	5			
4		13	5			
5		13	5			
6		13	5			
7		13	5			Variation 23° W. during the first
8		13	5			12 hours, and afterwards 25° W.
9		14	—			Position by observation—
10		14	—			Lat. 51° 30·2′ N.
11		14	5			Long. 14° 20′ W.
12		14	5			Find the set and drift of the current.

Ans. D. lat. 69·2′ N. ; dep. 149·0′ W. ; Course N. 65° W., dist. 164 m. ; lat. in 51° 7′ N. ; long. in 14° 6′ W. : Set N. 21° W., drift 24·5 m. by inspection.

DEVIATION CARD.

Ship's Head by Compass.			Ship's Head Magnetic.		
Compass.		Deviation.	Compass.		Deviation.
		°			°
0	North.	7 W.	0	North.	4 W.
10	N. 10 E.	1 E.	10	N. 10 E.	0
20	N. 20 E.	6 E.	20	N. 20 E.	4 E.
30	N. 30 E.	11 E.	30	N. 30 E.	7 E.
40	N. 40 E.	16 E.	40	N. 40 E.	11 E.
50	N. 50 E.	19 E.	50	N. 50 E.	14 E.
60	N. 50 E.	22 E.	60	N. 60 E.	17 E.
70	N. 70 E.	25 E.	70	N. 70 E.	20 E.
80	N. 80 E.	27 E.	80	N. 80 E.	22 E.
90	East.	25 E.	90	East.	24 E.
100	S. 80 E.	24 E.	100	S. 80 E.	25 E.
110	S. 70 E.	23 E.	110	S. 70 E.	25 E.
120	S. 60 E.	20 E.	120	S. 60 E.	24 E.
130	S. 50 E.	18 E.	130	S. 50 E.	22 E.
140	S. 40 E.	16 E.	140	S. 40 E.	19 E.
150	S. 30 E.	14 E.	150	S. 30 E.	18 E.
160	S. 20 E.	11 E.	160	S. 20 E.	15 E.
170	S. 10 E.	8 E.	170	S. 10 E.	11 E.
180	South.	6 E.	180	South.	8 E.
190	S. 10 W,	3 E.	190	S. 10 W.	4 E.
200	S. 20 W.	1 W.	200	S. 20 W.	0
210	S. 30 W.	4 W.	210	S. 30 W.	5 W.
220	S. 40 W.	7 W.	220	S. 40 W.	11 W.
230	S. 50 W.	11 W.	230	S. 50 W.	17 W.
240	S. 60 W.	15 W.	240	S. 60 W.	23 W.
250	S. 70 W.	19 W.	250	S. 70 W.	27 W.
260	S. 80 W.	22 W.	260	S. 80 W.	29 W.
270	West.	26 W.	270	West.	36 W.
280	N. 80 W.	28 W.	280	N. 80 W.	28 W.
290	N. 70 W.	29 W.	290	N. 70 W.	26 W.
300	N. 60 W.	29 W.	300	N. 60 W.	24 W.
310	N. 50 W.	29 W.	310	N. 50 W	21 W
320	N 40 W	25 W	320	N 40 W	19 W
330	N. 30 W.	21 W.	330	N. 30 W.	16 W.
340	N. 20 W.	17 W.	340	N. 20 W.	12 W.
350	N. 10 W.	12 W.	350	N. 10 W.	9 W.
360	North.	7 W.	360	North.	4 W.

N.B.—For examination purposes it will be better to work to seconds of arc in latitude and longitude, and not to the nearest mile as in the above examples, as it might throw the answer outside the $1\frac{1}{2}$ miles margin allowed.

FINDING THE LATITUDE

The latitude of a place on the earth's surface, being its distance from the equator, either north or south, is measured by an arc of a meridian contained between the zenith and the equinoctial, the zenith and the equinoctial corresponding on the *celestial* sphere to the position of the place and the equator upon the *terrestrial* sphere; hence, if the distance of any heavenly body from the zenith when on the meridian, and its declination, to the northward or southward of the equinoctial, be known, the latitude may thence be found.

The latitude of a place may also be defined as the declination of the zenith, and is equal to the altitude of the pole above the horizon of the place.

The best and most simple method of finding the latitude at sea is from an observed altitude of a heavenly body when on the *meridian*. From the altitude the zenith distance (90°—alt.) is known, and the object's declination being found from the Nautical Almanac, and corrected for Greenwich apparent time, the distance of the object both from the zenith and equinoctial is also known; consequently the distance of the zenith from the equinoctial—which is the distance of the observer from the equator—is at once determined.

The way in which the latitude of the place of observation is deduced from the meridian altitude and declination of a heavenly body is readily seen as follows:

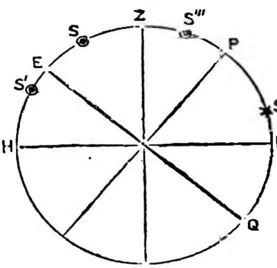


Fig. 1

In fig. 1 the circle is the meridian of the observer; H H′ his rational horizon; and Z his zenith; E Q is the celestial equator or equinoctial; and P the elevated pole, supposed, in this case, the *north* pole; also E Z is the latitude of the zenith, and hence of the observer.

(1) Taking S to be the place of the sun on the meridian; H S is the sun's altitude, bearing in this case *south;* and S Z his zenith distance; which is N., because the sun bears south; also E S is the sun's N. declination, measured from the equator at E; then, since the zenith distance and declination are of the *same* name, both *north* of the equator—

$$E\,Z = E\,S + S\,Z, \text{ or Lat.} = \text{zen. dist.} + \text{decl.}$$

(2) When S′ is the place of the sun, then E S′ is the sun's declination *south* of the equator; H S′ is the altitude, and S′ Z the zenith distance *north;* hence zenith distance and declination being of *contrary names*—

$$E\,Z = S'\,Z - E\,S', \text{ or Lat.} = \text{zenith distance} - \text{declination.}$$

(3) With the sun between the zenith (Z) and the pole (P), then H′ S‴ is the altitude,

and, $E\,Z = E\,S''' - S'''\,Z$; or Lat. = declination — zenith distance.

Thus far the object observed has been taken as *above* the pole; but in certain cases the object may be *below* the pole; and now we must first show that the *elevation of the pole* above the horizon of any place is always *equal to the latitude* of that place—

$$E\,Z = \text{Lat.}; \text{ also } Z\,H' = E\,P, \text{ each being a quadrant};$$

$$\text{or, } E\,Z + Z\,P = Z\,P + P\,H'$$

taking away Z P (the co-latitude) from each side of the equation,

$$\text{then (Lat.) } E\,Z = P\,H', \text{ the elevation of the pole.}$$

Also, for latitude by the meridian altitude of an object *below* the pole: let S″ be the object below the pole; then H′ S″ is its meridian altitude, and S″ P is its co-declination or polar distance therefore

$$P\,H' = H'\,S'' + S''\,P$$

i.e. Latitude (or the elevation of the pole) = altitude + polar distance.

It is here assumed that the *north* pole is the elevated one; hence for the *southern hemisphere* write south for north, and north for south, and the illustrations remain the same.

The same may be illustrated in the following manner—

In fig. 2 the circle is the rational horizon, N Z S the meridian, and Z the zenith of the observer and W Q E the equinoctial; then Z Q is the latitude, being the distance of the observer's zenith from the equinoctial.

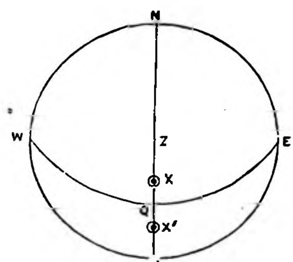


Fig. 2

(1) Taking X to be the place of the sun on the meridian, S X is the sun's altitude (bearing in this case *south*), and Z X his zenith distance, which is N. because the sun bears south; also Q X is the sun's N. declination measured from the equinoctial at Q. Then

$Z\,Q = Z\,X + Q\,X$ or Lat. = zenith distance + declination,

the rule when zenith distance and declination have the *same* name (in this case both being *north*). The latitude is also *north*, because Z is north of Q.

(2) When X′ is the place of the sun, then Q X′ is the sun's declination *south*, S X′ the altitude also *south* and Z X′ the zenith distance *north*. Then

$Z\,Q = Z\,X' - Q\,X'$ or latitude = zenith distance — declination.

the rule when zenith distance and declination have *different* names. The latitude is *north*, because Z is north of Q.

(3) With the sun's place North of Z, then Z Q or latitude = declination — zenith distance.

When the observer's position remains stationary, the meridian altitude of a fixed star is its greatest altitude ; but, as the sun, moon, and planets constantly change their declination, their greatest altitudes may be reached either before or after the meridian passage. In like manner, if the observer's position is rapidly changing in latitude, then also the greatest altitude of any heavenly body may not be the meridian altitude.

Latitude by the Meridian Altitude of the Sun

It is near, but *rarely exactly*, noon when the sun's meridian altitude is observed, because, if approaching him, you continue to raise the altitude after he has actually dipped and passed the meridian, hence it may be from 1 to 3 or 4 minutes past noon p.m. ; on the same principle, when receding from the sun he appears to dip before he comes to the meridian, and the sight may be observed a minute or more before noon a.m.

When the sun is approaching the meridian, for a quarter of an hour or so before noon continue to observe the altitude, using the tangent screw, until it is found to decrease or *dip ;* the greatest altitude attained is the *meridian altitude.*

In the case of a heavy sea the depression of the horizon is constantly changing, and it is therefore impossible to keep the sun's image in constant contact with the horizon. Having, then, the watch set to the time of *apparent* noon from the a.m. observation for time and the run of the ship in the interval, observe and read off separate altitudes in quick succession until they begin to decrease. The greatest is then taken as the meridian altitude, or, more accurately, the mean of the greatest.

RULE.—1. For apparent time at Greenwich. To apparent time at ship, *i.e.*, oh. om. os., apply the longitude in time, adding if longitude is west, but subtracting if longitude is east, putting the date back one day in east longitude.

2. Take the sun's declination (Nautical Almanac, p. I. of month) and correct it by the " Var. in 1 h." (Nautical Almanac, p. I.), always working from the *nearest noon*, *i.e.*, multiplying the " Var. in 1h." by the hours and decimal of an hour of the *longitude in time.*

In West longitude $\begin{cases}\text{Declination increasing, add the correction.}\\ \text{Declination decreasing, subtract the correction.}\end{cases}$

In East longitude $\begin{cases}\text{Declination increasing, subtract the correction.}\\ \text{Declination decreasing, add the correction.}\end{cases}$

3. Correct the observed altitude of the sun in the following order: index error $\frac{+}{-}$; dip—(Table of Dip) ; semi-diameter (Nautical Almanac) $\frac{+\text{L L}}{-\text{U L}}$; refraction—(Mean Refr. Table) ; and parallax + (Table of Par. in Alt.) ; the application of these quantities gives the *true* altitude of the sun's centre.

N.B.—If you use the *sun's correction* (Sun's Corr. Table), it combines refraction and parallax.

4. For the *zenith distance*, subtract the true altitude from 90°, which name N. or S., *contrary to the bearing* of the sun.

It is well to remember that if the zenith (the point over the observer's head) is north of the object *observed*, the zenith distance is N.; if south of the object, the zenith distance is S. The observer and his zenith have always the same name, N. or S.

5. For the *latitude*:—Add the zenith distance and *corrected* declination together when they are both N. or both S.; the result will be the latitude of same name as both; but subtract the less from the greater when they are of contrary names (*i.e.*, one N. and the other S.), and the result will be the LATITUDE of the *same name as the greater*.

Obs.—Should the declination be 0, the *zenith distance* is the latitude. Should the zenith distance be 0, the *declination* is the latitude. An altitude can never exceed 90° unless by some great error in the sextant.

Brief Rule for Sea-Use.—At sea you will adopt a shorter method of finding the latitude; but there is no necessity to guess or estimate corrections when you have in your Epitome, "Sun's Total Corr." Table *for the correction of the altitude*; the correction is taken out by *inspection* and without any trouble, but pay strict attention to the precepts at the top or bottom of the Table. Two examples will show how close this short method comes to the rigorous one; but *never adopt* the absurd and *often very erroneous method* of subtracting the observed altitude from 89° 48′ to get the zenith distance; it is frequently very *un*true.

If the horizon under the sun is obstructed by land, take the dip from Table of Dip at different distances from the observer.

1. Take the sun's declination (Nautical Almanac, p. I.), and correct it for the longitude, carefully noting its name, N. or S.

2. To the observed altitude of the sun's lower limb, apply the *index error* of the sextant + or —, and the correction from "Sun's Total Corr." Table (an *upper limb* observation cannot be worked by it); this gives the *true* altitude of the sun's centre.

3. Subtract the true altitude from 90°, and finish off the calculation as explained in paragraphs 4 and 5.

To draw the figure for latitude by meridian altitude to scale.—With the chord of 60° describe the circle N W S E to represent the rational horizon. Draw the diameter W Z E and it will represent the prime vertical, and at right angles to W Z E draw N Z S, the observer's meridian. To draw the equinoctial take in the compasses the secant of the co-latitude, and with one leg on E. or W. cut the meridian, or meridian extended, with the other leg, and the point reached will be the centre of the equinoctial, which draw and mark Q. To draw the parallel of declination lay off from Z in the direction of the elevated pole the semi-tangent of the sum of co-latitude and polar distance; also lay off from Z in the direction of the object the semi-tangent of the difference of the polar distance and co-latitude; bisect this line and

the centre of the line will be the centre from which to draw the parallel of declination with half the length of the bisected line as radius. To locate the pole take the semi-tangent of the co-latitude in the compasses, and with one leg on Z the other leg will find the pole, to the north of Z in N. latitude, to the south of Z in S. latitude ; mark this point P, and the figure is complete.

Example.—May 10th, 1890, in longitude 114° 3′ W., the observed altitude of the sun's lower limb was 67° 14′ 20″ bearing south ; index error + 1′ 20″ ; height of eye 20 feet ; required the latitude.

	D.	H.	M.	S.
App. T. at Ship	10	0	0	0
Long. in Time	+	7	36	12
App. T. Green.	10	7	36	12

	°	′
Long.	114	3 W.
		4
6,0)	45,6	12
Long. in time	7h. 36m. 12s.	

		°	′	″
Dec. (N.A. p. I. May 10d.)		17	40	54 N.
Hly. Var. 39·09″	Corr.	+	4	57
7·6	Corr. Dec.	17	45	51 N.
60′)297·084				
4′ 57″				

Short Sea Method

	°	′
Obs. Alt.	67	14·3 S.
I. E.	+	1·3
Sun's Total Corr.	+	11·0
True Alt.	67	26·6
	90	00
Zen. Dist.	22	33·4 N.
Corr. Decl.	17	45·7 N.
Latitude	40	19·1 N.

Obs. Alt. Sun's lower limb	67°14′20″ S.
Index Error	+ 1 20
	67 15 40
Dip	— 4 23
App. Alt. L.L.	67 11 17
Sun's S.D. (N.A. p. II.)	+ 15 52
App. Alt. Sun's centre	67 27 9
Refraction	— 24
	67 26 45
Parallax in Alt.	+ 3
True Alt. Sun's centre	67 26 48 S.
	90
Zen. Dist.	22 33 12 N.
Corr. Declination	17 45 51 N.
Latitude	40 19 3 N.

Explanation of Figure

N W S E	Rational horizon.
W Z E	Prime vertical.
W Q E	Equinoctial.
d d	Parallel decl.
P	Pole.
Z	Zenith.
X	Sun.
Q X	Sun's decl.
Q Z	Latitude.
X S	Altitude
Z X	Zen. Dist

Q X + Z X = Q Z ∴ lat. = Zen. Dist. + Decl.

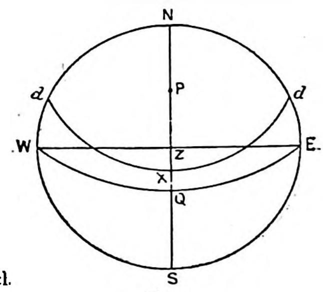


Fig. 3.

Example.—September 23rd, 1890, in longitude 168° 19′ E., the observed altitude of the sun's *upper limb* was 88° 59′ 20″ bearing north; height of eye 18 feet, index error—2′ 10″; required the latitude.

	D.	H.	M.	S.
App. T. at Ship, Sept.	23	0	0	0
Long. in Time		11	13	16 E.
App. T. Gr.	22	12	46	44

Long. 168°19′=11h. 13m. 16s.=11·2h.

Dec. (N. A. p. I. Sept. 23d). 0° 9′ 16″S.
Var. in 1h. 58″·48×11·2h.=—10 55*
Corr. Declination 0 1 39 N.

Ind. err.	—	2′	10″
Dip	—	4	9
Corr.	—		1
Semi-diam.	—	15	59
Total corr.	—	22	19
Obs. alt.	88°	59	20 N.
	88	37	1
	90		
Zen. dist.	1	22	59 S.
Decl.	0	1	39 N.
Lat.	1	21	20 S

Obs. Alt. Sun's up. limb	88°	59′	20″N.
Index Error	—	2	10
	88	57	10
Dip. of horizon	—	4	9
App. Alt. Sun's U.L.	88	53	1
Sun's semi-diam. (N.A.p.II.)	—	15	59
App. Alt. Sun's centre	88	37	2
† Sun's corr.	—		1
True Alt. Sun's centre	88	37	1 N.
	90		
Zen. Dist.	1	22	59 S.
Corr. Declination	0	1	39 N.
Latitude	1	21	20 S.

* The correction being subtractive and *greater than the declination*, take the declination from the correction, and change the name from S. to N.

† The sun's correction combines refraction and parallax.

Examples for Practice

The following examples for practice are all worked by the rigorous method—

Example 1.—October 11th, 1890; in longitude 18° 2′ W.; the observed meridian altitude of the SUN's lower limb being 26° 53′ 10″, bearing south of the observer; the height of the eye 17 feet; and the index error of the sextant 2′ 40″ to subtract; find the latitude.

Ans. Decl. 7° 7′ 34″ S.; Lat. 55° 51′ 38″ N.

Example 2.—April 3rd, 1890; in longitude 20° 59′ E.; the observed meridian altitude of the SUN's lower limb being 60° 22′ south of observer; height of eye 21 feet; and the index error of the sextant 2′ 48″ to add; find the latitude.

Ans. Decl. 5° 22′ 58″ N.; Lat. 34° 47′ 7″ N.

Example 3.—August 30th, 1890; in longitude 129° 15′ W.; the meridian altitude of the SUN's lower limb was 57° 18′ 30″, the observer being north of the sun; the height of his eye 18 feet; and index error 0′ 45″ +; required the latitude.

Ans. Decl. 8° 48′ 32″ N.; Lat. 41° 18′ 5″ N.

Example 4.—December 3rd, 1890; in longitude 63° 18′ E.; the meridian altitude of the SUN's lower limb was 64° 45′ 15″ north of the observer; the height of his eye being 20 feet; and index error 1′ 10″ to subtract; required the latitude.

Ans. Decl. 22° 7′ 30″ S.; Lat. 47° 11′ 55″ S.

Example 5.—March 20th, 1890; in longitude 101° 30′ W.; the meridian altitude of the SUN's lower limb was 89° 42′ 40″ north of the observer; the height of his eye being 22 feet; and the index error 2′ 24″ to add; required the latitude.

Ans. Decl. 0° 3′ 10″ N.; Lat. 0° 0′ 18″ S.

Example 6.—September 23rd, 1890; in longitude 168° 10′ E.; the meridian altitude of the SUN's lower limb was 84° 48′ 50″, the zenith being south of the sun; the height of the observer's eye 24 feet; and the index error 0′ 40″ to subtract; required the latitude.

Ans. Decl. 0° 1′ 39″ N.; Lat. 4° 59′ 4″ S.

Example 7.—January 15th, 1890; in longitude 149° 50′ E.; the meridian altitude of the SUN's upper limb was 33° 14′ 55″, the observer being north of the sun; the height of his eye 14 feet; and the error of the instrument 2′ 30″ to add; required the latitude.

Ans. Decl. 21° 9′ 20″ S.; Lat. 35° 54′ 33″ N.

Example 8.—March 10th, 1890; in longitude 89° 30′ W.; the meridian altitude of the SUN's lower limb was 14° 28′ 35″ S.; the height of the eye being 30 feet; the distance of the land under the sun 1¼ miles; and the index error 1′ 15″ to add; required the latitude.

Ans. Decl. 3° 54′ 16″ S.; Lat. 71° 37′ 19″ N.

Latitude by the Meridian Altitude of a Fixed Star

RULE.—Take out the star's declination from the "Apparent Places of Stars" in the Nautical Almanac, if it is at hand, as it saves time and calculation.

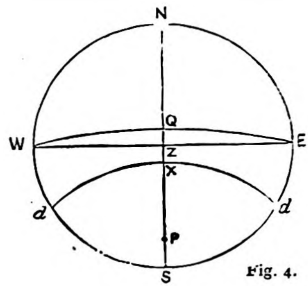
To the *observed* altitude of the star apply the index error, if any; then subtract the dip and refraction (or subtract the *correction* taken from "Total Star's Correction" Table), and the remainder will be the star's *true* altitude.

For the *zenith distance* subtract the true altitude from 90°, to be called north or south according as the observer is north or south of the star at the time of observation.

For the *Latitude*.—If the zenith distance and declination be both N. or both S., add them together; the result will be the latitude of same name as both; but if one be N. and the other S., subtract the less from the greater, and the difference will be the latitude of the same name as the greater.

Example.—July 16th, 1890; about 3 o'clock in the morning, the meridian altitude of the star Fomalhaut (α Piscis Aust.) was 73° 36′ south of the observer, the height of his eye being 24 feet, and index error 1′ 40″ to add; required the latitude.

Obs. alt. of Fomalhaut	73°	36′	0″	S.
Index error	+	1	40	
	73	37	40	
Star's corr. =	—	5	0	
True alt. of Fomalhaut	73	32	40	
	90			
Zenith distance	16	27	20	N.
Star's decl. (N.A.)	30	12	4	S.
Latitude	13	44	44	S.



Explanation of fig. same as fig. 3.

Fig. 4.

$$QX - ZX = QZ = \text{the latitude,}$$
or
$$\text{Latitude} = \text{Dec.} - \text{Zen. Dist.}$$

Examples for Practice

Example 1.—April 6th, 1890; the meridian altitude of the star Regulus (α Leonis) was 50° 14′ 20″) south of the observer, the height of his eye being 18 feet, and index error 1′ 15″ to add; required the latitude.

Ans. Lat. 52° 19′ 36″ N.

Example 2.—December 26th, 1890; the meridian altitude of the star Sirius (α Canis Majoris) was 36° 28′ 30″, the observer being north of the star, the height of his eye 16 feet, and index error —0′ 45″; required the latitude.

Ans. Lat. 37° 3′ 27″ N.

Example 3.—March 25th, 1890; the meridian altitude of the star Antares (α Scorpii) was 71° 49′ 45″ N., the height of the eye being 22 feet, and the index error of the instrument +2′ 10″; required the latitude.

Ans. Lat. 44° 24′ 13″ S.

Latitude by the Meridian Altitude of a Planet

Under the head of the given planet (in Nautical Almanac under "Mean Time") you will find the time of the *meridian passage* at Greenwich, which will only differ from the time of transit at ship by a few minutes; but you do not require this for the correction of the declination.

Rule.—1. Convert the longitude into time.

2. Take the declination from the Nautical Almanac (for ast. date), under the heading "At transit at Greenwich," and correct it for the longitude in time, by the "Var. of Dec. in 1 Hour of Long."

Correcting the declination by the "Var. in Dec." given in the Nautical Almanac, you apply the correction as for the Sun.

3. The horizontal parallax and semi-diameter are also given in Nautical Almanac under " transit at Greenwich."

4. Correct the planet's altitude as follows:—Index error $\pm$, dip —, refraction —, parallax in altitude (Table of Parallax for Planets) +

5. Subtract the true altitude from 90°, and name the *zenith distance* N. or S., contrary to the *planet's bearing.*

6. For the *Latitude.*—If the zenith distance and declination be both N., or both S., add them together for latitude of same name as both ; but if one be N. and the other S. subtract the less from the greater, and the difference will be the latitude of the same name as the greater.

Example.—February 21st, 1890, at about 5h. 30m. a.m. at ship, in longitude 109° 45′ E., suppose the meridian altitude of the planet Mars (centre) to be 62° 47′ 40″, the observer being south of the planet, and the height of his eye 18 feet, with the index error 1′10″ to add ; required the latitude.

The ship time being a.m. the data must be taken from the Naut. Alm., p. 278, on Feb. 20th, the astronomical day.

Long. 109° 45′ in time=7h. 19m. E.	
Var. in 1h. of long.	17″
7h. 19m.=	7·3
	51
	119
	6,0)12,4·1
Decl. increasing	— 2 4
Decl. (N.A.)	18° 9′ 37″S.
Corr. decl.	18 7 33 S.

Obs. Meridian Alt. of Mars.		62°	47′	40″N.
Index error	+		1	10
		62	48	50
Dip of horizon	—		4	9
Apparent alt. of Mars		62	44	41
Refraction	—			29
		62	44	12
Parallax in alt. for Planets	+			4
True alt. of Mars		62	44	16
		90		
Zen. dist.		27	15	44 S.
Decl. at time of transit		18	7	33 S.
Latitude		45	23	17 S.

Explanation of Figure.

N W S E	Rational horizon.
N Z S	Observer's mer.
W Q E	Equinoctial.
Z	Zenith of observer.
X	Planet.
X N	Mer. Alt.
Z X	Zen. Dist.
Q Z	Lat.

Q Z = Z X + Q X or Lat. = Zen. dist. + Dec.

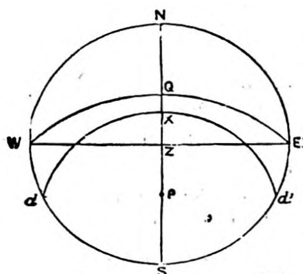


Fig. 5.

Examples for Practice

Example 1.—January 1st, 1890, at about 9h. 35m. a.m. at ship, in longitude 25° 30′ W., suppose the meridian altitude of the planet Venus (centre) to be 46° 38′ 30″ S., the height of the observer's eye being 24 feet, with index error 1′ 25″ to subtract; required the latitude.

Ans. Lat. 26° 38′ 58″ N.

Example 2.—September 12th, 1890, p.m. at ship, in longitude 36° E., suppose the meridian altitude of the planet Jupiter (centre) to be 26° 44′ 35″ S., the height of the observer's eye 18 feet, and the index error of the sextant 1′ 20″ to add; required the latitude.

Ans. Lat. 42° 59′ 13″ N.

Example 3.—March 6th, 1890, p.m. at ship, in longitude 50° W., suppose the meridian altitude of the planet Saturn (centre) to be 53° 22′, the observer being north of the planet, and the height of his eye 20 feet, with index error of the sextant 1′ 10″ to subtract; required the latitude.

Ans. Lat. 50° 5′ 45″ N.

Latitude by the Meridian Altitude of the Moon

Rule 1.—From page IV. of the month in the Nautical Almanac find the mean time of the moon's passing the meridian of Greenwich *on the astronomical day,* which reduce to the time of the *meridian passage* at ship (*see* p.245). Then, with this time and the ship's longitude, find the corresponding Greenwich mean time.

2. From pages V. to XII. of the month in the Nautical Almanac find the moon's declination, and reduce it to the mean time at Greenwich by the "variation" in 10m. given with it in the Nautical Almanac.

3. From page III. of the month in the Nautical Almanac take out the moon's semi-diameter and horizontal parallax, and reduce them to the mean time at Greenwich; to the semi-diameter apply the augmentation, from Table D; also reduce the horizontal parallax by Table E.

4. From the observed altitude (corrected for index error, if any) subtract the dip, and to the remainder add the moon's augmented semi-diameter, when the lower limb is observed, or subtract it if the upper limb be taken; the result will be the apparent altitude of the moon's centre, from which subtract the refraction; and then add the parallax in altitude; the result will be the *true* altitude of the moon's centre.

5. Finally, for the *zenith distance* and the *latitude*, proceed as for the sun.

N.B.—This is the rigorous Rule; but this precision is unnecessary for sea purposes; therefore proceed as follows:

Brief Rule for Sea-Use.—Reduce the time of the Greenwich meridian passage by Table "Correction of Moon's Meridian Passage" for time of

meridian passage at ship, to which apply the longitude in time, for the Greenwich date.

Correct the Moon's declination for Greenwich mean time.

Estimate approximately the semi-diameter, and horizontal parallax, from Nautical Almanac, p. III.

From the *observed*, find the *true* altitude in the usual way, using Moon's Correction Table for the joint effect of parallax and refraction ; then—

For zenith distance and latitude, proceed as for the sun.

Example.—January 27th, 1890 ; p.m. at ship ; latitude by dead reckoning 49° ; long. 31° 30′ W. ; the meridian altitude of the MOON's lower limb being 50° 20′ 30″ south of the observer ; height of eye 23 feet ; required the latitude.

	H.	M.	
Mer. Pass. 27th	5	55·3	Mer. pass. on ast. date and following date because the long. is West.
„ „ 28th	6	40·7	
Daily var. or interval	0	45·4	

Long. 31° 30′ W. = 2h. 6m. = 2·1h.

$$\frac{\text{Daily var. } 45{\cdot}4\text{m.} \times 2{\cdot}1}{24} = 4{\cdot}0\text{m. corr. for long.}$$

	H.	M.
(N A., p. IV.) moon's mer. Pass. at Gr. Jan. 27	5	55·3
Corr. for long. W.	+	4·0
☾'s mer. pass at ship	5	59·3
Long. in time W.	+2	6
M.T. at Green. Jan. 27d.	8	5·3

Moon's decl. at 8h.	10°	4′	29″·4	N.
Var. in 1m. 11″·68 × 5·3 =	+	1	1·9	
Corr. decl.	10	5	31·3	N.

$$\frac{23''{\cdot}8 \times 8{\cdot}1}{12} = 16''{\cdot}1 \text{ corr. of ☾'s hor. par.}$$

(N.A. p. III.) ☾'s hor. par., noon	56′ 23″·5			
Corr. for 8·1h.	— 16·1			
	56 7·4			
Reduction (Tab. E.)	— 6·3			
☾'s reduced hor. par.	56 1·1	P.L.	·5070	
Alt.	50° 30½′	sec.	·1966	
Par. in alt.	35′ 37″	P.L.	·7036	

Obs. alt. moon's L.L.	50°	20′	30″	S.
Dip	—	4	42	
App. alt. L.L.	50	15	48	
Semi-diam.	+	15	31	
App. alt. centre	50	31	19	
Refraction	—		48	
	50	30	31	
Par. in alt.	+	35	37	
☾'s true alt.	51	6	8	S.
	90			
Zen. dist.	38	53	52	N.
Decl.	10	5	31	N.
Latitude	48	59	23	N.

$$\frac{6''{\cdot}5 \times 8{\cdot}1}{12} = 4''{\cdot}4 \text{ corr. of ☾'s semi-diam.}$$

(N.A. p. III.) ☾'s semi-diam. at noon	15′ 23″·5	
Corr. for 8·1h.	— 4·4	
	15 19·1	
Augmentation (Table D.)	+ 11·6	
☾'s augmented semi-diam.	15 30·7	

$$Z☽ + Q☽ = QZ = \text{the latitude}$$

or

$$\text{Lat.} = \text{Dec.} + \text{Zen. dist.}$$

Explanation of fig. is the same as Fig. 3.

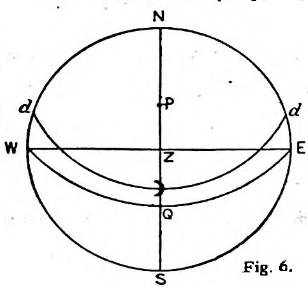


Fig. 6.

Or briefly thus—

Dec. at 8h.	10° 4′ 29″·4 N.	
„ 9h.	10 16 8 ·5 N.	
Diff. in 1h.	11 39	prop. log. 0·7118
Minutes after 8h.	5	prop. log. 1·0792
Correction	+ 58″	prop. log. 1·7910
Moon's decl.	10 4 29 N.	
☾'s corr. decl.	10 5 27 N.	

	D.	H.	M.
Moon's mer. pass. at Gr. Jan.	27	5	55
Corr. for long.			4
M.T. mer. pass. at ship		5	59
Long. in time		2	6
M.T. at Gr. Jan.	27	8	5

Mean between noon and midnight H.P. 56′ 11″ and semi-diam. 15′ 20″

Moon's obs. alt.	50°	20′·5
Dip. — 4′·7 } Semi-diam. + 15·3 }	+	10·6
Moon's corr.	+	34·8
True alt.	51	5·9 S.
	90	
Zen. dist.	38	54·1 N.
Decl.	10	5·4 N.
Lat.	48	59·5 N.

N.B.—No *brief* table for the moon, similar to that for the sun's *correction of altitude*) embracing *dip, semi-diameter, refraction* and *parallax*) would be of any value, unless it extended over many pages, and even then interpolation would be necessary. But the brief form of the problem, as here given, will generally be found sufficiently accurate for sea-use.

When the moon is near the equator the rate of change in the declination is so rapid that it is improbable you get the meridian altitude.

In cloudy weather the dark shadows projected on the water beneath the moon render the place of the horizon uncertain; in clear weather the upper edge of the illuminated part of the sea is the horizon.

Example.—February 28th, 1890; p.m. at ship; lat. by D.R. 38° 30′; long. 61° 30′ E.; the meridian altitude of the MOON's lower limb being

27° 5′ 30″, observer south of moon; index error 1′ 30″ to subtract; eye 26 feet; required the latitude.

	H.	M.	
Mer. Pass. 28th	7	46·2	Mer. pass. on ast. date and preceding date because long. is East.
,, ,, 27th	6	57·0	
Daily var. or interval	0	49·2	

Long. 61° 30′ E. = 4h. 6m. = 4·1h.

$$\frac{\text{Daily var. } 49{\cdot}2\text{m.} \times 4{\cdot}1}{24} = 8{\cdot}4\text{m. corr. for long.}$$

(N.A.) moon's decl., at 3h. (Feb. 28)	23°	35′	38″·4	N.
Var. in 1m.; 2″·1 × 31·8 =	+	1	6 ·8	
Corr. decl.	23	36	45 ·2	N.

	H.	M.
(N.A. p. IV.) moon's mer. pass. at Gr. Feb. 28	7	46·2
Corr. for long. E.	—	8·4
Moon's mer. pass. at ship	7	37·8
Long. in time E.	4	6
M.T. at Green. Feb. 28	3	31·8

$$\frac{1''{\cdot}7 \times 3{\cdot}5}{12} = 0''{\cdot}5 \text{ corr. of ☾'s semi-diam.}$$

(N.A.p.III.) moon's semi-diam. at noon	14′ 49″ ·1
Corr. for 3·5h.	— 0 ·5
	14 48 ·6
Augmentation (Table D.)	+ 6 ·5
Moon's augmented semi-diam.	14 55 ·1

$$\frac{6''{\cdot}3 \times 3{\cdot}5}{12} = 1''{\cdot}9 \text{ corr. of hor. par.}$$

(N.A. p. III.) moon's hor. par. at noon	54′ 17″·4	
Corr. for 3·5h.	— 1 ·9	
	54 15 ·5	
Reduction (Table E.)	— 4 ·1	
Moon's reduced hor. par.	54 11 ·4	P.L. ·5214
Alt.	27° 12′	sec. ·0509
Par. in alt.	48′ 11″	P.L. ·5723

Obs. alt. moon's L.L.	27°	5′	30″	N.
Ind. err.	—	1	30	
	27	4	0	
Dip	—	5	0	
App. alt. L.L.	26	59	0	
Semi-diam.	+	14	55	
App. alt. centre	27	13	55	
Refraction	—	1	51	
	27	12	4	
Par. in alt.	+	48	11	
Moon's true alt.	28	0	15	N.
	90			
Zen. dist.	61	59	45	S.
Decl.	23	36	45	N.
Latitude	38	23	0	S.

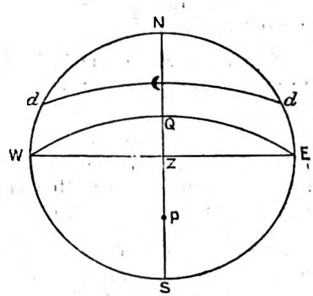


Fig. 7.

Explanation of Figure

☾ N = Altitude. Q Z = P S = Latitude.

☾ Q = Declination. Lat. = Zen. dist. — Declination.

The other parts of the fig. are the same as explained in Fig. 3.

Examples for Practice

Example 1.—June 28th, 1890; lat. by D.R. 51°; long. 105° 46′ W.; the observed meridian altitude of the MOON's lower limb being 53° 10′ bearing north; height of eye 20 feet; required the latitude.

Ans. Lat. 51° 13′ 35″ S.

Example 2.—September 28th, 1890; lat. by D.R. 32°; long. 178° 30′ E.; the observed meridian altitude of the MOON's lower limb being 54° 50′; observer north of the moon; eye 25 feet; required the latitude.

Ans. Lat. 31° 53′ 18″ N.

Example 3.—April 6th, 1890; a.m. at ship; lat. by D.R. 13° 20′; long. 81° W.; observed meridian altitude of MOON's lower limb 69° 40′ 30″; zenith north of moon; index error 1′ 40″ to subtract; eye 24 feet; required the latitude.

Ans. Lat. 13° 28′ 35″ N.

Example 4.—March 9th, 1890; a.m. at ship; lat. by D.R. 40°; long. 105° 30′ E.; the observed meridian altitude of MOON's lower limb being 49° 24′ bearing north; height of eye 23 feet; required the latitude.

Ans. Lat. 39° 51′ 11″ S.

Latitude by a Meridian Altitude BELOW *the Pole.*

When a star's declination and the latitude of a place are of the *same* name, both N., or both S., if the declination is *greater* than co-latitude; or, put otherwise, if the star's polar distance is *less* than the latitude of place; such stars are never below the horizon of the observer, and are called *circumpolar* stars; hence such stars pass the meridian both *above* and *below* the pole.

Similarly, when the latitude is higher than 66½°, the sun is above the horizon throughout the whole 24 hours *during part of the summer months of the hemisphere.*

Also, for the reason given above, when the moon's declination and latitude of the place are of the same name, during a part of every month the moon's altitude can be taken both above and below the pole when the polar distance is less than the latitude of observer.

N.B.—In an observation *below* the pole, the *lowest* altitude is the *meridian* altitude, and the latitude can be found as follows :—

RULE.—To the *true* altitude of the heavenly body add its polar distance (90° — Decl.); the sum will be the latitude of the same name as the declination.

Obs.—The altitude is to be corrected in the usual way; but, as it will be low, note the state of the barometer and thermometer, and apply the necessary correction to the refraction (Table "Correction of Mean Refraction").

Obs.—For the Sun.—The time being apparent midnight, the Greenwich date, apparent time, will be 12h. *plus* longitude W., but 12h *minus* longitude E.; for which Greenwich apparent time correct the sun's declination.

Obs.—For the Moon.—Use the *lower* meridian passage, and correct it in the same way as the time of the upper passage.

Example.—June 28th, 1890; at 12h. p.m., in longitude 40° E., the meridian altitude of the sun's lower limb below the pole was 6° 30′; height of the observer's eye being 20 feet; required the latitude.

	D.	H.	M.
App. time at ship, June	28	12	0
Long. in time E.	—	2	40
App. time at Gr.	28	9	20

(N.A.) Sun's Decl.			
June 28d.	23°	17′	6″ N.
7″·21 × 9·3 = corr.	—	1	7
Corr. decl.	23	15	59
	90		
Sun's pol. dist.	66	44	1

Obs. alt. sun's L.L.	6°	30′	0″
Dip.	—	4	23
	6	25	37
Semi-diam.	+	15	46
	6	41	23
Ref.	—	7	57
	6	33	26
Par.	+		9
Sun's true alt.	6	33	35
„ Pol. dist.	66	44	1
Latitude	73	17	36 N.

Short Method

Obs. alt. sun's L.L.	6°	30′
Sun's corr.	+	3·6
True alt.	6	33·6
Polar dist.	66	44
Latitude	73	17·6 N.

Explanation of Figure

dXd Parallel of declination.
P Pole.
⊙ N Sun's alt. below the pole.
⊙ P Sun's polar dist.
Z Q The latitude,
or
Latitude = polar dist. + the altitude
N P, the altitude of the Pole, is the Latitude.

The other parts of the fig. are the same as explained in Fig. 3.

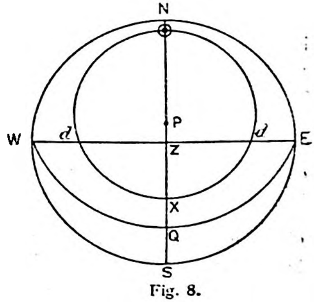


Fig. 8.

Example.—July 14th, 1890; at about 3h. 20m. a.m. (twilight), the altitude of the star DUBHE (α Ursæ Maj.) when on the meridian below the pole, was 21° 14′, height of the observer's eye being 19 feet, and the index error + 1′ 30″; required the latitude.

By Table "Apparent Time of Principal Stars passing the Meridian of Greenwich" the time of Dubhe's passing the meridian above the pole is 3h. 20m. p.m.; therefore 12h. after that time, or about 3h. 20m. a.m., it will be on the meridian below the pole.

(N.A.) decl. of Dubhe,	
July 14	62° 20′·9 N.
	90
Pol. dist. Dubhe	27 39·1

Obs. alt. of Dubhe	21° 14′
Ind. err.	+ 1·5
	21 15·5
Star's total corr. for 19ft.	— 6·7
True altitude	21 8·8
Star's pol. dist.	27 39·1
Latitude	48 47·9 N.

Explanation of Figure

X *d d*	Star's parallel of declination.
X	Star on mer. below the pole.
P X	Star's polar dist.
XN	Star's altitude.

P X + X N = the latitude,

or

Latitude = polar dist. + true alt.

N P, the Altitude of the Pole, is the Latitude.

The other parts of the fig. are the same as explained in Fig. 3.

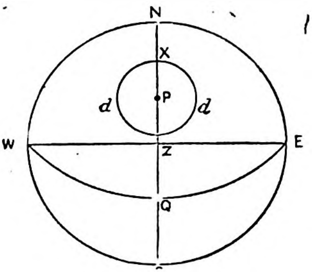


Fig. 9.

Examples for Practice

Example 1.—July 18th, 1890; at 12h. p.m., in long. 75° W., the altitude of the sun's lower limb, when on the meridian below the pole, was 8° 26′ 20″, the height of the observer's eye being 19 feet, and the index error + 3′ 15″; required the latitude.

Ans. Lat. 77° 42′ 2″ N.

Example 2.—August 19th, 1890; at about 2h. 20m. a.m., the star α Crucis being on the meridian below the pole, its altitude was observed to be 17° 32′ 10″, the height of the observer's eye being 26 feet, and the index error — 2′ 25″; required the latitude. Declination of the star 62° 29′ 37″ S.

Ans. Lat. 44° 52′ 3″ S.

Degree of Dependence.—In fine weather an observation of the sun when *on the meridian* should not be in error 2′. A star or planet taken at *twilight* might have the same error, no more; but on a dark night it might be 3′ or 4′; there is always more or less uncertainty about the moon. The accuracy of the approximate result will then depend on the accuracy with which the various corrections are made.

LATITUDE BY THE REDUCTION TO THE MERIDIAN

When there is a probability that the meridian altitude may be lost, owing to clouds, or other causes, altitudes of the sun may be taken *near noon* and the times noted by watch, regulated to local time from the a.m. observations for time, and reduced by a correction due to the difference of longitude in the interval.

This observation for latitude should be limited to altitudes taken within a given time from the meridian (*see* Table below); for unavoidable errors occur in the time as determined at sea, and the error in the latitude produced by an error in the time is considerable when the observations are made outside the prescribed limits.

Thus the term "near the meridian" has a specific signification; and speaking in general terms, the *number of minutes in the time from noon should not exceed the number of degrees in the sun's meridian zenith distance; or the number of minutes of time in the Meridian distance should never exceed the number of minutes of arc in the reduction.* The meridian distance should not exceed the limits in the following table, which is computed to give the number of minutes of meridian distance, when an error of *half a minute* in the time will produce an error of 1′ in the reduction.

Latitude.	Decl. and Lat. *same* name.					Decl.	Decl. and Lat. *different* names.				
	24°	20°	15°	10°	5°	0°	5°	10°	15°	20°	24°
°	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
0	0 14	0 11	0 8	0 5	0 3	0 0	0 3	0 5	0 8	0 11	0 14
5	0 11	0 8	0 5	0 3	0 0	0 2	0 5	0 7	0 11	0 14	0 16
10	0 8	0 6	0 3	0 0	0 3	0 5	0 8	0 11	0 14	0 17	0 19
15	0 6	0 3	0 0	0 3	0 6	0 8	0 11	0 14	0 17	0 19	0 22
20	0 3	0 0	0 3	0 6	0 9	0 11	0 14	0 17	0 20	0 22	0 25
25	0 0	0 3	0 6	0 9	0 12	0 15	0 17	0 20	0 23	0 25	0 28
30	0 4	0 7	0 10	0 12	0 15	0 18	0 21	0 23	0 26	0 29	0 31
35	0 8	0 10	0 13	0 16	0 19	0 22	0 25	0 27	0 30	0 33	0 35
40	0 12	0 14	0 17	0 20	0 23	0 26	0 29	0 31	0 34	0 37	0 39
44	0 17	0 18	0 21	0 24	0 27	0 30	0 33	0 35	0 39	0 42	0 44
48	0 22	0 23	0 27	0 28	0 31	0 34	0 38	0 41	0 44	0 47	0 49
52	0 28	0 30	0 32	0 34	0 37	0 40	0 44	0 48	0 51	0 54	0 56
56	0 33	0 36	0 38	0 40	0 43	0 46	0 50	0 53	0 56	0 59	1 1
60	0 40	0 43	0 46	0 48	0 50	0 54	0 57	1 0	1 2	1 5	1 7
64	0 50	0 52	0 54	0 55	0 57	1 0	1 5	1 10	while visible.		
68	1 2	1 5	1 6	1 6	1 8	1 10	1 15	1 20			

To use the Table.—With the approximate latitude in the side column and the declination at the top, having regard to the precept as to *name*, find the time; this is the *time from noon*, or meridian distance in time, within the limits of which you must keep when finding the latitude by the *reduction to the meridian*. See also Ex-Mer. Table in Norie's Table.

The necessary elements for the solution of the problem are:

The hour angle, which in the case of the sun is the apparent time from noon.

The declination of the object.

The altitude, and by some methods the latitude by account.

The following methods are those best adapted to the computation of the *reduction*:—

METHOD I.—*For all practical purposes at sea, you do not require the logarithms to more than four places when using this method of obtaining the reduction.*

Formula—Sin. $\frac{1}{2}$ r = hav. $h \times$ cos. $l \times$ cos. $d \times$ co-sec. $(l \pm d)$, where r is the reduction, h the mer. dist., l the latitude, and d the declination.

$(l \pm d)$ = zenith distance obtained from latitude by D.R. and declination.

Add together the following:—

Log. hav. meridian distance.
Log. co-sine of the latitude by dead reckoning.
Log. co-sine of the declination, and
Log. co-secant of the meridian zenith distance by dead reckoning.

N.B.—For *the zenith distance deduced from* dead reckoning *proceed as follows*:—With latitude and declination of the *same* name, take their *difference*; with latitude and declination of *different* names, take their *sum*; *the result will be the zenith distance by* dead reckoning.

The sum of the logs. (rejecting *tens* from the index) will be the Log. sine of half the *reduction*, which take out, and multiply by 2, for the whole *reduction*, to be *added* to the true altitude *off* the meridian.

The *reduced* altitude taken from 90° gives the zenith distance, to which apply the declination as in the usual problem of finding the latitude by meridian altitude.

Obs. 1.—The investigation of method I. admits of two other practical solutions: for example, if, to the form already given, the log. of 2, which is 0·301030, be added as a *constant*, or, if instead of log. haversine of meridian distance the log. rising be taken and the index increased by 5, then the result will be the log. sine of the *reduction* without the necessity of "multiplying by 2," as stated above.

Obs. 2.—Or again, if, to the form already given, the *constant* 5·61546 be added, the result will be a log., the nat. number of which will be the *reduction* in seconds of arc, to be added (as before) to the *true* alt.

If the observation is made *below the pole*, the *reduction* is to be *subtracted from* the true *altitude*, for the *reduced* altitude, which, added to the object's polar distance, gives the latitude.

METHOD II.—Navigators have long been accustomed to use the log.

rising and Table of natural sines, etc., but this method should not be preferred to Method I.

Formula—

$$N = \text{Vers. } h \times \cos. l \times \cos. d$$
$$\text{Cos. } z = N + \sin. a$$

where h is the mer. dist., l the latitude, d the declination, z the mer. zen dist., and a the true altitude.

Add together the—

Log. rising of the time from noon ;
Log. co-sine of the latitude by account ;
Log. co-sine of the declination.

The sum (rejecting *tens* in the index) will be the log. of a natural number, which take out.

Under this natural number put the natural sine of the true altitude and take their sum, which will be the natural co-sine of the true meridian zenith distance.

N.B.—The natural sine is taken only to five places, because the index of the log. rising is adapted to no more, and is quite sufficient. But if great accuracy is required the natural sine may be taken to six places, if a decimal place be added to the natural number, or if 1 be added to the index of the log. rising.

Name the meridian zenith distance N. or S. according to the position of the observer, and get the latitude as by the meridian altitude problem.

If the observation is made *below the pole* the natural *number* must be *subtracted* from the natural sine of true altitude ; and the meridian zenith distance must be subtracted from 90° for the *altitude below the pole ;* or the *altitude* can be taken direct from the natural sine column.

The DIRECT METHOD, although longer, has the advantage of being independent of the latitude by account and is mathematically sound. The problem is solved by Napier's Rules for right angled spherical triangles, as will be seen in the following examples.

The latitude found is that for the time and place of observation ; hence to be brought to noon it will require to be corrected for the course and distance run in the interval between the time of observation and noon.

For the subordinate computations proceed as follows—

For the Sun

1. For the apparent time at ship, using watch time and error on apparent time. Write down the month and day with the hours, minutes, and seconds by watch or chronometer ; apply the given error, adding if slow, subtracting if fast. Also turn the D. longitude made good, since the error on apparent time was ascertained, into time, and apply it, adding if East, subtracting if West. The result is the apparent time at ship.

2. For the Greenwich Date.—Under the ship date, apparent time, write the longitude in time; add if W.; subtract if E. The result is Greenwich date apparent time.

3. For the Declination.—Use "Nautical Almanac," p. I., of given month and correct it for the Greenwich date apparent time.

4. For the True Altitude.—Correct the observed altitude for index error, if any, dip, semi-diameter, refraction, and parallax in the usual way.

5. For the Time from Noon at Ship.—If p.m the apparent time at ship is the time from noon; if a.m., the apparent time at ship is to be subtracted from 24 hours; the result will be time from noon or easterly hour-angle

6. If the time by chronometer with its error on Greenwich mean time be given, apply the error and get the correct mean time at Greenwich. Take out the declination and equation of time from p. II. of the "Nautical Almanac." and correct them for the correct mean time at Greenwich.

To the mean time at Greenwich apply the longitude in time; subtract W. longitude, add E. longitude; then apply the equation of time according to the precept on p. II. of the "Nautical Almanac"; the result will be apparent time at ship. If it be p.m. it will be the time from noon, but if it be a.m. subtract it from 24 hours to get the time from noon.

When the sun is the object, the time from noon must always be an apparent interval.

For a star or planet you will require the hour-angle, which find as follows—

If the mean time at ship be given, apply the longitude in time and get the mean time at Greenwich. Take out the sidereal time from the "Nautical Almanac" and correct it for Greenwich mean time; then to the mean time at ship add the sidereal time, and the result will be the R.A.M.

Take the difference between the R.A.M. and the R.A. of the star, or planet, and the result will be the star or planet's hour-angle; E. when the R.A.M. is the lesser; W. when the R.A.M. is the greater.

If apparent time at ship be given find the Greenwich apparent time and correct the apparent sun's R.A. for Greenwich apparent time, and add it to apparent time at ship for R.A.M. The hour-angle is then found in exactly the same way as above.

If the mean time at Greenwich be given, find the mean time at ship by means of the longitude, adding if E., subtracting if W.; correct the sidereal time for the Greenwich date and add it to the mean time at ship for the R.A.M. The difference between the R.A.M. and the object's R.A. is the hour-angle.

It will be observed that when apparent time is given the apparent sun's R.A. is used to find the R.A.M., and when mean time is given the mean sun's R.A. or sidereal time is used.

When a planet is observed its R.A. and declination will require correcting for the Greenwich date.

Example.—January 28th, a.m. at ship; lat. by dead reckoning 44° 8′ N.; long. 32° 42′ W.; the observed altitude of the sun's lower limb when near the meridian was 26° 50′ South of the observer; height of eye 22 feet; time by watch Jan. 28d. 1h. 50m. 20s., which had been found 2h. 31m. 54s. fast on apparent time at ship; the difference of longitude made to eastward was 18′ after the error on apparent time was determined; required the latitude by the Reduction to the Meridian.

	D.	H.	M.	S.
T. by watch Jan.	28	1	50	20
Fast		2	31	54
Approx. T. ship „	27	23	18	26
Cor. for run 18′ E.		+	1	12
Apparent T. at ship „ „		23	19	38
Long. in time W.		2	10	48
App. T. at Gr. Jan. 28		1	30	26

	H.	M.	S.
	24	0	0
App T. at ship	23	19	38
Time from noon		40	22
Obs. alt. sun's L.L.	26°	50′	0″ S.
Dip	—	4	36
	26	45	24
Semi-diam.	+	16	16
	27	1	40
Ref. and par.	—	1	44
Sun's true alt.	26	59	56
Red.	+	41	00
Mer. alt.	27	40	56 S.
	90	00	00
Mer. zen. dist.	62	19	4 N.
Dec.	18	6	30 S.
Lat.	44	12	34 N.

Var. in 1h.	=39″·8		
1h. 30m.	1·5		
Corr.—	59·70		
(N. A. p. I.) Decl.	18°	7′	29·4
Corr.decl.	18	6	29·7 S.

		M.	S.			
Time from noon		40	22		L. hav.	7·88850
Lat. by D.R.	44°	8′	0″	N.	L. cos.	9·85596
Decl.	18	6	30	S.	L. cos.	9·97794
Z.D. by acc.	62	14	30		L. cosec.	0·05310
Half Reduction		20	30		L. sin.	7·77550
			2			
Reduction	+	41	0			

The circle represents the rational horizon, Z the centre is the zenith, N Z S the meridian of the observer, W Q E the equinoctial, P the pole (N), X the position of the celestial body near the meridian, X Z the zenith distance deduced from the observation, X′ the place of the body when on the meridian, X′ Z the mer. zen. dist., the difference between X Z and X′ Z is called the *reduction*.

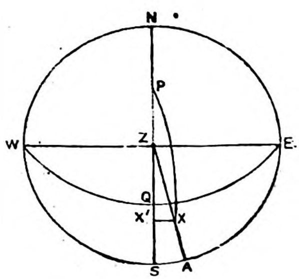


Fig. 1.

The object of the problem is to find X′ Z the meridian zenith distance, and thence the latitude Z Q.

The following is the same example by Direct Method.

THE DIRECT METHOD.

Given: Hour angle P = 40m. 22s.; P X (polar distance) = 108° 6′ 30″, and Z X (zenith distance) = 63° 0′ 4″; to find the latitude by ex-meridian altitude.

In Fig. 1. X is the object, PX the polar distance, Z X the zenith distance, the angle X P X′ (P) the hour angle, X X′ an arc of a great circle at right angles to the meridian, W Q E the equator.

Then Q Z is the latitude and is equal to the difference of the arcs Z X′ and Q X′.

Fig. 1.—In the right-angled spherical triangle P X′ X, right angled at X′, given ∠ P and side P X, to find side P X′.

$$\text{Cos. P} = \tan \text{P X}' \cot. \text{P X} \qquad \therefore \text{Tan. P X}' = \frac{\cos. \text{P}}{\cot. \text{P X}}$$

	M.	S.		+		
P	40	22		Cos.	9·993228	
				—		
P X	108°	6′	30″	Cot.	9·514563	
				—		
	71	37	34	Tan.	10·478665	
P X′ =	108	22	26			

In this case the angle found is to be subtracted from 180° to find P X′ because the tangent is minus.

In the same triangle given ∠ P and side P X′, to find side X′ X, the perpendicular from X on the meridian.

$$\text{Sin. P X}' = \cot. \angle \text{P} \quad \tan. \text{X}' \text{X} \qquad \therefore \text{Tan. X}' \text{X} = \frac{\sin. \text{P X}}{\cot. \text{P}}$$

P X′	108°	22′	26″	Sin.	9·977275
	M.	S.			
P	40	22		Cot.	10·749636
X′ X	9°	35′	13″	Tan.	9·227639

In the right-angled spherical triangle Z X′ X, given Z X and X′ X, to find Z X′.

$$\text{Cos. Z X} = \cos. \text{Z X}' \quad \cos. \text{X}'\text{X} \qquad \therefore \text{Cos. Z X}' = \frac{\cos. \text{Z X}}{\cos. \text{X}' \text{X}}$$

Z X	63°	0′	4″	Cos.	9·657030
X′ X	9	35	13	Cos.	9·993892
Z X′	62	35	13	Cos.	9·663138

To find the Latitude.

P X′ — Z X′ = Co-latitude

Latitude = 90° — Co-latitude

P X′	108°	22′	26″
Z X′	62	35	13
Co-latitude	45	47	13
	90	00	00
Latitude in	44	12	47 N.

Example.—September 30th, 1890; p.m. at ship; in lat. 48° 43′ S.; long. 22° 6′ W.; the observed altitude of the *sun's* lower limb near the meridian was 43° 31′ 40″ north of observer; height of the eye 23 feet; time by chronometer 1h. 55m. 32s., which was 6m. 52s. fast on mean time at Greenwich; required the latitude.

	D.	H.	M.
Ship T. nly.	30	0	0
Long 22° W.	=	1	28
Gr. T. nly.	30	1	28

	D.	H.	M.	S.
Time by chron. Sept.	30	1	55	32
Error			6	52 fast
M.T. at Green.	30	1	48	40
Long. 22° 6′ W.	=	1	28	24
M.T. at ship	30	0	20	16
Eq. T.	+		10	5
App. T. at ship	30	0	30	21

	M.	S.
(N.A. p. II.) Eq. T.	10	3·6
0·81 s. × 1·8 =	+	1·5
Corr. Eq. T. +	10	5·1

Decl. (p. II.)	2°	53′	5″ S.
58″·3 × 1·8 =	+	1	45
Corr. Decl.	2	54	50 S.

Formula—

$$\text{Sin. } \frac{R}{2} = \text{Hav. P cos. } l, \text{ cos. } d, \text{ cosec. Z D}$$

where R = red, P hour angle, l lat. by D. R., d the dec., Z D zen. dist. by D. R.

	M.	S.
T. from noon	30	21
Lat. by D.R.	48°	43′ S.
Decl.	2	55 S.
Z.D. by acc.	45	48
Half Red.		13′ 50″
		2
Reduction		27 40

Hav.	7·64126
Cos.	9·81940
Cos.	9·99944
Cosec.	10·14453
Sin.	7·60463

	°	′	″
Obs. alt. sun's L.L.	43	31	40 N.
Dip	—	4	42
	43	26	58
S.D.	+	16	1
	43	42	59
Sun's corr.	—		54
T. alt.	43	42	5
Red	+	27	40
Red. alt.	44	9	45 N.
	90		
Mer. Z.D.	45	50	15 S.
Decl.	2	54	50 S.
Lat.	48	45	5 S. at obs.
Run		3	0
Lat.	48	42	5 S. at noon

And suppose the ship, between noon and the time of observation, had made a *true* course S.W. ¾ W. distant 5 miles, then the difference of latitude being 3′, she was in lat. 48° 42′ S. at noon.

Explanation of Fig 2.

N W S E	Rational horizon.
W Z E	Prime vertical.
W Q E	Equinoctial.
N Z S	Mer. of observer.
P X	Polar dist.
Z X	Zenith dist. observed
Z X′	Zenith dist. on meridian.
Q Z = P S	The latitude.
P Z	Co-latitude.

Reduction = (Z X ~ Z X′) additive to true alt.

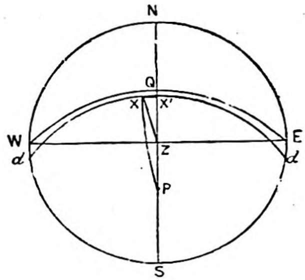


Fig. 2.

The same example by Direct Method

In Fig. 2, from X drop a perpendicular on the meridian, which mark X′. then in the right-angled spherical triangle P X′ X, right-angled at X′, given ∠ P and side P X to find side P X′.

$$\text{Cos. P} = \text{cot. P X tan. P X}' \qquad \therefore \text{Tan. P X}' = \frac{\cos \text{ P}}{\text{cot. P X}}$$

	M.	S.			
P	30	21		Cos.	9·996181
P X	87°	5′	00″	Cot.	8·707140
P X′	87	3	30	Tan.	11·289041

In the same triangle, given ∠ P and side P X, to find side X′ X.

$$\text{Sin. P X}' = \text{cot.} \angle \text{P tan. X}'\text{X} \qquad \therefore \text{Tan. X}' \text{X} = \frac{\sin \text{ P X}'}{\text{cot. P}}$$

P X′	87°	3′	30″	Sin.	9·999427
		M.	S.		
∠ P		30	21	Cot.	10·875473
X′ X	7°	34′	39″	Tan.	9·123954

In the right-angled spherical triangle Z X′ X, given Z X and X′ X to find Z X′

$$\text{Cos. Z X} = \text{cos. Z X}' \text{ cos. X}' \text{ X} \qquad \therefore \text{Cos. Z X}' = \frac{\text{cos. Z X}}{\text{cos. X}' \text{ X}}$$

Z X	46°	17′	55″	Cos.	9·839415
X′ X	7	34	39	Cos.	9·996190
Z X′	45	48	51	Cos.	9·843225

To find the Latitude.

P X′ — Z X′ = co-latitude and Latitude = 90° — co-latitude.

P X′	87°	3′	30″
Z X′	45	48	51
Co-latitude	41	14	39
	90	00	00
Latitude at obs.	48	45	21 S.

Example.—December 16th, 1890; a.m. at ship; lat. by D.R. 50° 47′ N.; long. 20° 4′ W.; time by chronometer 8h. 33m. 42s., which was 11m. 38s. fast on mean time at Greenwich; the observed altitude of Spica (α Virginis) near the meridian was 28° 6′ bearing south; height of eye 24 feet; required the latitude by the reduction to the meridian.

Formula—

Sin. R = hav. P cos. *l* cos. *d* cosec. Z D + log. 2

where R = reduction, P = hour angle, *l* = lat. D.R., *d* dec.,

Z D zenith dist. by D.R.

	D.	H.
Mer. pass. by Norie's Tab.	15	19
Long. 20° W. =		1
Approx. Green. time	15	20

	M. S.		
		Constant	·30103
*'s Mer. dist.	37 54	Hav.	7·83386
Lat. by D.R.	50° 47′ N.	Cos.	9·80089
*'s decl.	10 35½ S.	Cos	9·99255
Z.D.	61 22½	Cosec.	0·05662
Red. +	33′ 12″	Sin.	7·98496

	D.	H.	M.	S.
Time by chron.	15	20	33	42
Fast		—	11	38
M.T. Gr. Dec.	15	20	22	4
Long. 20° 4′ W.		1	20	16
M.T. at ship		19	1	48
Sid. T. Gr.		17	36	22·5
Accel.			3	20·7
R.A.M. or Sid.T. at ship		12	41	31·2
*'s R.A.		13	19	25·2
*'s H.A.			37	54 E.

Obs. alt.	28°	6′	00″ S.
Star's Total Corr.	—	5	30
T. Alt.	27	59	30
Red.	+	33	12
Red. Alt	28	32	42 S.
	90		
Z.D.	61	27	18 N.
Decl.	10	35	21 S.
Lat.	50	51	57 N.

Explanation of fig. same as in preceding problem.

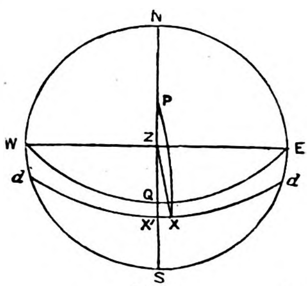


Fig. 3

Example.—July 3rd, 1890; a.m. at ship; lat. by D.R. 54° 2′ S.; long 60° 24′ W.; the observed latitude of α Centauri near the meridian *below the pole* was 25° 1′ 50″; height of eye 26 feet; mean time at Greenwich by chronometer (corrected for error and rate) was July 2d. 23h. 4m. 16s.; required the latitude by the reduction to the meridian.

Sin. R = hav. P cos. *l* cos. *d* cosec. Z D + log. of 2

where R = Reduction, P hour angle, *l* lat. D.R., *d* dec., Z D zen. dist.

	D. H. M. S.
M.T. at Green. July 2	23 4 16
Long. 60° 24′ W.	4 1 36
M.T. at ship	19 2 40
(N.A. p. II.) sid. T.	6 41 54·3
Acceleration	3 47·4
Sid. T. at ship	1 48 21·7
α Centauri R.A. (N.A.)	14 32 10·2
„ H.A.	11 16 11·5 W.
	12
*'s mer. dist. below pole	43 48·5

	M. S.		
		Const.	·30103
*'s mer. dist.	43 48·5	Hav.	7·95940
Lat. by D.R., S.	54° 2′	Cos.	9·76887
*'s decl., S.	60 23	Cos.	9·69390
	114 25		
	180		
*'s mer. Z.D.	65 35	Cosec.	0·04069
Reduction	— 19′ 58″	Sin.	7·76389
T. alt.	24° 55 0		
Red. alt	24 35 2		
*'s pol. dist.	29 36 59		
Lat.	54 12 1 S.		

α Centauri obs. alt.	25° 1′·8
Star's Total Corr.	— 6·8
True alt.	24 55

α Centauri decl. (N.A.)	60° 23′ 1″ S.
„ Pol. dist.	29 36 59

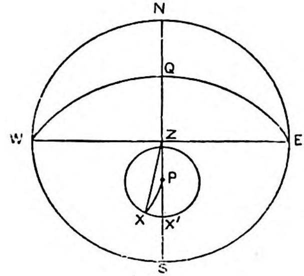


Fig. 4.

Explanation of Figure

The rational horizon, prime vertical, equinoctial and meridian of observer are the same as explained previously. The small circle is α Centauri's Parallel of Declination. P X polar distance, Z X observed zenith distance, Z X′, zenith distance when on the meridian, X P X′ hour angle or time from meridian passage at lower transit.

Reduction = (Z X′ ∽ Z X) to be subtracted from the true altitude because the altitude is least when on the meridian below the pole.

N.B.—The sum of lat. and decl. subtracted from 180° is the mer. zen. dist. by D.R. for an observation *below the pole*; the problem re-worked may be a few seconds more exact.

Or thus :

Formula—

Log. nat. no. = log. rising P + log. cos. l + log. cos. d – 20.

Nat. cos. mer. Z D. = nat. sin. true alt. — nat. no.

where P = hour angle, l = lat. by D.R., and d = declination.

	M. S.		
*'s mer. dist.	43 48·5	log. rising	3·26040
Lat. D. R.	54° 2′	cos.	9·76887
*'s decl.	60 23	cos.	9·69390
Nat. no.	— 529	log.	2·72317
*'s T. alt. 24° 55′ nat. sin.	42130		
Nat. cos.	41601	*'s mer. Z.D.	65° 25′
			90
		*'s alt.	24 35
		,, P.D.	29 37
		Lat.	54 12 S.

Take the difference between natural number and natural sine, because the observation is below the pole.

Examples for Practice

Example 1.—February 24th, 1890 ; p.m at ship ; lat. by D R. 58° 58′ S. ; long. 56° 8′ W. ; the observed altitude of the sun's lower limb when near the meridian was 39° 36′ bearing north ; height of eye 26 feet ; time by watch 1h. 0m. 20s., which had been found 20m. 4s fast on apparent time at ship ; the difference of longitude made to eastward was 32′ after the error on apparent time was determined ; required the latitude by the reduction to the meridian.

Ans. Lat. 58° 52′ 8″ S.

Example 2.—May 10th, 1890 ; at 9h. 34m. 40s. a.m. by watch at ship ; lat. by D.R. 38° 8′ S. ; long. 22° 40′ E. ; the observed altitude of the sun's lower limb when near the meridian was 33° 32′, observer south of sun ; height of eye 25 feet ; the watch had been found 1h. 51m. 4s. slow on apparent time at ship, but ship had made 16′ difference of longitude to eastward since the error for time had been determined ; required the latitude by the reduction to the meridian.

Ans. Lat. 38° 6′ 10″ S.

Example 3.—November 10th, 1890 ; p.m. at ship ; lat. 59° 50′ N. ; long. 32° 46′ W. ; the observed altitude of the sun's lower limb near the meridian was 11° 53′ 20″, bearing south ; height of eye 23 feet ; time by chronometer 3h. 31m. 26s., which had been found 2h. 30m. 56s. fast on apparent time at ship ; but the ship had made 24′ difference of longitude to westward since the error for time had been ascertained ; required the latitude.

Ans. Lat 59° 48′ 26″ N.

Example 4.—March 16th, 1890; a.m. at ship; lat. 45° 37′ S.; long. 168° 20′ E.; observed altitude of sun's lower limb near the meridian was 45° 40′ 25″; observer south of sun; index error of sextant 2′ to subtract; eye 22 feet; time by chronometer 11h. 59m. 54s., which was 25m. 58s. slow on mean time at Greenwich; required the latitude.

Ans. Lat. 45° 31′ 46″ S.

Example 5.—August 15th, 1890; a.m. at ship; lat. by D.R. 42° 50′ S.; long. 176° 4′ W.; observed altitude of sun's lower limb near the meridian 32° 43′ 15″ bearing north; height of eye 17 feet; time by chronometer August 15d. 11h. 20m 49s (astronomical time), which was 8m 51s. fast on mean time at Greenwich; required the latitude by the reduction to the meridian.

Ans. Lat. 42° 37′ 50″ S.

Example 6.—January 28th, 1890; a.m at ship; lat 48° 3′ N.; long 155° 16′ W.; observed altitude of sun's lower limb near the meridian was 22° 50′ 10″ bearing south; index error 1′ 20″ to subtract; height of eye 24 feet; time by chronometer 9h. 40m 55s which was 7m. 15s. slow on mean time at Greenwich; required the latitude.

Ans. Lat. 48° 12′ 14″ N.

Example 7.—October 18th, 1890; a.m at ship; lat. 44° 52′ N.; long. 45° 44′ W.; observed altitude of Procyon (α Canis Minoris) near the meridian 50° 10′; zenith N. of star; height of eye 26 feet; time by chronometer 8h 27m 20s, which was 3m. 42s. fast on mean time at Greenwich; required the latitude by the reduction to the meridian.

Ans. Lat. 45° 2′ 6″ N.

Example 8.—December 8th, 1890; a.m at ship; lat. by account 49° 10′ N.; long. 47° W; observed altitude of Sirius (α Canis Majoris) near the meridian 24° 15′ south of the observer; height of the eye 20 feet; index error of the sextant 2′ 3″ to be subtracted Time by chronometer 5h. 1m. 13s, which was correct for mean time at Greenwich; required the latitude.

Ans. Lat. 49° 10′ 11″ N.

Example 9.—June 5th. 1890; about 4h a.m. at ship; lat. by account 39° 50′ N.; long. 167° 30′ E.; observed altitude of Jupiter near the meridian 32° 22′ S.; height of the eye 23 feet; time by chronometer 4h. 31m. 28s., which was 6m. 30s. slow on mean time at Greenwich; required the latitude.

Ans. Lat. 39° 50′ 30″ N.

Example 10.—December 8th, 1890; p.m. at ship; lat. by D.R. 15° 11′ N.; long. 12° 4′ W.; the observed altitude of Dubhe (α Ursæ Majoris) *near the meridian below the pole* was 23° 50′; height of eye 25 feet; mean time at Greenwich by chronometer (corrected for error and rate) was December 8d. 5h. 48m. 20s.; required the latitude by the reduction to the meridian.

Ans. Lat. 50° 59′ 43″ N.

This problem is useful when the moon is the body observed, and when the ship is steaming at a rapid rate.

It has already been stated on p. 365 that the maximum altitude of the sun, moon, and planets is not always the meridian altitude. If the change in declination is in the direction which would raise the altitude, and the rate of change in declination is greater than the rate of fall in the altitude after the object has passed the meridian, the altitude must increase, and this increase will continue until the rate of change in declination is exactly equal to the rate of fall in the altitude. The maximum altitude will occur at this instant. In a similar manner, if a vessel is approaching the celestial body with a high speed, the vessel will be continually changing its horizon, and the altitude of the body will increase until the rate of speed of the vessel in latitude is the same as the rate of fall in the altitude Should these—the rate of change in the declination, and the rate of speed in latitude—combine together so that they both tend to raise the altitude, it will be easily understood that the maximum altitude will take place many minutes after the body has passed the meridian, and that it may differ considerably from the meridian altitude. Should the combination produce a diminution of the altitude, and if this rate of diminution is greater than the rate of rise in the altitude, then the maximum altitude may occur several minutes before the body comes to the meridian of the observer. As vessels at the present time reach a speed of more than 30 knots an hour, it becomes necessary, for safe navigation in all vessels of high speed, to be able to calculate the correct latitude by using the maximum for the meridian altitude ; or to calculate at what time before or after the time of maximum altitude the body was on the meridian of the observer. *This problem, therefore, becomes a special case of* THE REDUCTION TO THE MERIDIAN.

For Hour-Angle, or Time from Meridian

$$\textit{Formula}\text{—Sin. } h = \frac{\dfrac{c''}{900}}{\text{cos. lat.} \times \text{cos. dec.} \times \text{cosec. (lat.} \mp \text{dec.)}}$$

For the Correction of the Altitude

$$\textit{Formula}\text{—Sin. } 2\,(A \sim a) = \frac{\left(\dfrac{c''}{900}\right)^2}{\text{cos. lat.} \times \text{cos. dec.} \times \text{cosec. lat.} \mp \text{dec.)}}$$

(Lat. $\pm$ dec.) is the zenith distances computed from latitude by D.R. and declination.

c'' represents the combined change in declination and rate of speed in latitude in seconds, in one minute of time.

The speed in knots per hour in a meridional direction is equal to the speed in seconds (") per minute. Thus, if a vessel is going due south 24 knots an hour, this is equivalent to 24" per minute.

RULE.—Enter the *Traverse Table* with the true course, and speed per hour, in the distance column, and take out the number in the latitude column; this will be the change of latitude in seconds (") per minute.

Take from the Nautical Almanac in the case of the sun or planet the " Var. in 1h." and divide it by 60 for the change in declination in seconds (") per minute. In the case of the moon, take the " Var. in 10m." and move the decimal point one figure to the left for the change in declination in seconds (") per minute. For a star there is no change in declination.

If the vessel and the celestial body are moving in opposite directions, that is, the vessel going S. and the celestial body N., or *vice versa*, add together the change in latitude and change in declination in seconds (") per minute. If the vessel and the celestial body are moving in the same direction, that is, both going N., or both going S., take the difference between the change in latitude and change in declination in seconds (") per minute. Call this sum or difference (c).

Find the Greenwich time corresponding to the time of meridian passage, and take from the Nautical Almanac the declination of the body and correct if for the Greenwich time. Add the latitude and declination if of opposite name, but subtract them if of the same name.

To find the time from the Meridian.—From the logarithm of (c) subtract the logarithm of 900, call this (x). Add together L cos. latitude, L cos. declination, and L co-sec. (latitude $\pm$ declination), reject all *tens*, and call the sum of the logarithms (y). Then subtract (y) from (x), and the remainder is the L sin. of the time from the meridian when the maximum altitude occurs.

To find the correction of the maximum altitude.—To the L sin. found in the previous part, add (x), the sum is the L sin. of twice the correction: *or*, the correction can be found as in *reduction to the meridian.*

To find if the maximum altitude is before or after the time of meridian passage.—When the vessel is going towards the celestial body the time will be *after* the time of meridian passage, except when the body is moving in the same direction as the vessel, and the rate of change in declination is greater than the rate of speed in latitude. When the vessel is going away from the celestial body, the time will be *before* the time of meridian passage, except when the body is moving in the same direction as the vessel, and the rate of change in declination is greater than the rate of speed in latitude.

Always add the correction to the maximum true altitude; the sum is the meridian altitude at the place where the altitude was observed. Then find the latitude in the usual way, *using the declination corrected for the time from the meridian.*

Example.—A vessel is steaming due south at the rate of 30 knots an hour; what will be the meridian distance of the moon at the time of her maximum altitude, and the correction to be applied to the maximum altitude to reduce it to the meridian altitude? Lat. by D.R. 50° N., decl. 1° N. "Var. in 10m." 175″ N. Declination increasing.

If the maximum true altitude is 41° 5′ 39″ S., what is the latitude at the time of observation, and at the time of meridian passage?

Change in latitude	30″	S.	per minute.
,, declination	17 ·5	N.	,,
Total change (c)	47 ·5		,,

(c) 47″·5	log. 1·676694	lat.	50° N.	cos.	9·808067
900	log. 2·954243	dec.	1 N.	cos.	9·999934
		diff.	49	co-sec.	10·122220
(x)	8·722451				
(y)	9·930221		(y)		9·930221
Mer. Dist. 14m. 13s.	sin. 8·792230				
	(x) 8·722451				
$A - a$ 11′ 15″	sin. 7·514681				
Corr. 5 37					

As the vessel is going towards the moon, the moon attains her maximum altitude 14m. 13s. *after* the time of meridian passage, and the correction of the altitude is 5′ 37″.

The moon's declination will have increased in the time 17″·5 × 14·22 = 248″·85 = 4′ 9″, thus making it 1° 4′ 9″ N.

The vessel is altering her latitude 30″ per min.; in 14m. 13s. her latitude will have altered 14·22 × 30″ = 426″·6 = 7′ 7″.

Max. tr. alt.	41°	5′	39″	S.
Corr.	+	5	37	
Mer. alt.	41	11	16	
	90			
Mer. Z.D.	48	48	44	N.
Dec.	1	4	9	N.
Lat. at sights	49	52	53	N.
D. lat.		7	7	N.
Lat. at mer. pass.	50	0	0	N.

Examples for Practice

Example 1.—December 19th, 1890; p.m. at ship; in lat. by acct. 57° 5′ N., long. 3° W.; steering S. 20° W. (true) 20 knots an hour; the moon's maximum altitude of the lower limb was observed to be 31° 34′ 50″ S., *i.e.* —2′ 5″; height of the eye 20 feet; required the latitude when the moon was on the meridian of the ship, and also at the time of taking the altitude.

Ans. Latitude when on the meridian 57° 8′ 25″ N.; latitude at the time of taking the altitude 57° 4′ 15″ N.

Example 2.—April 1st, 1890, in lat. by acct. 51° 20′ N.; long. 9° 30′ W., steering S. 10° W. (true) 25 knots an hour; the sun's maximum altitude of the lower limb was observed to be 43° 7′ 0″ S., *i.e.* +1′ 10″; height of the eye 18 feet; required the latitude at the time of observation.

Ans. Latitude 51° 18′ 13″ N.

Example 3.—October 1st, 1890; in lat. by acct. 48° 30′ N., long. 5° 15′ W., steering north (true) 30 knots an hour; the sun's maximum altitude of lower limb was observed to be 38° 5′ 0″ S., *i.e.* —2′ 20″; height of the eye 21 feet; required the latitude at the time of observation.

Ans. Latitude 48° 28′ 4″ N.

LATITUDE BY AN ALTITUDE OF THE POLE STAR OUT OF THE MERIDIAN

Finding the latitude by *Polaris*, or the Polar star, is a form of the reduction to the meridian, for which, however, *special* tables which simplify the computation are provided in the Nautical Almanac. When you have no Nautical Almanac you can use Tables "Latitude by Altitude of Pole Star."

RULE.—1. You must know the sidereal time of observation, *i.e.* the right ascension of the meridian at the place; for which purpose you may have the apparent time at ship, the mean time at ship, or the mean time at Greenwich, and the longitude; with which you can find the required sidereal time of observation.

2. To the *observed* altitude of the star apply the index error, dip, and refraction in the usual way; the result will be the *true* altitude, from which subtract the *constant* 1′, for the *reduced* altitude.

3. Turn to Tables I., II., and III. (pp. 483-485 in the Nautical Almanac for 1890) made expressly for the Pole star. The *first* correction will be additive or subtractive according to the sidereal time; the *second* and

third corrections are always to be added. The application of the three corrections to the *reduced* altitude gives the latitude.

The nearer *Polaris* is to the meridian, either above or below the pole, when the observation is taken, the less will be the error in latitude arising from an error in the time. The time of the star's passing the meridian can be obtained from Table "Apparent Time of Principal Stars Passing Meridian of Greenwich," and the "Explanation."

By Brief Rule and Tables "Latitude by Altitude of Pole Star."—For the sidereal time of observation at place, *see* paragraph 1 above.

To solve the problem without the aid of the special tables in the Nautical Almanac, or Tables "Latitude by Altitude of Pole Star" proceed as follows:—

1. Find the Greenwich date. Correct the altitude as usual.

2. Take out the right ascension and declination of Polaris for the Greenwich date. Find the polar distance and reduce it to seconds, which call (p).

3. Correct the R. A. M. S., to which add Mean time at ship. This will be the sidereal time at place. From the sum subtract the star's right ascension, the result is the hour angle, which call (h).

4. Add log. of p to the log. of co-sine h, the result will be the first correction in seconds.

If the sidereal time at ship be between 6 and 18 hours add the correction to the altitude, if otherwise subtract the correction.

5. Find $\frac{1}{2}$ sine 1" (p sine h)^2 tangent a. To log. p add log. sine h, square this and add log. tangent of a and the constant log. of ($\frac{1}{2}$ sine 1") 4·384545 to the product.

The result is the second correction, always additive.

The formula is $l = a - p \cos. h + \frac{1}{2} \sin. 1'' (p \sin. h)^2 \tan. a$

where l = latitude.
p = polar distance in seconds.
h = the hour angle.
a = the altitude.

The usual method of finding the latitude is shown in the following examples.

From the *observed* get the *true* altitude, applying index error, and using Table "Star's Total Correction."

Then to the *true* altitude apply the correction from Table "Latitude by Altitude of Pole Star," *adding* or *subtracting*, as directed by the sign *plus* or *minus*. A small further correction from Table "Correction of Latitude by Altitude of Pole Star," always *additive*, will give the latitude.

Example.—May 21st, 1890, at 10h. 12m. p.m. *apparent* time at ship; in long. 22° 30′ W.; the observed altitude of *Polaris* was 50° 18′ 10″; height of eye 20 feet; index error +1′ 10″; required the latitude.

	D.	H.	M.
App. T. at ship, May	21	10	12
Long. in time		1	30
App. T. at Gr.		11	42

			S.
(N.A. p. I.) Var. in 1h.			10·02
11h. 42m. =			11·7
		6,0)	11,7·234
	+	1	57·2
(N.A. p. I.) sun's R.A.	3	52	40·4
Corr. sun's R.A.	3	54	37·6
App. T. at ship	10	12	
R.A.M. or Sid. T. at ship	14	6	37·6

		°	′	″
Obs. alt. Polaris		50	18	10
I.E	+		1	10
		50	19	20
Dip 4′ 23″, Ref. 48	−		5	11
True alt.		50	14	9
Subtract	−		1	
Reduced alt.		50	13	9
1st corr.	+	1	15	16
2nd corr.	+			3
3rd corr.	+			54
(By N.A.) latitude		51	29	22 N.

		°	′
By Brief Rule, bringing forward T. alt.	....	50	14
Lat. by Alt. of Pole Star Tables	+	1	15
	+		0
Lat.		51	29 N.

If any interpolation be necessary when taking out the 1st corr. it can be done at sight.

d d	Parallel of dec. (much exaggerated).
A	First point of Aries.
P	The pole.
X	Polaris.
A X	R.A. of Polaris.
A *d* Z	R.A.M. or sid. time of obs.
P x′	The correction, plus to true alt.
Z Q	= P N = the latitude or
Lat.	= P *x*′ + alt.

The other parts of fig. are the same as in previous Fig.

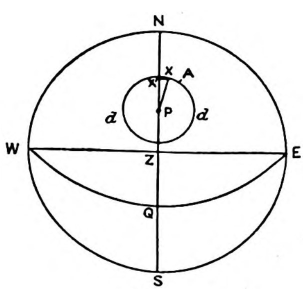


Fig 1.

Example.—October 26th, 1890, a.m. at ship; long. 175° 42′ E.; the observed altitude of *Polaris* was 58° 0′ 30″; height of eye 27 feet; index error of sextant 1′ 50″ to subtract; time by chronometer 5h. 10m. 2s., which was 9m. 40s. slow on mean time at Greenwich; required the latitude.

	D.	H.	M.	S.
Chron. Oct.	25	5	10	2
			9	40
M.T.G.	,,	5	19	42
Long.		11	42	48 E.
M.T.S.	,,	17	2	30
Sid. T. (N.A. p. II.)		14	15	18·11
Acceleration for 5h. 19m. 42s.			+	49·28
			+	3·12
			+	·12
Sid. T. of obs. at ship or R.A.M.		7	18	40·63

Polaris obs. alt.	58°	0′	30″
I.E.	—	1	50
	57	58	40
Dip	—	5	5
	57	53	35
Ref.		—	35
	57	53	0
Subtract	—	1	
	57	52	0
1st corr.	+	0	14
2nd corr.	+	1	23
3rd corr.	+		31
(By N.A.) Latitude	57	54	8 N.

By Brief Rule, T. alt.	57°	53′
Star Tables }		0
Lat. by Alt. of Pole Star }		1·3
Lat.	57	54·3 N.

Explanation of the Figure

N W S E	Rational horizon.
W Z E	Prime vertical.
A	First point of Aries.
X	Pole star (Polaris).
A X	R.A. of Polaris.
A X Z	Sid. time of obs. or R A.M.
Q Z = P N = Latitude.	

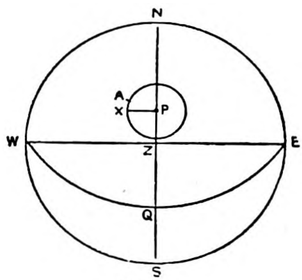


Fig. 2.

The small circle is the parallel of declination much exaggerated.

The figure shows that when the star's hour angle is 6h. E. or W. there is practically no correction to apply, the true altitude being, for all practical purposes, the latitude. When the star is on the meridian above or below the pole, the star's polar distance is the correction, plus to true altitude when below the pole, minus to true altitude when above the pole.

Examples for Practice

Example 1.—September 6th, 1890, in long. 38° 30′ W., at 11h. 51m. p.m., apparent time at ship, suppose the altitude of the Polar star to be 30° 30′ 25″, the height of the observer's eye being 22 feet, and the index error +2′ 20″; required the latitude.

Ans. 29° 24′·7 North.—*By N.A. Method*, 29° 25′ 7″ N.

Example 2.—March 12th, 1890, in long. 45° 30′ E., at 3h. 30m. a.m., mean time at ship, suppose the altitude of the Polar star to be 70° 26′ 30″, the height of the observer's eye being 18 feet, and index error —1′ 30″; required the latitude.

Ans. 71° 32′ North.—*By N.A. Method*, 71° 31′ 35″ N.

Example 3.—November 4th, 1890, in long. 150° 18′ E., at 10h. 23m., p.m., mean time at ship, the observed altitude of the Polar star was 53° 10′ 20″, height of eye 19 feet, index error +2′ 10″; required the latitude.

Ans. 51° 50′·7 North.—*By N.A. Method*, 51° 51′ 8″ N.

Example 4.—At ship March 22nd, 1890, p.m., in long. 40° 30′ W.; the observed altitude of the Polar star was 48° 30′; height of eye 26 feet; time by chronometer March 22d. 10h. 4m. 30s., which was 8m. 10s. fast on mean time at Greenwich; required the latitude.

Ans. 48° 24′·8 North.—*By N.A. Method*, 48° 24′ 28″ N.

Example 5.—August 3rd, 1890, at 10h. 28m. 12s. p.m., mean time at ship; long. 35° 2′ W.; the observed altitude of Polaris off the meridian was 35° 10′ 15″; index error of sextant 1′ 45″ to subtract; height of eye 17 feet; required the latitude.

Ans. By N.A. Method, 35° 3′ 52″ N.

Example 6.—November 23rd, 1890, a.m. at ship; long. 168° 44′ W.; mean time at Greenwich by chronometer (corrected for error and rate) was November 23d. 0h. 11m. 36s.; the observed altitude of the Polar star off the meridian was 48° 15′; height of eye 25 feet; required the latitude.

Ans. By N.A. Method, 47° 28′ 8″ N.

NOTE.—The method of finding the latitude by the pole star, though confined to the northern latitudes, is very useful at sea, as it is available at all times when the star is visible, and the horizon is sufficiently distinct; it also does not require a more accurate knowledge of the time than is usually possessed on board ship. Consequently the chief error will depend on the capacity of the observer in taking an altitude, and the state of the weather, in conjunction with the visibility of the horizon. By using the Nautical Almanac method, the first *correction* always gives the latitude within 2′ of accuracy.

THE CORRECTIONS OF THE COMPASS

BY AMPLITUDES, TIME-AZIMUTHS AND ALTITUDE-AZIMUTHS

THE Compass needle points to the *magnetic* pole of the earth, which, not being coincident with the *true* pole, the result is a varying angle (according to locality), which is called the Variation of the Compass. Before the time of Columbus variation had not been recognised, but we now know there is a slow progressive alteration of the position of the needle with respect to the true meridian ; it moves towards the west, until it arrives at its maximum on that side ; it then returns, passes over the true meridian, and moves easterly, until it arrives at its maximum towards the east ; it then returns as before. When first noticed in London, there was about a point of easterly variation ; this had decreased to *zero* in 1657 ; the needle then moved westward until it attained a maximum of $24\frac{1}{2}°$ W. in 1816 ; now (1914) it is less than 15°, progressing to eastward, and the line of *no* variation will in years to come again coincide with the true meridian—a complete cycle of changes through east and west occupying about 320 years.

DEFINITIONS

Variation of the compass is the angle which measures the difference between the true and magnetic meridians.

Deviation of the compass is the deflection of the needle to the right or left caused by local attraction, generally in the ship or in the cargo. It measures the deflection of the needle from the magnetic meridian.

Compass error is the combined effect of both variation and deviation, the algebraic sum of which is the error, and measures the total deflection of the needle from the true meridian.

The methods of finding the true bearing of a celestial object are by—

Amplitude, Altitude azimuth, Time azimuth.

The elements required are—

Time, Latitude, Declination.

The error of the compass can also be found by comparison with objects on land whose true bearing is known, and the deviation by comparing the magnetic and compass bearings.

The true bearing of a point of land or any conspicuous object can be found in combination with a celestial object.

AMPLITUDE

The method by an amplitude consists in observing the compass bearing of the sun, or other heavenly body when its centre is in the *true* horizon ; that is, at its *rising* or *setting*. Since the object can only be referred to the *visible* horizon, and being subject to *vertical displacement* due to refraction, parallax, and dip, an observation taken near the visible horizon requires a small correction.

Refraction causes objects to appear in the horizon when, on an average, they are 33′ below. A star, if you can really recognise it, may be taken when it is 33′ *plus* the dip above the horizon. The moon, owing to its irregular disc and large horizontal parallax, is a very unfavourable object.

You can find the true amplitude (bearing) of the sun without computing it, from Amplitude Table, which is accurate within a degree.

The Observation.—For the sun, which has an appreciable disc, no preparation is needed except being ready at the compass a few minutes in advance, and keeping the sight vanes (attached to the compass) pointed in the right direction. Then when it is estimated that the lower edge of the sun is about his semi-diameter above the horizon, observe the bearing.

There should also be noted with this observation, as well as with all others for determining the error of the compass and the deviation, the ship's head by Standard Compass, and the angle of heel, if any, to starboard or port.

RULE.—1. *For the Greenwich Date, and the Declination of the heavenly body.*—With the time at ship and longitude, find the corresponding time at Greenwich; and to that Greenwich time reduce the object's declination, taken from the Nautical Almanac.

2. *For the True Amplitude.*—Under the latitude write the corrected declination; then, to the log. secant of the latitude add the log. sine of the declination; their sum (rejecting *index* 10) will be the log. sine of the true amplitude, which take out in degrees and minutes.

The true Amplitude is to be named:
From east if the object is rising, but from west if the object is setting.
Towards N. for N. declination, towards S. for S. declination.

If the declination is 0, the true amplitude is *true* east at rising, *true* west at setting; no computation is required.

For latitude 0 the declination is the true amplitude.

3. *For the Error of the Compass.*—Under the true amplitude write the *observed* amplitude, reckoned from E. or W. as the case may be.

If both are N., or both S., take their difference; if one is N. and the other S., take their sum; the result in each case will be the error of compass.

Then looking from the centre of the compass in the direction of the observed amplitude—

Name error E. if the true is to the right of observed amplitude.
Name error W. if the true is to the left of observed amplitude.

4. *For the Deviation of the Compass.*—Under the error of the compass write the variation taken from the chart, or as given in the question, then—

Error and variation, both E. or both W., take their difference.
Error and variation, one E. and the other W., take their sum.

The resulting deviation will be of the *same name as the error* of compass; *unless* the error has been *subtracted* from the variation, in which case the deviation will be *E. when error is W.*, but *W. when error is E.*

Or, if the error of the compass is to the right of the variation, the deviation is east, but if to the left, the deviation is west.

Also, with error o, deviation is of the same amount as the variation, but of the opposite name; with variation o, the error is the deviation.

The circle (*see* Fig. 1) represents the rational horizon, Z the centre is the zenith, and N Z S the meridian of the observer. P is the elevated pole, W Q E the equinoctial, and W Z E the prime vertical. Then Z Q is the latitude and Z P the co-latitude of the observer. If X is the position of the sun on the horizon at setting W X is the amplitude.

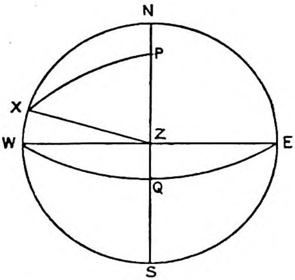


FIG. 1.

The problem is to find W X by means of the quadrantal spherical triangle P Z X (Z X being a quadrant or 90°), W X being the complement of the angle Z. In the rule given the co-lat. (l') and polar distance (p) have been modified to give latitude (l) and decl. (d).

In the diagrams, the point X′ represents the observed amplitude, so that it is instantly seen whether the true (X) is to the right or left of the observed (X′).

In the small diagrams N is the true north point, E is the point to which N is drawn by the combined action of the earth and the iron of the ship—*i.e.*, arc N E is the error of the compass. V is the point to which N is drawn by the magnetism of the earth alone—*i.e.*, arc N V is the variation of the compass; hence the iron of the ship must draw N from V to E—*i.e.*, arc V E is the deviation of the compass, and is easterly when E is on the right of V, and westerly when on the left looking from the angular point which represents the centre of the compass or centre of horizon.

The arc W X or the angle W Z X (fig. 1) is the amplitude, P X is the polar distance and P Z the co-latitude.

The triangle P Z X is quadrantal because Z X = 90°.

The circular parts are therefore the angles at Z and X and the complements of the sides P X, P Z, and the angle Z P X. In the quadrantal spherical triangle P Z X, fig. 1., given P Z and P X to find the angle P Z X.

P X is the middle part and P Z and angle Z the opposite parts.

$$\text{Sin. (comp. P X)} = \text{cos. P Z X,} \times \text{cos. (comp. P Z)}$$

$$\text{that is cos. P X} = \text{cos. P Z X,} \times \text{sin. P Z}$$

$$\therefore \text{Cos. P Z X} = \frac{\text{cos. P X}}{\text{sin. P Z}}$$

$$\text{or Sin. dec.} = \text{sin. amplitude} \times \text{cos. latitude}$$

$$\therefore \text{Sin. amp.} = \frac{\text{sin. dec.}}{\text{cos. lat.}}$$

$$= \text{sin. dec.} \times \text{sec. lat.}$$

which is the formula in common use.

Example 1.—April 26th, at 5h. 11m. a.m. apparent time at ship; lat. 41° 29′ N., long. 5° 45′ E., the observed bearing of the sun at rising was E. ½ N. Required the true amplitude, and error of the compass; also, supposing the variation to be 14° 50′ W., required the deviation of the compass on the direction of the ship's head.

	D.	H.	M.
App. T. ship, April	26	5	11 a.m.
		12	
„	25	17	11
Long. in time		—	23
App. T. at Gr., „	25	16	48

Ap. 26th, sun's decl. (N A. p. I.)	13°	35′	25″N.
48″·19 N. × 7·2h.	—	5	47
Corr. decl.	13	29	3 N.

N.B.—Decl. is corrected for 7h. 12m before noon of 26th.

Formula— Sin. amp. = sec. lat. × sin. dec.

Log. sin. amp. = log. sec. lat. + log. sin. dec.

Lat.	41° 29′	Sec.	0·125432
N. decl.	13 29½	Sine	9·367922
True amp.	18 8½	Sine	9·493354

True amplitude E.	18°	8′	30″	N.
Obs. amplitude E.	5	37	30	N. (E ½ N.)
Error of compass	12	31	0	W.
Variation	14	50	0	W.
Deviation	2	19	0	E.

Explanation of Fig. 2.

N W S E	Rational horizon.
P	The pole.
Z	The zenith.
l′	Co-latitude.
l	Latitude.
W Q E	The equinoctial.
W Z E	Prime vertical.
X	Sun rising.
X′	Bearing by compass.
E X	The rising amplitude.
P X or p	Polar dist.
Z P X	Easterly hour angle.

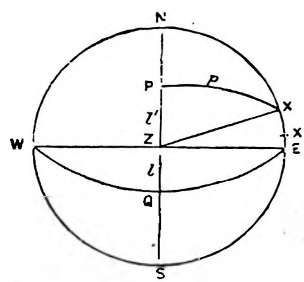


Fig. 2.

The large fig. shows the error is West, and the small fig. shows that the deviation is the difference between the variation and the error and is East.

Explanation of the small figure.

N	True North.
N V	Variation.
N E	Comp. error.
V E	Dev.

By Inspection.—Entering Amplitude Table with lat. 41½° and decl. 13½° the *true amplitude* is E. 18° 10′ N.

N.B.—The parallel of dec. in these figures has been omitted for the sake of clearness.

Example 2.—May 2nd, at 4h. 38m. p.m. apparent time at ship; lat. 51° 31′ S., long. 50° 15′ W., the sun's observed amplitude at setting was W. ¼ S. Required the true amplitude, and error of the compass; also, supposing the variation to be 11° 45′ E., required the deviation of the compass for the direction of the ship's head.

	D.	H.	M.	
App. T. ship. May	2	4	38	
Long. in time		3	21	W.
App. T. at Gr., ,,	2	7	59	

May 2nd. Sun's decl. (N.A. p. I.)	15°	26′	50″	N.
44″·5 × 8h.	+	5	56	
Corr. decl.	15	32	46	N.

Formula.—Sin. amp. = sec. lat. × sin. dec.

Log. sin. amp. = log. sec. lat. + log. sin. dec.

Lat.	51°	31′	Sec.	10·206009
N. decl.	15	33	Sin.	9·428263
True amp.	25	31	Sin.	9·634272

True amp. W.	25°	31′	0″	N.	
Obs. amp. W.	2	48	45	S.	(W. ¼ S.)
Error of comp.	28	19	45	E.	
Variation	11	45	0	E.	
Deviation	16	34	45	E.	

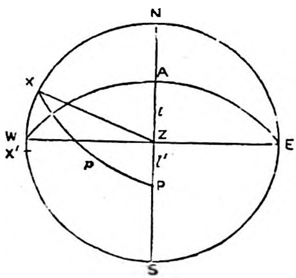
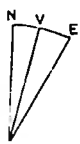


Fig. 3.

Explanation of fig. same as Fig 2.

The large fig. shows the error is East.



The small fig. shows that the deviation is East, and also that it is the difference between the error and the variation.

By Inspection.—Entering Amplitude Table with lat. 51½° and decl. 15½°, the *true amplitude* is 25° 26′ N.

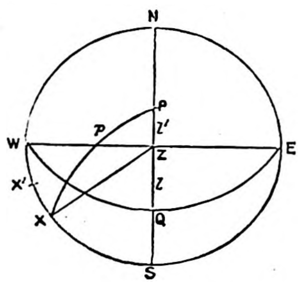
Example 3.—November 23rd, at 3h. 50m. p.m. mean time at ship; lat. 52° 46′ N., long. 49° 45′ W., the sun's observed amplitude at setting was S. 69° 30′ W. Required the true amplitude, and error of the compass; also, as the chart gives the variation 41° 20′ W. at that locality, required the deviation of the compass for the direction of the ship's head, being at the time on a W. ¼ N. course by compass; also give the true course the ship was making.

	D.	H.	M.
M.T. ship. Nov.	23	3	50
Long. in time		3	19
M.T. at Gr. ,,	23	7	9

Nov. 23rd, sun's decl. (N.A. p. II.)	20°	24′	37″ S.
31″ × 7·1h.	+	3	40
Corr. decl.	20	28	17 S.

Formula.—Same as Example 2.

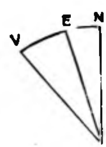
Lat.	52°	46′	Sec.	10·218200
S. decl.	20	28	Sin.	9·543649
True amp.	35	18	Sin.	9·761849



T. amp. W.	35°	18′	S.
Obs. amp. W.	20	30	S. (S. 69° 30′ W.) *
Error of comp.	14	48	W.
Variation	41	20	W.
Deviation	26	32	E. on W. ¼ N.

* By subtracting from 90° the letters change places.

By Inspection.—Entering Amplitude Table with lat. 52¾° and decl. 20½°, the *true amplitude* is W. 35° 10′ S.



Explanation of fig. same as Fig. 2.

The large fig. shows that the error is West, and the small fig. shows that the deviation is the difference between the variation and the error, and is East.

Ship's course by comp. W. ¼ N. = N.	87°	11′	W.
Error	14	48	W.
N.	101	59	W.

or

Ship's true course S. 78° 01′ W.

Example 4.—March 20th, at 6h. 0m. a.m. apparent time at ship; lat. 39° S., long. 144° W., the sun's observed amplitude was N. 84° E. Required the true amplitude, and error of the compass; also, the variation by chart being 3° 45′ W., required the deviation of the compass for the direction of the ship's head, the course at the time being east by compass; also give the true course the ship is making.

	D.	H.	M.	
App. T. ship, March	20	6	0	a.m.
		12	0	
	19	18	0	
Long. in time		9	36	
App. T. at Gr.	20	3	36	

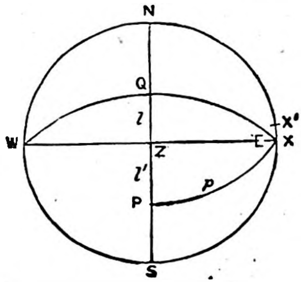
Mar. 20th, sun's decl. (N.A. p. I.)	0°	3′	31″ S.
59″·26 × 3·6h.	—	3	33″
Corr. decl.	0	0	2 N.

The sun being on the equator, rises at 6h. a.m., *true* east; no computation is required.

True amp. E.	0°	0′	N.
Obs. amp. E.	6	0	N. (N. 84° E.)
Error of comp.	6	0	E.
Variation	3	45	W.
Deviation	9	45	E. on East.

Ship's course by comp. being East and error of comp. being 6° E., makes true course E. 6° S. or S. 84° E.

The large fig. shows that the error is East, and the small fig. shows that the deviation is the sum of error and variation, and is also East.



Explanation of fig. same as Fig. 2.

Examples for Practice

Example 1.—February 13th at 6h. 49m. a.m. apparent time at ship; lat. 42° 27′ N., long. 139° 52′ W., the sun's observed amplitude at rising was east. Required the true amplitude, and error of the compass; also, supposing the variation to be 18° 40′ E., find the deviation for the direction of the ship's head.

Ans. T. Amp. E. 18° 2′ S.; Err. of comp. 18° 2′ E.; dev. 0° 38′ W.

Example 2.—June 1st a.m. at ship; lat. 58° 29′ N., long. 85° 40′ W., the sun was observed to rise by compass E. 15° 40′ N., when the chronometer which was correct for G. M. T. indicated 8h. 54m. 40s. Required the true amplitude, and error of the compass; also, supposing the variation to be 15° 10′ W., find the deviation for the direction of the ship's head.

Ans. T. Amp. E. 45° 58′ N.; Err. of comp. 30° 18′ W.; dev. 15° 8′ W

Example 3.—July 2nd at 9h. 29m. p.m. mean time at ship ; lat. 61° 42′ N., long. 56° 45′ W., the sun's setting amplitude was observed to be N. 2° E. Required the true amplitude, and error of the compass ; also, the variation from chart being 58° 30′ W., find the deviation for the direction of the ship's head.

Ans. T. Amp. W. 55° 29½′ N. ; Err. of comp. 36° 30½′ W. ; dev. 21° 59½′ E.

Example 4.—September 23rd at 6h. a.m. apparent time at ship ; lat. 48° 32′ S., long. 177° E., the sun's rising amplitude was observed to be E. ½ S. Required the true amplitude, and error of the compass ; also, the variation by chart being 17° 20′ E., find the deviation for the direction of the ship's head.

Ans. T. Amp. E. 0° 12′ N. ; Err. of comp. 5° 49½′ W. ; dev. 23° 9½′ W.

Example 5.—June 21st at 5h. 43m. p.m. apparent time at ship ; lat. 9° 57′ S., long. 92° E., the sun was observed to set by compass S. 88° 30′ W. Required the true amplitude, and error of the compass ; also, the variation by chart being 0°, find the deviation and the true course, the ship's head by compass being N.E.

Ans. T. Amp. W. 23° 50′ N. ; Err. of comp. and dev. 25° 20′ E. ; ship's true course N. 70° 20′ E.

Example 6.—November 2nd at 4h. 53m. p.m. mean time at ship ; lat. 39° 45′ N., long. 11° 14′ W., the sun's observed amplitude at setting was W. by S. ¾ S. Required the true amplitude and error of the compass ; also, the variation by chart being 21° 30′ W., find the deviation and the ship's true course, the ship's head by compass being S. ¼ E.

Ans. T. Amp. W. 19° 33′ S. ; Err. of comp. 0° 8′ E. ; dev. 21° 38′ E. ; ship's true course S. 2° 41′ E.

BY TIME AZIMUTH

For the Observation.—Any heavenly body sufficiently bright to be seen through the sight vanes of the standard compass may be employed in time azimuths with more or less convenience. When taking the bearings, note the times by a watch the error of which on *local time* is known. The altitude should not be too high for convenient bearings to be taken.

For the Computation you require—the object's hour angle, the latitude, and thence the co-latitude, the object's declination, and thence its polar distance.

For the Sun.—Find the Greenwich date, correct the declination and thence find the polar distance ; the sun's hour angle is the apparent time (at place) past noon if p.m., or the approximate time subtracted from 24h. if a.m. If mean time be given you will have to apply the equation of time to get apparent time.

For a Star.—Find the Greenwich date, and get the mean sun's right ascension if mean time be given, but if apparent time be given get the apparent sun's right ascension and correct it for apparent time at Greenwich.

For Star's Hour Angle.—To mean time at place add the mean sun's right ascension, and from the sum subtract the star's right ascension : the result is the hour angle W., which if less than 12h. is the argument required. But if the hour angle W. exceed 12h. subtract it from 24h. for the hour angle E. If apparent time at place be given add the apparent sun's right ascension for right ascension of the meridian, and subtract the star's right ascension for star's hour angle.

If the *Moon* or *Planet* be used, the object's right ascension and declination will require correction for the Greenwich date. The hour angle is obtained as for a star.

In Fig. a, let P = hour angle, p the polar distance, and l' the co-latitude and X Aurigæ (Capella).

Then—

$$\text{Tan. } \tfrac{1}{2}(Z + X) = \frac{\cos. \tfrac{1}{2}(p - l')}{\cos. \tfrac{1}{2}(p + l')} \cot. \frac{P}{2}$$

$$\text{Tan. } \tfrac{1}{2}(Z \sim X) = \frac{\sin. \tfrac{1}{2}(p - l')}{\sin. \tfrac{1}{2}(p + l')} \cot. \frac{P}{2}$$

Instead of dividing by cos. $\frac{1}{2}(p + l')$ multiply by sec.
Instead of dividing by sin. $\frac{1}{2}(p + l')$ multiply by co-sec.

The sum of $\dfrac{Z + X}{2}$ and $\dfrac{Z \sim X}{2}$ is the greater angle.

The difference of $\dfrac{Z + X}{2}$ and $\dfrac{Z \sim X}{2}$ is the lesser angle.

Special Cases.

When $\frac{1}{2}(p + l') = 90° \; Z = 90°$
When $\frac{1}{2}(p - l') = \;\; 0° \; X = \;\; 0°$

When P = 0 the object is on the meridian
When P = 12 hours the object is again on the meridian, below the pole.

When the polar distance is greater than the co-latitude Z = the sum of $\dfrac{Z + X}{2}$ and $\dfrac{Z \sim X}{2}$; when it is less Z = the difference of $\dfrac{Z + X}{2}$ and $\dfrac{Z \sim X}{2}$.

When the co-latitude is the greater the lesser angle is the azimuth.

When $\frac{1}{2}(p + l')$ exceeds 90° the secant is negative and therefore tan. $\frac{1}{2}(Z + X)$ is negative, in which case the supplement must be taken for $\frac{1}{2}(Z + X)$.

RULE.—Take half the sum of co-latitude and polar distance and call it $\frac{1}{2}(p + l')$.

Take half the difference of co-latitude and polar distance and call it $\frac{1}{2}(p - l')$.

Take half the hour angle and call it $\dfrac{P}{2}$.

Example.—January 25th, at 2h. 10m. a.m. mean time at ship; lat. 49° 2′ N., long. 35° 30′ W.; find the true bearing of α Aurigæ (*Capella*); also if Capella bore N. 64° 30′ W. by compass, and the variation by chart is 36° 40′ W. Required the error of the compass, and the deviation for the direction of the ship's head.

	D.	H.	M.
M.T. ship Jan.	24	14	10
Long. in time		2	22 W.
M.T.G.		16	32

	H.	M.	S.
M.T. ship	14	10	0
Sid. T. (N. A. p. II.)	20	15	2·05
Accel. for		2	37·70
16h. 32m.			5·26
R.A. of mer.	10	27	45
Capella's R.A.	5	8	34
" H.A.	5	19	11 W.
$\frac{P}{2}$	2	39	35·5
In arc	39°	53′	52

Capella's decl.	45°	53′	12″ N.
" N.P.D.	44	6	48
Co-lat.	40	58	0
Sum	85	4	48
Diff.	3	8	48
$\frac{1}{2}(p+l)$	42	32	24
$\frac{1}{2}(p-l')$	1	31	21

Formula—

$$\text{Tan.}\ \tfrac{1}{2}(Z+X) = \frac{\cos.\ \frac{1}{2}(p-l')}{\cos.\ \frac{1}{2}(p+l')}\ \text{cot.}\ \frac{P}{2} \qquad \text{Tan.}\ \tfrac{1}{2}(Z \sim X) = \frac{\sin.\ \frac{1}{2}(p-l')}{\sin.\ \frac{1}{2}(p+l')}\ \text{cot.}\ \frac{P}{2}$$

$$\angle Z, \text{ the azimuth} = \tfrac{1}{2}(Z+X) + \tfrac{1}{2}(Z \sim X)$$

$\frac{1}{2}(p+l')$	42°	32′	24″	Sec	10·13266	. . . . .	Co-sec.	10·16997
$\frac{1}{2}(p-l')$	1	34	24	Cos.	9·99984	. . . . .	Sin.	8·43910
$\frac{P}{2}$	39	53	52	Cot.	0·07773	. . . . .	Cot.	0·07773
$\frac{1}{2}(Z+X)$	58	21	30	Tan.	10·21023	$\frac{1}{2}(X-Z)$ 2° 47′	Tan.	8·68680

$\frac{1}{2}(Z+X)$	58	21	30	
Z = N.	01	8	30 W.	Capella's true bearing.
N.	64	30	0 W.	" Bearing by compass.
Err. of Comp.	3	21	30 E.	
Var.	36	40	0 W.	
Dev.	40	1	30 E.	

Explanation of Fig. a.

N W S E	Rational horizon.
N Z S	Observer's meridian.
W Z E	Prime vertical.
W Q E	Equinoctial.
P X	Polar distance.
P Z	Co-latitude
Z X	Zenith distance.
P	Hour angle.
X	Position of Capella.
P Z X	The Azimuth.

The small circle is Capella's parallel of declination, and shows that the star is circumpolar.

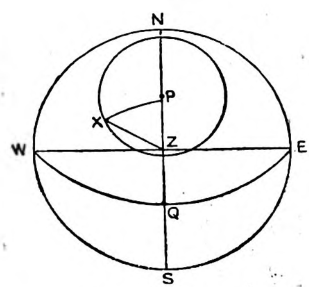


Fig a.

N.B.—The problems here given are only worked to the nearest half-minute of arc.

Examples for Practice

Example 1.—August 12th, at 11h. 56m. 50s. mean time at Greenwich by chronometer (corrected); lat. 51° 49′ N., long. 17° W., Benetnasch (η Ursæ Majoris) bore by compass N. 19° 30′ W. Find the star's true azimuth and the error of the compass; also, the variation being by chart 30° 30′ W., find the deviation and the true course, which was E.S.E. by compass.

Ans. Star's T. Az. N. 49° 43′ W.; Err. of comp. 30° 13′ W.; Dev. 0° 17′ E.; True course N. 82° 17′ E.

Example 2.—March 3rd, at 2h. 2m. a.m. mean time at ship; lat. 15° 42′ N., long. 85° 10′ E., α Centauri bore by compass S. by E. Find the star's true azimuth and the error of the compass; also, the variation being 1° 40′ E., find the deviation for the direction of the ship's head.

Ans. Star's T. Az. S. 13° 12′ E.; Err. of comp. 1° 57′ W.; Dev. 3° 37′ W.

Example 3.—January 3rd, at about 5h. a.m. ship time; lat. 0°, long. 29° 15′ W., when the chronometer indicated 6h. 40m. 20s. which was 3m. 40s. slow for Greenwich mean time, Pollux (β Geminorum) bore by compass N. 64° 30′ W. Required the star's true azimuth, and the error of the compass; also, if the variation is 16° W., find the deviation and the true course, the ship heading S. ½ E. by compass.

Ans. Star's T. az. N. 58° 7′ W.; Err. of comp. 6° 23′ E.; Dev. 22° 23′ E.; True course S. 0° 45½′ W.

Example 4.—June 29th, at about 11h. p.m. ship time; lat. 47° 29′ 21″ S., long. 160° W. Find the true bearing of α Gruis when the chronometer indicated 9h. 20m. 0s. which was correct for mean time at Greenwich; also if the star bore S. 67° E. by the compass, and the variation is 13° 30′ E. by chart. Required the error of the compass, and the deviation for the direction of the ship's head.

Ans. T. Az. S. 61° 51′ E.; Err. of comp. 5° 9′ E.; Deviation 8° 21′ W.

Example 5.—July 22nd, at 1h. 4m. a.m. apparent time at ship; lat. 68° 42′ N., long. 19° W., the sun bore by compass N.E. Required the true azimuth of the sun; and if the deviation for the direction of the ship's head was known to be 10° E. Required the variation of the compass.

Ans. T. Az. N. 14° 59′ E.; Var. 40° 1′ W.

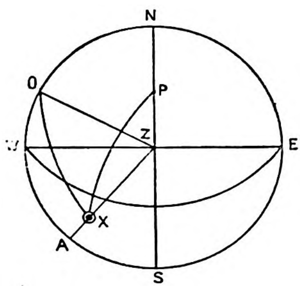
All azimuths can be worked out, expeditiously and with few figures, by Tables A, B and C: they are the A, B and C (Azimuth) Tables of Rosser's "Stellar Navigation." Thus, the Azimuth of Capella as given is briefly found as follows:—

Table A for lat. and hour angle gives	− ·21
" B for decl. and hour angle gives	+ 1·06
	+ 0·85

And in Table C + 0·85 under lat. 49° give Capella's True Azimuth N. 61° W.

N.B.—Azimuths and Amplitudes of Celestial objects whose dec. does not exceed 23° N or S. are usually found by "Inspection" from Davis's and Burdwood's Azimuth Tables, when the dec. exceeds 23° use the above-mentioned, A, B and C Azimuth Tables in Norie's Nautical Tables.

To obtain the true bearing of an object with the aid of the sextant, when the object is on the horizon.



The diagram is on the plane of the horizon,

P Z X or N A is the Sun's azimuth,

O is the object whose true bearing is desired,

X is the position of the sun.

Having the time, latitude and declination, the time azimuth is computed: thus the angle N Z X is known.

At the same time measure the distance O X with the sextant, and take the sun's altitude. When the object is on the horizon Z O = 90°, Z X = zenith distance and O X the distance as measured by the sextant (± the semidiameter), from which the angle O Z X may be computed by the following formula:—

In the quadrantal spherical triangle O Z X, the quadrantal side being O Z, given side O X, and side Z X to find angle O Z X.

$$\text{Cos. } OX = \text{sin. } ZX \times \text{cos. } OZX$$
$$\therefore \text{Cos. } OZX = \text{cos. } OX \div \text{sin. } ZX$$

And the true bearing of O = N Z X — O Z X.

In practice, the best way to find the true bearing of a point, if the sun's alt. is not observed, is to measure the horizontal angle between the sun and the required point by the "Pelorus," and apply it to the sun's true bearing taken from the Azimuth Tables; the result will be the true bearing of the point observed.

BY ALTITUDE-AZIMUTHS

The Observation.—Any heavenly body (sun, moon, star or planet) may be used in this problem ; and the method is the same for each. Take several bearings of the celestial object with the Standard Compass, bisecting it each time if it have a sensible disc, taking its altitude simultaneously, and also noting the times with a watch. The mean of the bearings is the compass azimuth ; the mean of the set of altitudes is the corresponding observed altitude ; and the mean of the times is the corresponding watch time of the observation.

As a general rule, an object should be selected which is relatively low in altitude (say from 20° to 40°), not only that, in being seen through the sight vanes directly rather than by reflection, the compass azimuth is more reliable, but because the condition is more favourable for a reliable true azimuth.

For the Computation.—Whatever celestial object you use, you require the polar distance of the object, the latitude of the place of observation, and the altitude of the object.

RULE.—1. *For the Greenwich Date, and the Declination of the heavenly body.*—With the time at ship and longitude, find the corresponding time at Greenwich ; and to that Greenwich time reduce the object's declination, taken from the Nautical Almanac. The mean time at Greenwich by chronometer can be advantageously used for the correction of the Nautical Almanac elements.

2. *For the Polar Distance.*—Subtract the corrected declination from 90° when latitude and declination are both N., or both S. ; if latitude and declination are one N. and the other S. add 90° to the declination. For latitude 0, subtract declination from 90, thus assuming the latitude and declination to have the same name.

3. *For the True Altitude* correct the observed altitude as required.

4. *For the True Azimuth.*—Add together the polar distance of the object, the latitude of the place, and the true altitude of the object ; divide the sum by 2, for the half-sum, and take the *difference* between this half-sum and polar distance.

Then, add together the—

secant of the latitude,
secant of the altitude,
cosine of the half-sum, and the
cosine of the difference.

The sum of these four logarithms is the log. haversine of the true azimuth, that is—

Formula—

$$\text{Hav. Z} = \frac{\cos. s \times \cos. (s - p)}{\cos. a \times \cos. l} \quad \text{where } s = \frac{a + l + p}{2}$$

a = Alt.
l = Lat.
p = Polar dist.

and substituting the reciprocals of cos. a and cos. l we have

$$\text{Hav. Z} = \sec. a \times \sec. l \times \cos. s \times \cos. (s - p)$$

and

$$\text{Log. hav. Z} = \log. \sec. a + \log. \sec. l + \log. \cos. s + \log. \cos. (s - p) - 30$$

The true azimuth is named from south towards E. or W. in north latitude, and from north towards E. or W. in south latitude.

East with an increasing altitude.
West with a decreasing altitude.

If the true and the observed azimuths are not of the same name make them so by subtracting one of them from 180°.

When on the Equator and the declination N., name azimuth from S.; with declination S., name it from N.

When the latitude and declination are both 0, the object is moving in the *prime vertical*, and will be *true* east while the object's altitude is increasing, and *true* west when it is decreasing.

5. *To find the Error of the Compass.*—Under the *true* azimuth write the *observed* azimuth or bearing, both reckoned from the same point, N. or S.

If both are E., or both W., take their difference; if one is E. and the other W., take their sum; the result in each case will be the error of compass.

Then, looking from the centre of the compass—

Name error E. if the true is to the right of observed azimuth.

Name error W. if the true is to the left of observed azimuth.

In the diagrams the point A′ represents the observed azimuth, so that it is instantly seen whether the true (A) is to the right or left of the observed (A′).

6. *To find the Deviation of the Compass.*—Under the error of compass write the variation, and proceed as directed in the Amplitude Problem.

Example.—January 20th. at 6h. 23m. a.m. mean time at ship; lat. 50° 42′ S., long. 30° 15′ W., the sun's bearing by compass was S. 35° 30′ E. Observed altitude of the sun's lower limb was 17° 7′ 45″; height of eye 27 feet. Required the true azimuth and error of the compass; and the variation by chart being 34° 30′ W., find the deviation of the compass for the position of the ship's head.

	D.	H.	M.	
M.T. at ship, Jan. 20th		6	23	a.m.
		12	0	
„ „	19d.	18	23	
Long. in time		2	1	W.
M.T. at Gr., „	19d.	20	24	

Jan.				
20th. sun's decl. (N.A. p. II.)	20°	4′	1″	S.
$32''{\cdot}79 \times 3{\cdot}6 =$	+	1	58	
Corr. decl.	20	5	59	S.
	90			
P.D.	69	54	1	

Sun's decl. corrected for 3h. 36m. before noon of 20th.

Sun's obs. alt.	17°	7′	45″
Dip 5′ 5″ / Ref. 3 4	—	8	9
	16	59	36
Semid.	+	16	17
Par.	+		8
T. alt.	17	16	1

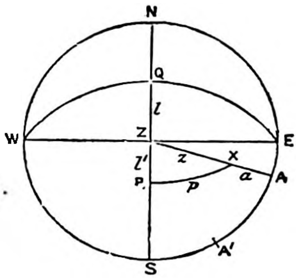
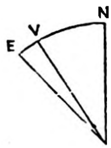


Fig. 1.

Formula—

$$\text{Hav. } Z = \frac{\cos. s \cos. (s - p)}{\cos. l \cos. a} \text{ where } s = \frac{a + l + p}{2}$$

Sun's P.D.	69°	54′	1″	
Lat.	50	42	0	Sec. 10·198335
Alt.	17	16	1	Sec. 10·020028
Sum	137	52	2	
s or ½-sum	68	56	1	Cos. 9·555638
(P.D. ∼ ½-sum)	0	58	0	Cos. 9·999938
T. azimuth N.	100°	52′	E.	Hav. 9·773939
	180			
„ S.	79	8	E.	
Obs. az. S.	35	30	E.	
Error of comp.	43	38	W.	
Variation	34	30	W.	
Deviation	9	8	W.	



Explanation of Fig. 1.

N W S E	Rational horizon.
N Z S	Observer's mer.
W Z E	Prime vertical.
W Q E	Equinoctial.
Z	Zenith.
P	Pole.
X	Position of sun.
l	Latitude.
l'	Co-latitude.
z	Zenith dist.
a	Altitude.
p	Polar dist
N Z A or P Z X	True azimuth.

The large fig. shows that the error is W. because A is to the left of A′.

In the small fig.—

N	True N.
N V	Variation.
N E	Error of compass.
V E	Dev.

Dev. = Error of compass — variation, and is W. because the error is to the left of the variation.

Example.—March 16th, at about 11h. p.m. at ship in lat. 35° 45′ N. the observed altitude of the star Dubhe east of the meridian was 24° 12′ 30″, bearing by compass N. 20° E.; height of the eye 20 feet. Required the star's true azimuth and error of the compass; if the variation is 12° W. what is the deviation of the compass for the direction of the ship's head?

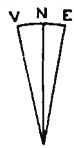
Star's obs. alt.	24°	12′	30″
D.p.	—	4	23
	24	8	7
Ref.	—	2	7
True Alt.	24	6	0

Star's decl.	62°	20′	43″ N.
	90		
P.D	27	39	17

Formula same as above.

P.D.	27°	39′	17		
Lat.	35	45	0	Sec.	10·090672
T. alt.	24	6	0	Sec.	10·039608
Sum	87	30	17		
½-sum	43	45	8	Cos.	9·858740
Rem.	16	5	51	Cos.	9·982629
Tr. Az. S.	150°	53′	E.	Hav.	9·971649
	180				
„ N.	29	7	E.		
Obs. az. N.	20	0	E.		
Err. of comp.	9	7	E.		
Var.	12	0	W.		
Dev.	21	7	E.		

Explanation of fig. same as Fig. 1



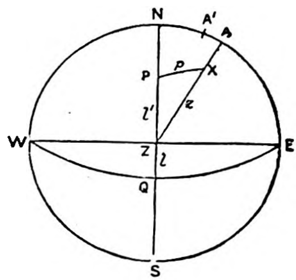


Fig. 2.

In the large fig.

N A True az.
N A′ Comp. az.

The error is E. because A is to the right of A′.

In the small fig.

N True N.
N V Variation.
N E Error of compass.
V E Dev.
Dev. = Error + variation, and is E. because error is to right of variation.

The hour angle is not required to find the Alt. Az. of a star by calculation, but it would be required to construct the fig. to scale and can be found by finding the R.A.M. and thence the Stars H.A.

Example.—October 5th at about 7h. 30m. a.m. at ship; lat. 26° 54′ N., long. by dead reckoning 168° E.: the observed altitude of the sun's lower limb was 18° 52′ bearing by compass east; height of eye 23 feet; time by chronometer 7h. 56m. 5s., which was 15m. 51s. (allowing error and rate) slow on mean time at Greenwich. Required the longitude, and error of chronometer on apparent and mean time at ship; also find the true azimuth and error of the compass, together with the deviation for the direction of the ship's head by compass, the variation by chart being 9° 40′ E.

	D.	H.	M.	
T. at ship, Oct.	5	7	30	a.m
„ „	4d.	19	30	
Long.		11	12	E.
Approx. Gr. date „ „		8	18	

	D.	H	M.	S.
T. by chron., Oct.	4	7	56	5
Slow		+	15	51
M.T. at Gr., „ „		8	11	56

Oct 4th, sun's decl. (N.A. p. II.)	4°	26′	6″ S.	Eq. of T. (N.A. p. II.)	M. 11	S. 18·98
57″·9 × 8·2		7	54·8	0·756s. × 8·2	+	6·20
Corr. decl.	4	34	0·8 S.	Corr. eq. of T. —	11	25·18
N.P.D.	94	34	0·8			

Formula—

$$\text{Hav. P} = \frac{\cos. s \sin.(s-a)}{\cos\ l \sin.\ p}$$

Formula—

$$\text{Hav. Z} = \frac{\cos. s \cos\ (s-p)}{\cos. l \cos. a} \text{ where } s = \frac{a+l+p}{2}$$

Sun's obs. alt.	18°	52′	0″				
Dip 4′ 42″ / Ref. 2 46	—	7	28				
	18	44	32				
Semid.	+	16	2				
Par	+		8				
(*a*) T. alt	19	0	42	. . . .		Sec	10·024361
(*l*) N. lat	26	54	0	Sec.	10·049734	Sec	10·049734
(*p*) N.P.D.	94	34	1	Co-sec.	10·001381		
Sum	140	28	43				
(*s*) ½-sum	70	14	22	Cos.	9·529033	Cos.	9·529033
(*s* − *a*)	51	13	40	Sin.	9·891895		
(*s* − *p*)	24	19	39	. . . .		Cos.	9·959616
	H.	M.	S.				
P	4	23	56	Hav.	9·472043	Hav.	9·562744
d.	24						
App. T. at ship, Oct. 4	19	36	4				
Eq. T.	—	11	25				
M.T at ship	19	24	39				
M.T. Gr., ...	8	11	56				
Long. in time	11	12	43				
Long.	168°	10′	45″E				

Z or T. Az.	S. 74°	23′E.	
(East)	S. 90	0 E.	
Error of comp.	15	37 E.	
Variation	9	40 E.	
Deviation	5	57 E.	

	H.	M.	S.		H.	M	S.
App. T. at ship	7	30	4	M.T at ship	7	24	39
T. by chron.	7	50	5	. . . .	7	50	5
Chron. fast on A.T.S.		20	1	Chron. fast on M.T.S		31	26

In the large fig.

S A = True az.
S A′ = Comp. az.
The error is E. because A is to the right of A′.

In the small fig.

N True North.
N V Variation.
N E Error of compass.
V E Dev.
Dev. = Error of compass — variation and is E. because the error is to the right of variation.

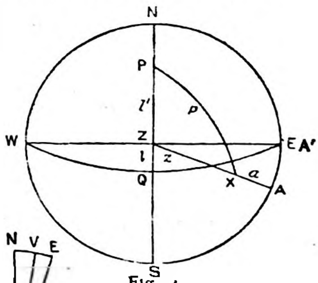


Fig. 4
Explanation of fig. same as Fig. 1.

The formula when using the zenith distance and co.-lat., is as follows:

$$\text{Cos.}^2 \frac{Z}{2} = \frac{\sin. s. \sin. (s \sim p)}{\sin. z. \sin. l'}, \text{ where } s = \frac{z + l' + p}{2} \text{ and}$$

z = zenith dist., l' = co-lat., and p = polar dist.

$$\text{Cos. } \frac{Z}{2} = \sqrt{\frac{\sin. s \sin. (s \sim p)}{\sin. z \sin. l'}}$$

And substituting the reciprocals of sin. z and sin. l' we get,

$$\text{Cos. } \frac{Z}{2} = \sin. s \sin. (s \sim p) \text{ co-sec. } z \text{ co-sec. } l'$$

and

$$\text{Log. cos. } \frac{Z}{2} = \tfrac{1}{2} \left\{ \text{L. sin. } s + \text{L. sin. } (s \sim p) + \text{L. co-sec. } z + \text{co-sec. } l' - 20 \right\}$$

In the spherical triangle P Z X, fig. 3, given the three sides p, l' and z to find angle Z, the azimuth. Using the above formula we get the following result—

Lat.	26° 54′		Alt.	19° 0′ 42″		Polar Dist.	94° 34′ 1″
	90 00			90 0 0			
Co.-lat.	63 6		Zen. dist.	70 59 18			

p	94°	34′	1″		
l'	63	6	0	Cosec.	10·049734
z	70	59	18	Cosec.	10·024360
	2)228	39	19		
s	114	19	39	Sin.	9·959616
$s \sim p$	19	45	38	Sin.	9·529033
					2)19·562743
$\frac{Z}{2}$	52°	48′	34″	Cos.	9·781371
			2		
Z or Az. N.	105	37	8 E.		
	or				
S.	74	22	52 E.		

N.B.—When using the sides of the triangle the azimuth is always reckoned from the elevated pole.

Examples for Practice

Example 1.—January 31st at 4h. 29m. 10s. p.m. mean time at ship; lat. 37° 8′ S., long. 40° 18′ E., the observed altitude of the sun's lower limb was 30° 48′ 40″ bearing by standard compass N. 50° 30′ W.; height of eye 21 feet. Required the sun's true azimuth and error of the compass; also, the variation being 28° 30′ W., find the deviation for the direction of the ship's head.

Ans. T. Az. N. 88° 51′ W.; Err. of comp. 38° 21′ W.; Dev. 9° 51′ W.

Example 2.—February 26th at 7h. 10m. 30s. a.m. mean time at ship; lat. 56° 48′ S., long. 136° 7′ E.. the observed altitude of the sun's lower limb was 15° 5′ 10″ bearing by standard compass E. 1° 30′ S.; height of eye 26 feet. Required the sun's true azimuth and error of the compass; also, the variation by chart being 3° 30′ E., find the deviation for the direction of the ship's head.

Ans. T. Az. N. 82° 49′ E.; Err. of comp. 8° 41′ W.; Dev. 12° 11′ W.

Example 3.—April 15th at 8h. 26m. a.m. mean time at ship; lat. 40° 59′ S., long. 29° 40′ W., the observed altitude of the sun's lower limb was 19° 10′ 20″ bearing by standard compass S. 87° 30′ E.; height of eye 18 feet. Required the sun's true azimuth and the error of the compass; also, the variation by chart being 7° 40′ W., find the deviation for the direction of the ship's head.

Ans. T. Az. N. 57° 2′ E.; Err. of comp. 35° 28′ W.; Dev. 27° 48′ W.

Example 4.—August 23rd a.m. at ship; lat. 37° 40′ N., long. 144° 52′ W., when the chronometer (corrected for error and rate) indicated mean time at Greenwich August 23d. 6h. 17m. 33s. (*Astronomical time*), the sun's observed altitude was 37° 15′ 40″ bearing by compass S. 74° 30′ E.; height of eye 20 feet. Required the sun's true azimuth and error of the compass; also, the variation by chart being 15° 20′ E., find the deviation for the direction of the ship's head.

Ans. T. Az. S. 73° 49½′ E.; Err. of comp. 0° 40½′ E.; Dev. 14° 39½′ W.

Example 5.—May 5th at 6h. 52m. a.m. apparent time at ship; lat. 39° 40′ N., long. 140° 41′ W., the observed altitude of the sun's lower limb was 20° 14′ 40″ bearing by compass E. 1° 30′ N.; height of eye 23 feet. Required the sun's true azimuth and the error of the compass, also the variation by chart being 18° 30′ E.. find the deviation for the direction of the ship's head, which being a N.W. course by compass, give the true course.

Ans. T. Az. N. 85° 17′ E.; Err. of comp. 3° 13′ W.; Dev. 21° 43′ W.; True course N. 48° 13′ W.

Example 6.—November 17th at about 4h. 22m. p.m. at ship; lat. 6° 15′ N., long. by dead reckoning 158° 30′ E., the observed altitude of the sun's lower limb was 17° 6′ 50″ bearing by standard compass S. 64° 40′ W.; height of eye 24 feet; time by chronometer 6h. 2m. 56s. which (allowing for error and rate) was 15m. 23s. fast on mean time at Greenwich. Required

the longitude, together with the sun's true azimuth and the error of the compass; also, the variation by chart being 7° 20′ E. at the ship's position, find the deviation and the true course, the ship's compass course at the time being N.W.

Ans. Long. 158° 41′ 30″ E.; T. Az. S. 67° 51½′ W.; Err. of comp. 3° 11½′ E.; Dev. 4° 8½′ W.; True course N. 41° 48½′ W.

Example 7.—January 1st at about 7h. 26m. p.m. at ship; lat. 49° 42′ N., long. by dead reckoning 14° W., the observed altitude of Pollux (β Geminorum) was 26° 4′ bearing by standard compass N. 78° 30′ E.; height of eye 28 feet; time by chronometer 8h. 30m. 20s. which was (by error and rate) 9m. 56s. fast on mean time at Greenwich. Required the longitude, together with the star's true azimuth and the error of the compass; also, the variation by chart being 25° 40′ W. at the ship's position, find the deviation and the true course, the ship's compass course being E. ¼ N.

Ans. Long. 13° 40′ 30″ W.; T. Az. N. 76° 25′ E.; Err. of comp. 2° 5′ W.; Dev. on E. ¼ N. = 23° 35′ E.; True course E. 4° 54′ N.

Example 8.—January 11th at about 5h. 15m. p.m. at ship; lat. 48° 27′ N., long. by dead reckoning 28° W., the observed altitude of Vega (α Lyræ) was 28° 1′ bearing by standard compass N. 27° 30′ W.; height of eye 26 feet; time by chronometer 6h. 53m. 56s. which (allowing for error and rate) was 6m. 56s. slow on mean time at Greenwich. Required the longitude, together with the star's true azimuth and the error of the compass; also, the variation by chart being 32° 30′ W. at the ship's position, find the deviation and the true course, the ship's compass course at the time being N.W. by W.

Ans. Long. 28° 12′ 45″ W.; Star's T. Az. N. 62° 6′ W.; Err. of comp. 34° 36′ W.; Dev. 2° 6′ W.; True course S. 89° 9′ W.; or about W. 1° S.

LONGITUDE BY CHRONOMETER

(1) BY SUN'S ALTITUDE; OR (2) BY STAR'S ALTITUDE

Longitudes at sea are determined by computing the hour-angle of a heavenly body the altitude of which has been measured by a sextant, and through this hour angle obtaining the local time for comparison with the Greenwich time by chronometer.

Thus longitude becomes the difference between time at place and time at Greenwich at the same instant.

The Greenwich time is ascertained from the *chronometer*, which has previously been regulated, and its error and rate tabulated; the daily rate being properly applied gives the Greenwich time at any instant.

In determining the local time by means of the object's altitude above the sea-horizon let the time be noted by watch. For greater precision, observe several altitudes in quick succession, noting the time of each, and take the mean of the altitudes as corresponding to the mean of the times. But in taking the mean of several observations in this way it must not be forgotten that we assume that the altitude varies in proportion to the time, which is theoretically true only in the exceptional case where the observer is on the equator and the object's declination is zero. It is, however, practically true for an interval of a few minutes when the heavenly body is not too near the meridian.

Best Position of a Heavenly Body for determining the Time at Place by an Altitude of the object.—When the azimuth of the heavenly body is 90°; that is, when it is on the *prime vertical*, bearing *true* east or west; the error in time will be the least possible, since, for an object in that position—(1) it rises and falls fastest, allowing its altitude to be observed with the greatest precision; also (2) the error in the hour angle, corresponding to a small error in the altitude, is least; and (3) the error in the hour angle, corresponding to a small error in the latitude, vanishes.

But no object can reach the prime vertical unless its declination is of the same name as the latitude of the place, and even then observations when the object is *nearly* east or west must not be carried so far as to include observations at very low altitudes where anomalies in the refraction may produce serious errors.

Tables giving the time when an object is on the prime vertical, that is, when it bears east or west, and its altitude thereon, are given towards the end of Norie's Nautical Tables.

When the latitude and declination are of opposite names the object will not be on the observer's prime vertical, but will be nearest to it when rising or setting; therefore the altitude should, in this case, be taken as soon as it exceeds 10° or 12°.

For the Sun, when you are in the opposite hemisphere, it is not practicable to observe in the most favourable position, hence choose the position as near to it as possible, but not too low; and remember that, *generally*, for any object, throughout the interval between the best position and the meridian the nearer the object is to the meridian the more unfavourably is it situated for the purpose of computing the time from an altitude. In the *tropics*, with latitude and declination of same name, proximity of the *sun* to the meridian has little effect on the hour angle.

The stars and planets are good objects at *twilight* and dawn, and the moon by day when in a favourable position.

Longitude by Chronometer and Sun's Altitude.

RULE—1. Take any odd number of altitudes of the sun, with the corresponding times by chronometer; take the *mean* of the altitudes, and also of the times, *i.e., take the sum of each and divide by the number of observations.*

At sea, the first thing to note is—*does the time by chronometer* require to be increased by 12 hours, in order to express the Greenwich time astronomically?—Your ship time and longitude will indicate this (*see* p. 233).

2. *For the Greenwich Date, mean time.*—To the mean of the times ascertained as above (par. 1), and expressed astronomically, apply the *original error*, by *addition* if *slow*, by *subtraction* if *fast*.

Then multiply the *daily rate* by the number of days and parts of a day that have elapsed since the original error on Greenwich was determined; the product, which is called the *accumulated rate*, being *added* to the above sum or remainder, if the chronometer be *losing*, or *subtracted* from it if *gaining*, the result will give the mean time at Greenwich, for which all the elements from the Nautical Almanac must be corrected (*see* also pp. 124-6).

3. *For the Declination.*—Take the declination from the Nautical Almanac, p. II. of given month, and correct it (by Var. in 1h. p. I. of Almanac) for the Greenwich date, mean time (*see* p. 238).

For the Polar Distance.—Subtract the corrected declination from 90° when latitude and declination are both N., or both S.; if latitude and declination are one N. and other S., add 90° to the declination. For latitude 0, subtract declination from 90°.

4. *For the True Altitude*, correct the mean of the observed altitudes for dip, semidiameter, refraction, and parallax (*see* p. 254).

5. *For the Equation of Time.*—For the given day take the equation from Nautical Almanac, p. II. of month; also take "Var. in 1h." for same day from p. I. of month; multiply the "Var. in 1h." by the hours and tenths of Greenwich time for the *correction* of equation (*see* p. 239).

Also, take special notice, on p. I. of Nautical Almanac of the day, when the equation changes; in the column the change is marked by a strong dash —— and at top thus $\frac{\text{add.}}{\text{sub.}}$ or $\frac{\text{sub.}}{\text{add.}}$

6. *For the Hour Angle.* Write down in succession the true altitude, the latitude, and the polar distance. Take the sum of these quantities, which

divide by 2, for the half-sum; lastly, subtract the true altitude from the half-sum for the remainder.

Then add together—

The secant of the latitude	(Table Log. Sines, Co-sines, etc.).
„ co-secant of the polar distance,	„
„ cosine of the half-sum, and	„
„ sine of the remainder.	„

The sum of these four logarithms (rejecting *tens* in the index) will be the L Haversine of the hour-angle, *i.e.*, distance of the object from the meridian, in time.

7. *For the Apparent Time at Ship.*—If the altitude was observed in the forenoon, *i.e.* a.m. at ship, subtract the hour angle from 24 hours and before the remainder write the astronomical ship date, which will be one day less than the civil date; or, to save subtracting from 24 hours, take the hour angle from the bottom of the Table.

For an altitude in the afternoon, *i.e.* p.m. at ship, the hour angle is the correct time, before which write the ship date unaltered :—

And thus you have the ship date, apparent time or the apparent time at ship.

8. *For the Mean Time at Ship.*—Under the apparent time at ship write the equation of time, which is to be added or subtracted as directed in Nautical Almanac, p. I. of given month. The result will be the mean time at ship.

9. *For the Longitude.*—Under the ship date, mean time, write the Greenwich date, mean time; take the less from the greater, *remembering that the value of the days in each must be considered as well as the hours.* The remainder will be the longitude in time, which turn into ..° ..′ ..″, and name—

East if Greenwich date, mean time, is less than ship date, mean time

West if Greenwich date, mean time, is greater than ship date, mean time.

10. *Comparison of Ship Time and Time by Chronometer.*—Now, if it be required to find the error of the chronometer upon apparent or mean time at ship, it is only necessary to bring down the *mean of times* shown by such chronometer at the time of observation, and the difference between that time and the apparent or mean time at ship will be the error, fast or slow of ship according as the time by Chronometer is greater or less than the ship's time.

The rare occasions on which the latitude is 0° and the declination of the object is 0, the zenith distance converted into time is the hour-angle.

Example 1.—May 19th at about 3h. p.m. at ship; in long. by D.R. 56° W.; the following altitudes of the sun's lower limb were observed, with the corresponding times by chronometer, which was *fast* 3m. 18s. on Greenwich mean noon on May 1st and *gaining* 7·8s. per day:

At noon the latitude by sun's meridian altitude was 41° 31′ N., since which time the ship had run N.W $\frac{1}{4}$ N. (*true*) 31 miles; height of eye 18 feet.

Required the true longitude.

	D H M		
Ship time, May	19 3 0		Green. time agrees approximately with time by chron.
D. R. long in time	3 44	W.	
Approximate Green time, May	19 6 44		

Time by chron.	H. M. S.		Obs. alt. Sun's L.L.			s
	6 58 40	....		44° 7′ 10″		
	6 59 36	....		43 57 20	Daily gain	7·8
	7 0 51	....		43 44 35	Days from May 1-19	18
Sum of times	3)20 59 7		Sum of alts.	3)131 49 5		624
						78
Mean of times	6 59 42		Mean of alts.	43 56 22		
Original error	— 3 18		Dip	— 4 9		60)140·4
	6 56 24			43 52 13	Gain in 18 days	2m. 20·4s.
Accum. rate	— 2 23		S.D.	+ 15 50	Gain in 7 hours	+ 2·3s.
M.T.G., May 19	6 54 1			44 8 3	Accum. rate 2m. 22·7s. fast	
			Ref and par.	— 53		
			Tr. alt.	44 7 10		

	S. M.		
Eq. of time (p. II N.A.)	3 44·6	Sun's decl. (p. II. N.A.)	19° 49′ 10″ N.
0·12s. × 6·9h. =	— 0·8	32″ × 6·9h. =	+ 3 41
Equat. at Green. M.T. =	3 43·8	Sun's decl. at Green. M.T.	19 52 51 N.
			90
		Sun's polar distance	70 7 9

Latitude of ship at noon	41° 31′ N.
Diff. lat. (N.W. $\frac{1}{4}$ N. 31m.) =	+ 23 N.
Lat. when sights were taken	41 54 N.

Formula— $$\text{Hav. P} = \frac{\cos. s \sin. (s - a)}{\cos. l \sin. p}$$

and substituting the reciprocals of cos. l and sin. p

we get hav. P = sec. l, co-sec. p, cos. s, sin. $(s—a)$

Log. hav. P = log. sec. l + log. co-sec. p + log. cos. s + log. sin. $(s—a)$ — 30

where l = the latitude, a = alt., p = polar dist., $s = \frac{l + a + p}{2}$

Sun's true altitude	44°	7′	10″		
Ship's latitude	41	54	0	Sec.	0·128245
Polar distance	70	7	9	Co-sec.	0·026686
Sum	156	8	19		
$\frac{1}{2}$ sum	78	4	9	Cos.	9·315405
Remainder	33	56	59	Sin.	9·746996
	H.	M.	S.		
App. T. at ship, 19d.	3	11	42	Hav.	9·217332
Equat. of time	—	3	44		
M.T. at ship, May 19d.	3	7	58		
M.T. at Green., May 19d.	6	54	1		
Longitude in time	3	46	3	= 56° 30′ 45″ W. long.	

LONGITUDE BY CHRONOMETER

Explanation of Fig. 1.

The circle N W S E	Rational Horizon.
W Z E	The Prime Vertical.
W Q E	The Equinoctial.
N Z S	Meridian of Observer.
P X (p)	Polar distance.
Z X (z)	Zenith distance.
P Z (l')	Co-latitude.
Z Q (l)	The Latitude.
X A (a)	The Altitude.
Z P X	Hour angle.
P Z X	The azimuth.

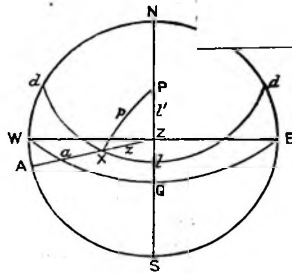


Fig. 1.

Using the sides of the spherical triangle Z P X proceed as follows:

Example 1*a*.—In the spherical triangle Z P X, Fig. 1, given z = zenith distance, l' = co-latitude, and p = polar distance to find $\angle$ P—

$$(1)\quad \text{Cos. } \frac{P}{2} = \sqrt{\frac{\text{sin. } s \text{ sin. } (s - z)}{\text{sin. } p \text{ sin. } l'}},\ \text{where } s = \frac{z + l' + p}{2}$$

and substituting the reciprocals of sin. p and sin. l'—

$$(2)\ \text{we get}\quad \text{Cos. } \frac{P}{2} = \sqrt{\text{co-sec. } p \text{ co-sec. } l' \text{ sin. } s \text{ sin. } (s - z)}$$

And

$$(3)\ \log.\ \cos. \frac{P}{2} = \frac{1}{2}\left\{\log.\ \text{co-sec.}\, p + \log.\ \text{co-sec.}\, l' + \log.\ \sin.\, s + \log.\ \sin.\,(s - z) - 20\right\}$$

In (1) and (2) the trigonometrical functions are Natural.

In (3) the trigonometrical functions are logarithmic.

Altitude	44°	7′	10″
z =	45	52	50
Latitude	41°	54′	
l'	48	6	

z	45°	52′	50″		
l'	48	6	00	co-sec.	10·026686
p	70	7	9	co-sec.	10·026686
Sum	164	5	59		
s	82	2	59	sin.	9·995806
$s - z$	36	10	9	sin.	9·770978
					2)19·921715
				cos.	9·960857

	H.	M.	S.
$\frac{P}{2}$	1	35	51
			2
$\angle$ P or app. time at ship 19d.	3	11	42
Equation of time	—	3	44
Mean time at ship 19d.	3	7	58
Mean time Greenwich 19	6	54	1
Long. in time	3	46	3
	60		
	4)226	3	0
Longitude in	56	30	45 W.

Example 2.—June 26th, at about 7h. 10m. a.m. at ship; in long. by D.R. 18° W.; the following altitudes of the sun's lower limb were observed, with the corresponding times by chronometer, which had been found 3m. 48s. *slow* on mean noon at Greenwich on June 2nd and *gaining* 2·4s. per day:

The latitude at noon was 10° 45′ S., and the ship's *true* course was E.S E. with distance run 24 miles since the sights were taken: height of eye 24 feet.

Required the true longitude of the ship at time of sights, and brought to noon.

	D. H. M.		D. H. M.	
Ship time, June 26	7 10 a.m. =	June 25	19 10	Shows the 8h. by chron. must be 20h. at Greenwich.
D.R. long.			1 12 W.	
Approx. Green. time, June			25 20 22	

Time by Chron.	H. M. S.		Obs. Alt. Sun's L.L.	
	8 28 35	....		13° 8′ 30″
	29 20	....		20 45
	30 18	....		33 20
Sum of times 3)	88 13		Sum of alts. 3)	62 35
Mean of times	8 29 24		Mean of alts.	13 20 52
	12		Dip.	— 4 48
	20 29 24			13 16 4
Original error	+ 3 48		S.D.	+ 15 46
	20 33 12			13 31 50
Accum. rate	— 0 57		Ref. and par.	— 3 19
M.T.G., June 25d.	20 32 15		Tr. alt.	13 28 1

	S.
Daily gain	2·4
Days from June 2 to June 25	23
	72
	48
Gain in 23 days	55·2
Gain in 20h. 33m.	+ 2·1
Accum. rate	57·3s. fast

	M. S.
Eq. of time (p. II. N.A.)	2 33·2
0·52s. × 3·5h. =	— 1·8
Equat. at Green. M.T.	+ 2 31·4

Sun's decl. (p. II. N.A.)	23° 22′ 4″ N.	
5″.1 × 3.5h. =	+ 0 18	
Sun's decl. at Gr. M.T.	23 22 22	N.
	90	
Sun's polar distance	113 22 22	

Sun's decl. and Eq. of T. are corrected for 3½ hours before noon of June 26th.

Latitude of ship at noon	10° 45′	S.
Diff. lat (E.S E. 24m.) =	— 9·2	S.
Lat. when sights were taken	10 35·8	S.

Formula —

$$\text{Hav. P} = \frac{\cos.\ s \sin.\ (s-a)}{\cos.\ l \sin.\ p} \quad \text{where } s = \frac{l+a+p}{2}$$

a = Altitude
l = Latitude
p = Polar distance

Sun's true altitude	13°	28′	1″		
Ship's latitude	10	35	48	Sec.	0·007470
Polar distance	113	22	22	Co-sec.	0·037184
Sum	137	26	11		
½ sum	68	43	5	Cos.	9·559856
Remainder	55	15	4	Sin.	9·914691 w
	H.	M.	S.		
P	4	40	45	Hav.	9·519201
	24				
App. T. at ship, 25d.	19	19	15		
Equat. of time	+	2	31		
M.T. at ship, 25d.	19	21	46		
M.T. at Greenwich, 25d.	20	32	15		
Longitude in time	1	10	29	= 17° 37′ 15″	W. long at sights
E.S.E. 24m. = Dep. 22·2 = D. long.				22 30	E.
				17 14 45	W. long. at noon

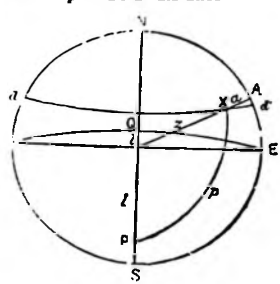


Fig. 2.

Example 3.—November 3rd, at about 8h. a.m. time at ship, when the south point of Trinidad Island (S. Atlantic) in lat. 20° 31′ S., long. 29° 19′ W., bore N. 60° W. (*true*) distant 8 miles, sights were taken for verification of position, and approximate error of chronometer as follows:

Mean of times by chronometer was 9h. 55m. 55s.; mean of altitudes of sun's lower limb 33° 9′ 10″; the chronometer was (allowing for error and rate) supposed to be 12m. 18s. fast on mean time at Greenwich: height of eye 24 feet. Find the approximate error of chronometer on mean time at Greenwich.

	H.	M.	S.
Ship T. Nov. 3rd. 8h. om. a.m. =	2	20	0
Approximate long. W. =		1	57
Approx. Gr. time		21	57

Hence the 9h. by chron. indicates Green. time Nov. 2d. 21h.

	H.	M.	S.
Mean of times by chron.	9	55	55
	12		
Nov. 2d.	21	55	55
Fast.	—	12	18
M.T. at Gr. by chron. „ 2d.	21	43	37

Mean of sun's alt. L.L.	33°	9′	10″
Dip	—	4	48
	33	4	22
Semid.	+	16	10
	33	20	32
Ref. — Par.	—	1	21
Sun's T. alt.	33	19	11

Decl. is corrected for 2·3h. before noon of Nov. 3rd.

S. point of Trinidad, lat. 20° 31′ S.	3rd decl. 15° 8′ 54″·6 S.	
Bearing N.60°W.8m.D. lat. + 4 S.	46″·8×2·3 — 1 47·6 S.	M. S.
Estimated lat. of ship 20 35 S.	Corr. decl. 15 7 7 S.	Eq. T.—16 20·9
	90	
	P.D. 74 52 53	

Bearing N. 60° W. 8m = Dep. 6′·9 = 7′·37 D. long. E.

Formula— $$\text{Hav. } P = \frac{\cos\ s\ \sin.\ (s - a)}{\cos.\ l\ \sin.\ p}, \text{ where } s = \frac{a + l + p}{2}.$$

T alt.	33°	19′	11	
Lat.	20	35	0	Sec. 0·028649
S.P.D.	74	52	53	Co-sec. 0·015298
Sum	128	47	4	
½-sum	64	23	32	Cos. 9·635693
Rem.	31	4	21	Sin. 9·712753
	H.	M.	S.	
Sun's H.A.	3	58	19	Hav. 9·392393
App. T. ship 2d.	20	1	41	
Eq. T.	—	16	21	
M.T. ship 2d.	19	45	20	
Long. 29° 11′·6	1	56	46	
M.T. at Gr. 2d.	21	42	6	
Time by chron. „	21	55	55	
Chron. fast on Gr. M.T.		13	49	

S. pt. of Trinidad, long.	29° 19· W.
D. long.	7·4 E.
Position of ship by bearing	29 11·0W

Explanation of Fig. 3 same as Fig. 1.

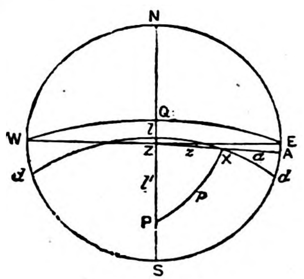


Fig 3.

FOR THE TIME AT SHIP AND LONGITUDE BY AN ALTITUDE OF A FIXED STAR, A PLANET, OR THE MOON

REMARKS.—When the hour angle of a heavenly body, or the time at ship, is to be determined by an altitude of a fixed star, or planet, or by the moon, the observation is the same as for the sun, already described on pages 419-420: the remarks there given also apply to the best position of the object, as well as to the taking of the mean of the times by chronometer or watch, and the mean of the altitudes.

The same method is also to be adopted in finding the error of the chronometer on approximate and mean time at ship.

For the true Altitude.—The observed altitude must also be corrected as already indicated on p. 258 for a fixed star; on p. 257 for a planet; and on p. 256 for the moon.

For the Mean Sun's Right Ascension, take out (from Nautical Almanac, p. II.) the sidereal time for the given date, and accelerate it for the Greenwich mean time: it is required for whichever object you observe (*see* also p. 247).

The other elements to be taken from the Nautical Almanac, and corrected for the Greenwich date, are as follow:

For a Fixed Star, the declination and right ascension are taken from the Nautical Almanac under the heading "*Apparent Places of Stars,*" for a given day, no correction of these elements being required.

For a Planet.—Turn to the Nautical Almanac under the heading of the given planet (Venus, Mars, Jupiter, or Saturn), and Greenwich *mean time,* where the declination and right ascension are given for every day, Greenwich *mean noon;* being hence a difference for 24 hours, you must correct it accordingly for the Greenwich date, mean time.

The planet's horizontal parallax is found under the heading of the given planet, "*at Transit at Greenwich.*"

Under the same heading, "*at Transit at Greenwich,*" you will also find the "Var. of R.A. in 1 hour of Long." and the "Var. in Decl. in 1 hour of Long."; these you can conveniently use for the correction of the given planet's right ascension and declination, in the same manner as you use the "Var. in 1 hour" in correcting the sun's declination.

For the Moon.—The declination and right ascension come from Nautical Almanac, pp. V. to XII., and have the "Var. in 10m." attached, through which each can be corrected for the Greenwich date, mean time (*see* p. 241). The moon's semi-diameter and horizontal parallax are given in Nautical Almanac, p. III., and require correction for Greenwich date, mean time; and also the first augmented for altitude (Table D.), and the second reduced for latitude (Table E.) in Norie's Tables.

To compute the Object's Hour Angle, or Meridian Distance in Time.—With the *true* altitude, the latitude and the *corrected* declination, you find object's hour angle as in the case of the sun (*see* p. 420).

You require the Westerly Meridian Distance.—If the *object* is *west of* the *meridian* the hour angle taken from the Haversine Table will be the westerly meridian distance, but if the *object* is *east of* the *meridian* subtract the hour angle from 24h. for the westerly meridian distance.

For the Mean Time at Ship.—To the object's westerly meridian distance add the object's corrected right ascension; the sum will be the right ascension of the meridian, or sidereal time of observation, from which

subtract the mean sun's right ascension, borrowing 24h. if necessary; the remainder will be the mean time at ship, before which write down the day.

For the Longitude.—The difference (as before) between the mean time at ship and mean time at Greenwich (*see* p. 421, paragraph 9) will be the longitude in time, which convert into arc.

Another Method of finding the Longitude.—To the Greenwich mean time add the mean sun's right ascension; the sum is the sidereal time at Greenwich. The difference between the sidereal time at ship or right ascension of the meridian and the sidereal time at Greenwich, is the longitude in time, which convert into arc.

Example 4.—January 5th, a.m. at ship, in lat. by D.R. 18° 17′ N., and long. by D.R. 55° 20′ W.; the following altitudes of the star *Procyon* (α Canis Minoris) were taken when it was *west* of the meridian; the height of the eye 19 feet. The chronometer was 10m. 20s. slow of Greenwich mean time; and the mean of times by ship's watch was 4h. 49m. 50s. a.m. 5th January. Required the longitude by chronometer.

Time by chron.	H. M. S.
	8 27 30
	28 50
	30 10
3)	86 30
Mean	8 28 50
	12
Jan. 4d.	20 28 50
Chron. slow	+ 10 20
M.T. Gr. 4d.	20 39 10

Obs. alts. Procyon.	
	24° 44′ 20″
	29 30
	14 10
3)	88 00
Mean	24 29 20
Dip	— 4 16
	24 25 4
Ref.	— 2 6
Star's T. alt.	24 22 58

	H. M. S.
Star's R. A. (N.A.)	7 33 32·8

	H. M. S.
Sid. T. (N.A. p. II.)	18 56 10·9
Accel. for 20h. 39m. 10s.	3 23·6
Mean sun's R.A.	18 59 34·5
	M. S.
Equat. of T. 5th Jan.	5 43·7
1.11s. × 3⅓h.	— 3·7
Corr. equat.	5 40
Star's decl. (N.A.)	5° 30′ 22″ N.
	90
Star's pol. dist.	84 29 38

(*See Fig.* 4). $$\text{Hav. P} = \frac{\cos. s. \sin. (s - a)}{\cos. l. \sin. p} \text{ where } s = \frac{a + l + p}{2}$$ a = Altitude, l = Latitude, p = Polar dist.

and substituting the reciprocals of cos. l and sin. p, we get—

$$\text{Hav. P} = \text{sec. } l. \text{ co-sec. } p. \cos. s. \sin. (s - a)$$

$$\text{Log. hav. P} = \text{log. sec. } l + \text{log. co-sec. } p + \text{log. cos. } s + \text{log. sin. } (s - a) - 30$$

T. alt.	24° 22′ 58″		
Lat.	18 17 0	Sec.	0·022497
P.D.	84 29 38	Co-sec.	0·002008
Sum	127 9 36		
s	63 34 48	Cos.	9·648309
$(s \sim a)$	39 11 50	Sin.	9·800711
	H. M. S.		
✳'s H.A.W.	4 24 27	Hav.	9·473525
„ R.A.	7 33 33		
Sid T. at ship	11 58 0		
Mean sun's R.A.	18 59 35		
M.T. at ship 4d.	16 58 25		
M.T. Gr. „	20 39 10		
Long. in time	3 40 45	=	55° 11′ 15″ W

Example 4. By Direct Method.

In the spherical triangle Z P X, Fig. 4, given side z = zenith distance, side l' = co-latitude, and side p = polar distance to find ∠ P, the hour angle.

$$\text{Cos. } \frac{P}{2} = \sqrt{\frac{\sin. s. \sin. (s - z)}{\sin. l'. \sin. p}}. \text{ where } s = \frac{z + l' + p}{2}$$

$$\text{Log. cos. } \frac{P}{2} = \tfrac{1}{2}\left\{\text{Log. co-sec. } l' + \text{log. co-sec. } p + \text{log. sin. } s + \text{log. sin. } (s - z) - 20\right\}$$

Alt.	24°	22′	58″	z	65°	37′	2″		
Zenith dist. (z)	65	37	2	l'	71	43	0	L. co-sec.	10·022497
				p	84	29	28	L. co-sec.	10·002061
					221	49	30		
Lat.	18°	17′		s	110	54	45	L. sin.	9·970406
Co-lat. (l')	71	43		$s - z$	45	17	43	L. sin.	9·851712
									2)19·846676
								L. cos.	9·923338

	H.	M.	S.
$\frac{P}{2}$	2	12	12·3
			2
* Hour angle W.	4	24	24·6
* R.A.	7	33	33
R.A.M.	11	57	57·6
R.A.M. ⊙	18	59	35
M.T. ship 4d.	16	58	22·6
M.T.G. 4d.	20	39	10
	3	40	47·4
	60		
	4)220m.	47·4s.	
Longitude in	55°	11′	51″ W.

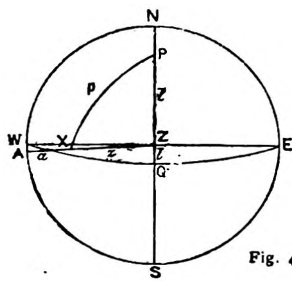


Fig. 4.

EXPLANATION OF FIG. 4.

N W.S.E	Rational Horizon.
Z	The Zenith of Observer.
W Z E	Prime Vertical.
W Q E	Equinoctial.
N Z S	Observer's Meridian.
P X (p)	Polar distance.
P Z (l')	Co-latitude.
Z X (z)	Zenith distance.
X A (a)	Altitude.
Z Q (l)	Latitude.
Z P X	Hour Angle.

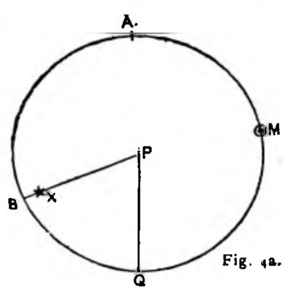


Fig. 4a.

EXPLANATION OF FIG. 4a.

A B Q M	Equinoctial.
P	Celestial Pole.
P Q	Observer's Meridian.
P X B	Meridian through Procyon.
Q P B	Star's Westerly Hour Angle.
A	First Point of Aries.
A B Q	Right Ascension of Meridian (R A M) or Sidereal time of observation.
A B	Star's Right Ascension.
A B Q M	Right Ascension of Mean Sun (R A M⊙).

Example 5.—April 22nd, p.m. at ship, lat. by D.R. 42° 9′ N.; long. by D.R. 11° 45′ West, when a chronometer which was 5m. 16s. fast for Greenwich mean time showed 9h. 8m. 46s., the observed altitude of the star *Arcturus* (α Bootis) *east* of the meridian, was 36° 53′ 30″, the error of the instrument being 3′ 0″ to add, and the height of the eye 23 feet.

Required the longitude of the ship.

	D	H.	M.	S.	
Ship T. by watch, Apr.	22	8	15	10	
Long.			47	0	W.
Approx. ast. T. at Gr.	„	9	2	10	

Since ship time and long. make Green. time 9h.; the 9h. by chron. are 9h.

	H.	M.	S.
Time by chron. Apr. 22d.	9	8	46
Chron. fast	—	5	16
M.T. at Green. 22d.	9	3	30

	H.	M.	S.
Sid. T (N.A.) (p. II.)	2	1	58·8
Accel. for 9h. 3m. 30s.		1	29·3
Mean sun's R.A.	2	3	28·1

Star's obs. alt.	36°	53′	30″
Index error	+	3	0
Corr. obs. alt.	36	56	30
Dip	—	4	42
	36	51	48
Ref.	—	1	16
Star's true alt.	36	50	32
	90		
Star's true Z.D.	53	9	28

	H.	M.	S.	
Star's R.A. (N.A.)	14	10	39·8	
Star's decl. (N.A.)	19°	45′	14″	N.
P.D.	70	14	46	
Lat.	42	9	0	
Co-lat.	47	51	0	

[*Formula same as last problem.*]

z	53°	9′	28″		
l'	47	51	00	Co-sec.	0·129953
p	70	14	46	Co-sec.	0·026340
	171	15	14		
s	85	37	37	Sin.	9·998734
$s-z$	32	28	9	Sin.	9·729850
	H.	M.	S.		2)19·884877
$\frac{P}{2}$	1	55	25	Cos. $\frac{P}{2}$	9·942439
P	3	50	50 E.		
* H.A.	20	9	10 W.		
„ R.A.	14	10	40		
Sid. T. ship	10	19	50		
M.S.R.A.	2	3	28		
M.T.S.	8	16	22		
M.T.G.	9	3	30		
		47	8		
Long.	11°	47′	00 W		

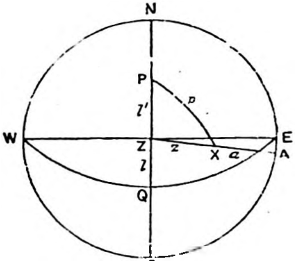


Fig 5

Explanation of Fig. 5 same as Fig. 1.

EXPLANATION OF FIG. 5a

The Circle	Equinoctial.
P	Pole.
A	First Point of Aries
A M	R.A.M. ☉
A M Q	R.A.M.
A M Q B	R.A. of Arcturus.
P Q	Observer's Meridian
P B	Meridian through Arcturus
Q P B	Easterly H.A. of X, the object.

Whenever the R. A. M. is less than the R. A. of the object the H. A. is Easterly.

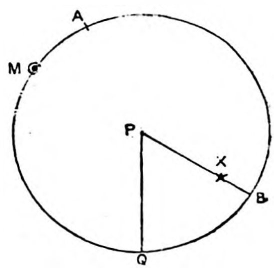


Fig. 5a.

Example 6.—September 13th, at about 7h. 30m. p.m. at ship; lat. by D.R. 39° 43′ S., long. by D.R. 158° E., when a chronometer which was 10m. 48s. (by error and rate) *slow* on Greenwich mean time showed 8h. 49m. 11s., the altitude of the planet Venus (centre) was 25° 4′ west of the meridian; height of eye 22 feet. Required the true longitude.

	D.	H.	M.
Ship T. Sept	13	7	30
D.R. Long. in time		10	32 E.
Approx. Gr. date, Sept.	12	20	58

	H.	M.	S.
T. by chron.	8	49	11
Slow	+ 10	48	
	8	59	59
	12		
M.T. Gr. Sept 12d.	20	59	59

Obs. alt.	25°	4′	0″
Dip.	—	4	36
	24	59	24
Ref.	—	2	2
	24	57	22
Par.	+		10
T. alt.	24	57	32

The R.A. and Dec. of Venus are corrected for nearest Gr. noon.

		S.
Var. of R.A. in 1h.		9·88
		3
Corr.	—	29·64
R.A. 13th	14 14	25·6
	H. M.	S.
Corr. R.A.	14 13	56

Var. of decl in 1h.	64″		
	3		
	192		
Corr	—	3′	12″
Decl. at noon	15°	59	28
Venus' corr. decl.	15	56	16 S.
	90		
„ S.P.D	74	3	44

	H.	M.	S.
Sid. T (N.A. p. II., Sept. 12)	11	25	46·32
Accl. for 20h.		3	17·13
„ „ 59m.			9·69
„ „ 59s.			·16
Mean sun's R.A.	11	29	13·30
M.T.G.	20	59	59
Sid. T. at Gr.	8	29	12·3

Formula—

$$\text{Hav. P} = \frac{\cos. s. \sin. (s - a)}{\cos. l. \sin. p}, \text{ where } s = \frac{a + l + p}{2}$$

a = Alt.
l = Lat.
p = Polar dist.

Venus' Alt.	24°	57′	32″		
l	39	43	0	Sec.	10·113953
p	74	3	44	Co-sec	10·017024
Sum	138	44	16		
s	69	22	8	Cos.	9·546074
(s — a) Rem.	44	24	36	Sin.	9·844967
	H.	M.	S.		
„ H.A.	4	42	7·8 W.	Hav.	9·522918
„ R.A.	14	13	56		
Sid. T. at ship	18	56	3·8		
„ at Gr.	8	20	12·3		
Long. in time	10	26	51·5		
Long.	156°	42′	52″ E.		

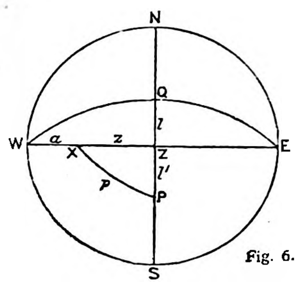


Fig. 6.

Explanation of Fig. 6 same as Fig. 1.

EXPLANATION OF FIG. 6a.

The Circle	Equinoctial.
P	Pole.
A	First Point of Aries.
P Q	Observer's Meridian.
P X B	Meridian through object.
M ☉	Mean Sun.
A G M	Mean Sun's R.A.
A G M B	R.A. of Venus.
Q B M A	R.A.M. or Sidereal time of observation.
Q P B	Planet's Westerly H.A.
A G	Sidereal at Greenwich or R.A. of Gr mer.

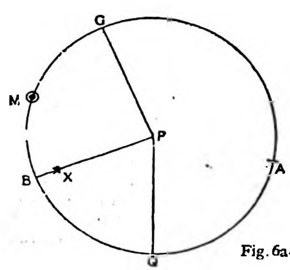


Fig. 6a.

Example 7.—April 30th, at 0h. 37m. 24s. a.m. by ship's watch; lat. 10° 7′ N., long. by D.R. 176° 10′ W.; when the mean of a set of times by chronometer was 12h. 17m. 47s., and the mean of a set of observed altitudes of the moon's lower limb was 33° 1′ 50″ west of the meridian; the chronometer had been found 2m. 12s. fast on mean noon at Greenwich on March 26th, and losing 7·4s. daily; height of eye 24 feet.

Required the longitude of the ship.

		D.	H.	M.	S.	
Ship time, April 30th, 0h. 37m. 24s. a.m	= April	29	12	37	24	
Long. 176° 10′			11	44	40	W.
Approx. ast. T. at Green.	,,	30	0	22	4	

Since ship time and long. make Green. time 0h.; the 12h. by chron. must be replaced by 0h.

	H.	M.	S.
Time by chron.	0	17	47
Fast	—	2	12
	0	15	35
Accum. rate	+	4	19
M.T. at Green. April 30d.	0	19	54

March	26
	5
April	30
Days	35

	S.
Daily loss	7·4
Days	35
	6,0)25,9·0

Accum. rate 4m. 19s. slow

Moon's decl.	9°	56′	39″·6 N.
11″·83 × 19·9	—	3	55 ·4
Corr. decl.	9	52	44 ·2 N.
N.P.D.	80	7	16

	H.	M.	S.
Moon's R.A.	11	19	5·0
1·94s. × 19·9	+		38·6
Corr. R.A.	11	19	43·6

	H.	M.	S.
Sid. T. (N.A. p. II.)	2	33	31·22
Accel. for 19m.			3·12
,, ,, 54s.			·15
Mean sun's R.A.	2	33	34·5
M.T. Green.	0	19	54
Sid. T. Green.	2	53	28·5

☾'s Semid.	15′	7″·5
Corr.	+	·1
Augm.	+	8 ·1
Red. Semid.	15	15 ·7

☾'s H. Par	55′	24″·7
Corr.	+	·5
	55	25 ·2
Red.	—	·2
Red. H.P.	55	25

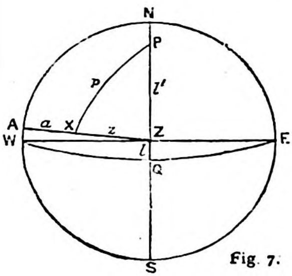


Fig. 7.

Moon's obs. alt. L.L.	33°	1′	50″		
Dip	—	4	48		
	32	57	2		
Semid.	+	15	16		
	33	12	18	*Formula same as*	
Moon's corr.	+	44	55	*in example 6*	
Moon's T. alt.	33	57	13		
l	10	7	0	Sec	0·006805
p	80	7	16	Co-sec.	0·006488
Sum	124	11	29		
s	62	5	45	Cos	9·670241
(*s*—*a*) Rem.	28	8	32	Sin.	9·673631

	H.	M.	S.		
Moon's H.A. W.	3	47	57·2	Hav.	9·357165
,, R.A.	11	19	43·6		
Sid. T. at ship, 29d.	15	7	40·8		
Sid. T. at Green., 30d.	2	53	28·5		
Long. in time	11	45	47·7	= 175° 26′ 55″ W.	

[*Explanation of Figs. same as before.*]

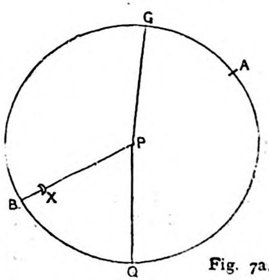


Fig. 7a.

Example 8.—January 31st, at about 4h. 20m. p.m. at ship; lat. by D.R. 37° 8′ S., long. by D.R. 40° E.; the mean of a set of times by chronometer was 1h. 39m. 38s., and the mean of a set of observed altitudes of the sun's lower limb was 30° 48′ 40″; height of eye 21 feet; the chronometer had been found 5m. 54s. slow on mean noon at Greenwich on December 17th, and losing 3·2s. daily. Find the longitude.

The Natural Haversine Method introduced into this Example is recommended in preference to any other.

Dec.	14 days	
Jan.	31·07	..
	45·07	..
	3·2	
	144·224 S.	
Acc. rate	2·24 +	

	D.	H.	M.	S.
Time by chron.	31	1	39	38
Slow		+	5	44
Approx. Gr. time		1	45	32
Acc. rate		+	2	24
Corr. Gr. time	31	1	47	56

	°	′	″
Dec.	17	18	27·6 S.
	—	1	15·9
Corr. dec.	17	17	11·7
	90	00	00
P.D.	72	42	48

Hly. Var.
42″·15
1·8
75″·870
1′ 15″·9

	M.	S.
Eq. time	13	43·54
	+	·64
	+ 13	44·18

Hly. Var.
s ·354
1·8
s ·6372

	°	′
Lat.	37	8
	90	0
Co-lat.	52	52

	°	′	″
Obs. alt.	30	48	40
Dip	—	4	29
	30	44	11
Refr.	—	1	36
	30	42	35
Parx.	+		8
	30	42	43
Semi-diam.	+	16	16
Tr. alt.	30	58	59
	90	00	00
Zen. dist.	59	1	1
p	72	42	48
l'	52	52	00
$(p \sim l')$	19	50	48

Formula—

Nat. hav. θ = nat. hav. z — nat. hav. $(p \sim l')$

L. hav. P = L. cosec. p + L. cosec. l' + L. hav. θ — 20

where P = hour angle: z, the zenith dist.; p, polar dist. and l', the co-lat.

Nat. hav. 0·24261

. . . . L. cosec. 10·02008
. . . . L. cosec. 10·09842

Nat. hav. 0·02970

θ Nat. hav. 0·21290 L. hav. 9·32820

	H.	M.	S.		
Hour angle	4	15	26	L. hav.	9·44670
Eq. time	+	13	44		
M.T. ship 31d.	4	29	10		
M.T. Gr. 31	1	47	56		
Long. in time	2	41	14		
Longitude	40	18	30 E.		

Examples for Practice

Example 1.—February 26th, at about 7h. a.m. at ship; lat. by D.R. 56° 48′ S.; long. by D.R. 135° 30′ E.; the mean of a set of times by chronometer was 10h. 6m. 25s., and the mean of a set of altitudes of the sun's lower limb was 15° 5′ 10″; height of eye 26 feet; the chronometer had been found 4m. 50s. fast on mean noon at Greenwich on January 1st, 1890, and losing 4·8s. daily. Find the longitude.

Ans. 136° 7′ 30″ E.

Example 2.—August 23rd, at about 8h. 30m. a.m. at ship; lat. by D.R. 37° 40′ N., long. by D.R. 144° W.; the mean of a set of times by chronometer was 5h. 53m. 16s., and the mean of a set of altitudes of the sun's lower limb was 37° 15′ 40″; index error of sextant 2′ 15″ to subtract; height of eye 20 feet; the chronometer had been found 17m. 30s. slow on mean noon at Greenwich on June 30th, and was 18m. 45s. slow on mean noon at Greenwich on July 30th. Required the longitude.

Ans. 144° 52′ 30″ W.

Example 3.—January 30th, at about 3h. 20m. p.m. at ship; lat. by D.R. 30° 36′ N.; long 170° E.; the mean of a set of times by chronometer was 4h. 0m. 15s., and the mean of a set of altitudes of the sun's lower limb was 24° 23′; height of eye 25 feet; the chronometer had been found 6m. 3s. fast on mean noon at Greenwich on November 2nd, 1889, and losing 3·5s. daily. Required the longitude.

Ans. 169° 31′ 15″ E.

Example 4.—April 15th, at about 3h. 12m. p.m. at ship; lat. by D.R. 44° 58′ S., long. by D.R. 73° E.; the mean of a set of times by chronometer was 10h. 33m. 1s., and the mean of a set of altitudes of the sun's lower limb was 19° 10′ 15″; height of eye 23 feet; the chronometer had been found 3m 4s. slow on mean noon at Greenwich on November 16th, and on January 23rd, 1890, it was 2m. 2s. fast on mean noon at Greenwich. Required the longitude.

Ans. 73° 26′ 30″ E.

Example 5.—November 29th, at about 3h. 25m. a.m. at ship; lat. 10° 31′ S., long. by D.R. 30° W.; the observed altitude of Aldebaran (α Tauri) west of the meridian was 30° 45′ 40″; height of eye 20 feet; time by chronometer was 5h. 29m. 57s., which (allowing for error and rate) was 3m. 53s. slow on mean time at Greenwich. Required the longitude.

Ans. 30° 35′ 30″ W.

Example 6.—March 3rd, at about 7h. 20m. p.m. at ship; lat. 8° 58′ N., long. by D.R. 60° 30′ E; the observed altitude of Regulus (α Leonis) was 30° 36′ 45″ E. of the meridian; height of eye 24 feet; time by chronometer was 3h 21m. 9s., which was 10m. 33s. fast (allowing for error and rate) on mean time at Greenwich. Required the longitude.

Ans. 61° 4′ 15″ E.

Example 7.—May 5th, at about 6h. 40m. a.m. at ship, in long. by account 140° 40′ W., when a chronometer indicated 4h. 3m. 54s., which had been found 3m. 4s. fast on mean noon at Greenwich on January 3rd, and on February 28th it was 2m. 4s. slow on mean noon at Greenwich; the observed altitude of the sun's lower limb was 20° 14′ 40″; height of eye 23 feet. Required the longitude at the time of observation, the latitude at noon on May 4th by observation being 39° 50′ N., and the ship has since sailed S. 83° W. (true) 82 miles.

Ans. 140° 41′ 15″ W.

SUMNER'S METHOD OF FINDING A SHIP'S POSITION AT SEA

Before proceeding with the calculations required in the solution of this problem, it may be as well that the navigator should understand the principle of the problem, and its value as a general method of finding a ship's position at sea.

Latitude alone, or Longitude alone, does not indicate the position of a place on the globe. Latitude merely shows that the place is somewhere on a small circle (a parallel) at a definite distance from the equator ; longitude merely shows that the place is somewhere on a great circle (a meridian) that makes a definite angle with another great circle which passes through a fixed conventional place of reference. To know the exact position of a place it is necessary to determine the point of the intersection of these two circles—that is, of the meridian with the parallel ; but this cannot always be done at sea, at any given or required instant, by any of the ordinary rules of nautical astronomy. The position may, however, be found by a combination of rules, or partly by computation and partly by projection ; or where a good point cannot be ascertained as that on which the ship is, a line may be found on or near to which she is known to be, and this at the time may be priceless : the position of the ship is thus determined by a method of utilising parts of circles which, in their completeness, would be *oblique* to the parallels and meridians.

When the declination of a celestial object coincides in amount and name with the latitude of a place on the terrestrial sphere, it must at some time, during the earth's rotation on its axis, appear in the zenith of that place ; it will do so when the object's hour-angle for the place is oh., that is, when it is on the meridian. When this occurs, the Greenwich time by chronometer being known, let it be taken as granted that the object is above the horizon of another place ; that its altitude is observed, and its zenith-distance consequently known. In Fig. I an object is vertical to the point S on the globe ; with S as a pole, and the observed zenith-distance S A as a polar-distance, describe a small circle : this is a *circle of position*, on some point of which the observation has been made, for from every point within or without this small circle a less or greater zenith-distance than S A would be observed at the instant of the object being at S. " If, then," as Chauvenet says, " the navigator can project this small circle upon an artificial globe, or chart, *the knowledge that he is upon this circle will be just as valuable to him in enabling him to avoid dangers as the knowledge of either his latitude alone or his longitude*

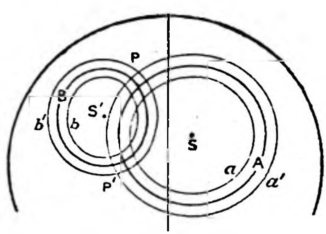


Fig. I

alone; since one of the latter elements only determines a point to be in a certain circl· without fixing upon any particular point of that circle."

The altitude of another celestial object S′, taken at the same time as the former, gives a second circle of position (*see* Fig. 1, B). The observer being in the circumference of each of these circles, must be at one of their points of intersection, at P or P′: there will be no difficulty in ascertaining which point is to be taken, as it will be sufficiently indicated by the dead reckoning

The circles to which reference has been made are such as they would appear when represented on the spherical surface of a globe, and they illustrate the principle of the problem. On a Mercator's chart, where the distance between the parallels is considerably augmented in the higher latitudes—in order to preserve the proportion that exists at different parts of the earth's surface between the meridians and the parallels—circles of position would be represented as elliptical figures (Fig. 2); perhaps we had better say as *curves of position*, which, to delineate properly, would require to be computed for every 5 cr 10 degrees: happily, in the projection of the problem, we only require a very small part of these curves, for which we assume two latitudes a few miles on each side of the latitude by *dead reckoning*. We may take the tangent (T) to the curve, or the chord (C); but our computation and projection will be the more perfect the more closely the chord and the tangent coincide—in fact, we shall then have the best *line of position*.

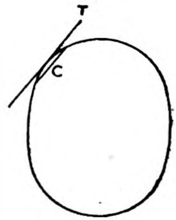


Fig. 2.

The data for the problem are (1) the *correct* Greenwich Date by chronometer; (2) simultaneous altitudes of two stars, or of a star and planet—which are by far the best objects to give the ship's position; when the sun alone is the object there must be an earlier and later altitude, with the course and distance carefully noted in the interval of the observations; (3) two assumed latitudes, the basis of which must be the latitude by D.R.; and, finally, (4) the elements from the Nautical Almanac, respecting which there is no excuse for taking them out inaccurately.

Simultaneous Altitudes

Simultaneous altitudes of two celestial objects are unquestionably the best for determining the position of a ship—which is thus got at once without any change of place, or interval of time for which to allow. With a good knowledge of the stars and planets two objects can be selected at pleasure, and in such relation to each other that the angle between their verticals shall be the best possible—something between 60° and 120°—and so develop a good point of intersection. If there be any doubt, a third star will give, with the two others, a *space* or triangle *of certainty*, within which the ship must be. Taken in the twilight—and how often may this be done when no sun has been visible all day—the altitudes, by a practised hand, ought to be obtained within a limit of 2′ to 3′, less rather than more.

When *assuming the two latitudes* it is generally sufficient to select them about 30′ or *less* on each side of the latitude by D.R.; but this will much depend upon what length of time has elapsed since the ship's position had

been previously determined. If the altitudes are simultaneous, and the lines of position (when computed and projected) intersect considerably beyond one or other of the assumptions, then take another latitude a little beyond that of the intersecting point, compute anew for this, and so project again. The position will be more accurately determined in this manner, for the latitude is an important element in the computation of the hour-angle. You will of course reject, as outside the requirements of the problem, in fact as erroneous, all that portion of the computation based on the most distant assumed latitude. But usually, in practice, the necessity for recomputing will rarely occur.

It is not, however, essential that the same assumed latitudes should be used in computing both lines of position; it is only more convenient to do so, as it saves some logarithms. In the case of two altitudes of the same object, as of the sun, where a course and distance have been made in the interval, if the course has been nearly north or south, it would be better to assume two latitudes differing from those used for the first observation, and such that they may be more in accordance with the altered position of the ship.

What we want to know is the *position* (latitude and longitude) *of the ship by projection on Mercator's chart*, after having made a few easy computations on the basis of the usual "chronometer problem"; the data being elements, some of which are exactly, and others nearly, correct, and among which are introduced certain assumptions derived from the estimated parallel on which the ship is found to be by the "dead reckoning." The rules, briefly stated, are as follows—

From an altitude of a celestial body taken at a given Greenwich time, to find the curve of position of the observer by projection on a Mercator's chart.—The circle of position, as delineated on the sphere, becomes, when transferred to Mercator's chart, a curve of position, which can only be laid down by a series of computed points. For any given altitude you can select any number of parallels of latitude crossed by the required circle. For each of these latitudes, with the true altitude deduced from the observed, and with the polar distance of the celestial body taken for the Greenwich time, compute the time at place, and thence the longitude by chronometer. Each latitude with its corresponding *longitude* gives a point in the circle of position. You may, by way of experiment, compute several (say ten or a dozen) such points for intervals of 30′ of latitude; then, having plotted these different points on Mercator's chart, you obtain, by joining them, a portion of the curve of position.

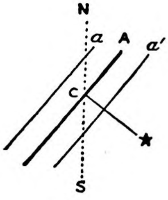


Fig. 3.

In practice it is generally sufficient to lay down only two points; for, the approximate position of the ship being known, two latitudes are selected such that the ship may be assumed to be between them.

A single altitude of a celestial object at any given Greenwich time, with its polar distance and two assumed latitudes, determines the elements for a line of position, A, which is plotted on the chart according to the respective latitudes and longitudes; if the data are correct, A is unquestionably a line on some part of which is the ship; if the altitude is assumed to be doubtful to the extent of 2′ or 3′, in one direction or the

other, this can also be shown. When the altitude is too *small*, the hour angle is too *great*; when the altitude is too *great*, the hour angle is too *small*. Hence by projecting the lines *a* and *a′* (Fig. 3), one on each side of A, and parallel with it, and to the extent of the error of altitude, we get a *zone*, or linear space, bounded by the lines *a* and *a′ within* which it will be safe to assume the ship's position to be.

If the altitudes of two objects have been taken at the same time, then, assuming the data to be correct, we at once determine the point by the intersection of the lines of position A and B (Fig. 4); but if, as in the case of A, the altitude which gives B is also doubtful, we project, as before, the lines *b* and *b′*, we thus get a space, indicated in the figure by the shaded quadrilateral, and which is determined by *a a′* in one direction and by *b b′* in the other. Within this space is the ship's position, and the area of the space is naturally more circumscribed than either zone. If we now assume a small error in the chronometer, we can delineate it around the quadrilateral; but as this gives no error in latitude we get a figure of a different form—a hexagon, which determines the limit of error of the point, and gives an area or surface of certitude within which lies the ship's position.

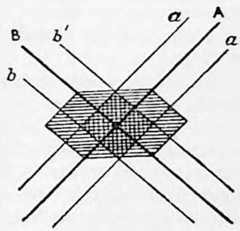


Fig. 4.

When the azimuthal angle between the lines of position is 90° the form of the quadrilateral will be that given in Fig. 4; it will change its outline considerably for smaller or greater angles; its area, nevertheless, defines the limit of error, though the exact position of the point within it is unknown (*see* also Fig. 5).

A position obtained by two altitudes, with an interval of time between the observations, is affected to the extent of the errors in the "dead reckoning" during the interval, and by errors in the altitudes.

The position determined by simultaneous altitudes of two stars, if the angle at the vertical is good—and this is a mere matter of selection—can only be affected to the extent of the errors of altitudes and those of the chronometer, and the navigator should never lose an opportunity of observing them.

It is evident from the nature of the projection that the most favourable case for the accurate determination of the intersection is that in which the lines of position intersect at right angles. Hence the two objects observed, or the two positions of the same object, should, if possible, differ about 90° in azimuth.

We give on next page the greatest errors, in miles, likely to arise on the point, for different values of the errors of altitude at different angles of the intersection of the lines of position.

Reference to the Table shows that an error of 1′ in the altitude will produce an error of position on the earth's surface equal to at least 1·4 miles even when the azimuthal difference of the lines is at its best (90°). When the angle is very small or very large, the error is proportionally greater; and the latitude and longitude will be more or less affected accordingly, the latitude most by observations made when the object is near the prime vertical, the longitude most by observations taken near the meridian.

Error of Altitudes.	Angle of intersection of lines of position.						
	90°	75°	60°	45°	30°	20°	10°
	m.	m.	m.	m.	m.	m.	m.
1'	1·4	1·6	2·0	2·6	3·9	5·8	11·5
2	2·8	3·3	4·0	5·2	7·7	11·5	22·9
3	4·2	4·9	6·0	7·8	11·6	17·3	34·4
4	5·7	6·6	8·0	10·5	15·5	23·0	45·9
5	7·1	8·2	10·0	13·1	19·3	28·8	57·4
	90°	105°	120°	135°	150°	160°	170°
	Angle of intersection of lines of position.						

Fig. 5 will illustrate this: B and A are lines of position projected for observations on each side of the prime vertical, with an azimuthal angle between them of about 30°; if both altitudes are correct, the intersection of B and A gives the correct position. For altitudes of equal errors—each too great and too small—the shaded quadrilateral defines the space within which must be the ship's position. If both altitudes are equally too great or too small the ship may be at the outermost part of the quadrilateral, to the right or left. If one altitude is too great, and the other equally too small, the ship's position may be at the uppermost or lowermost part of the quadrilateral; in which case the latitude will be most in error, and is likely to be so, for the observations having been made with the objects near the prime vertical, the longitude will, under such conditions, be but little affected.

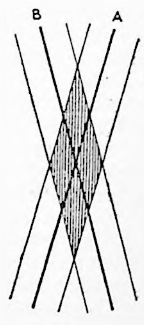
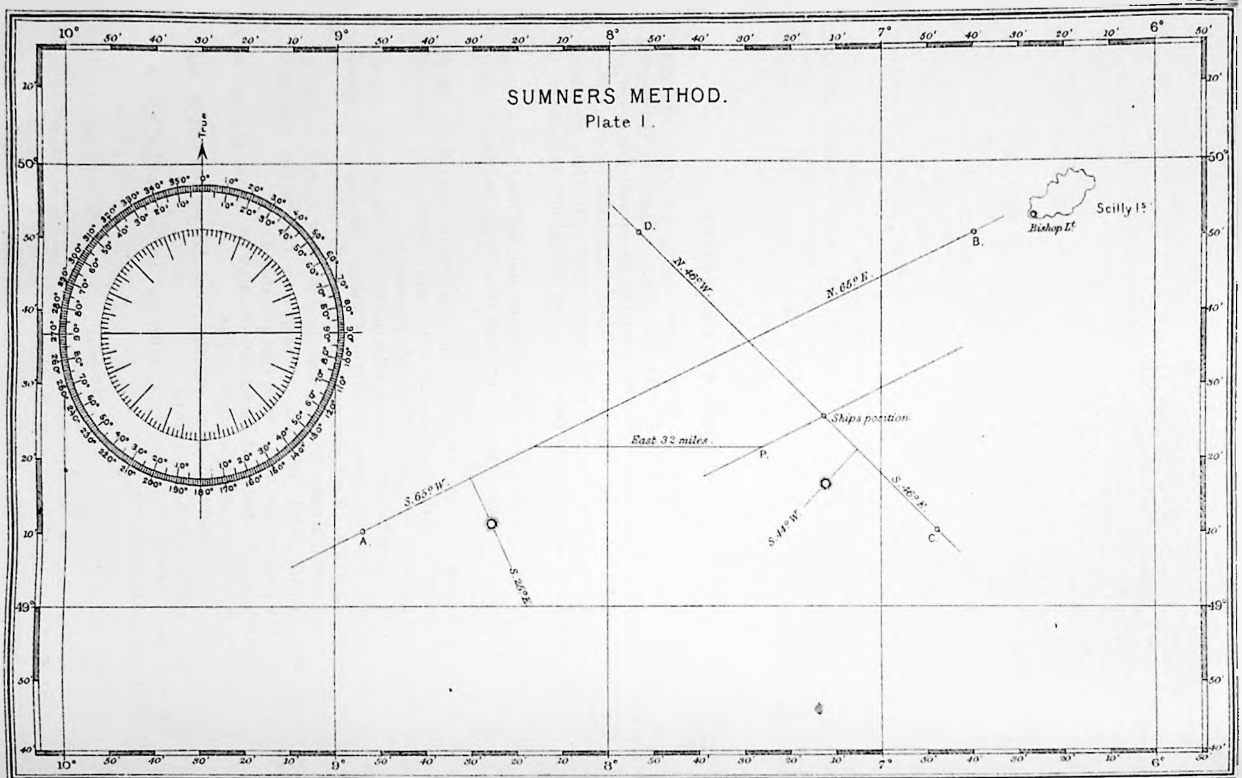


Fig. 5.

If you now turn the page, top to the side, and look at the Fig. with its length trending to right and left, you will see that the lines of position, B and A, indicate that the observations were made when the objects were on different sides of, and not far from, the meridian—the azimuthal difference being as before, 30°; in this case the longitude is much more likely to be affected than the latitude, and to a greater extent; but the quadrilateral still defines the space in some part of which is the ship, and it will continue to define it in whatever intermediate direction you turn the Fig., and though the latitude and longitude undergo change.

Is it judicious or safe to shape a course on the line of position?—Certainly it is. Habit has taught a good observer to properly estimate the errors of altitude, and due allowance will be made on this score. Having confidence in the chronometers, and knowing the errors of the compass by having lately taken time azimuths for the quadrant in which the courses may lie, then, should the line of position lead to the channel bound for, to a well-known point of land, to a light, or so near thereto that it shall be within visibility—what is there to fear? If, *en route*, you get a sounding, the position, though out of sight of land, is as well determined as by cross-bearings when coasting. The result of a sounding on or near the line of position, or the bearing of a distant inland object (while the coast is still invisible) is



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priceless. No doubt numerous methods of speedily verifying the ship's position, according to whether the course is parallel with, directly towards, or oblique to, the coast line, will at once suggest themselves to the intelligent man; but there must be no vacillation or half-heartedness—let a good look-out and the lead take the place of these.

Neither theory nor method is new; and the graphic construction has not been used to the development of all its excellence. It not only shows the limit of error, but enables us to combine terrestrial with astronomical observations. But it must not be lost sight of that we cannot use Sumner's Method without a partial recourse to the old methods, for what they are worth. These days of rapid steaming require, however, something at once more ready and certain than of old.

The latitude (and even the longitude) may also be found by computation, independently of the projection, and in this form Sumner's Method becomes that given long ago by Lalande. The distinctive feature of Sumner's Method, however, is that a single altitude taken at any time is made available for determining a line on the globe on which the ship is situated.

SUMNER'S METHOD BY PROJECTION

RULE.—Assume two latitudes 30 or 40 miles apart, but not more than 1°, one greater than the D.R. latitude, the other less. Find the Greenwich mean time for each observation. For each of the Greenwich dates correct the sun's declination and the equation of time, and from the observed altitudes get the true altitudes in the usual way; then with these elements, and the two assumed latitudes, obtain the corresponding longitudes as follows—

With the *first* true altitude, the sun's declination for the first G.M.T., and the *less* latitude, find the time at ship and thence the longitude appertaining to the *less* latitude. Call this position A.

With the *first* true altitude, the sun's declination for the first G.M.T., and the *greater* latitude, find the time at ship and thence the longitude appertaining to the *greater* latitude. Call this position B.

With the *second* true altitude, the sun's declination for the second G.M.T., and the *less* latitude, find the time at ship and thence the longitude appertaining to the *less* latitude. Call this position C.

With the *second* true altitude, the sun's declination for the second G.M.T., and the *greater* latitude, find the time at ship and thence the longitude appertaining to the *greater* latitude. Call this position D.

It is to be duly noted that both observations may be taken at a.m., or both at p.m., or one at a.m. and the other at p.m.

These remarks being understood, an example may be worked out in full and the problem explained in proceeding.

Example.—November 3rd, in lat. by account 49° 24′ N., long. 7° 12′ W., the following observations were made—

Ship Times nearly	Chron. Times	Obs. Alts. Sun's L.L.
H. M.	H. M. S.	
10 25 a.m.	10 40 3	21° 51′ 30″
2 55 p.m.	3 8 5	14 30 5

The true course of the ship and the distance sailed in the interval were

east 32 miles. The chronometer was correct for Greenwich mean time. The index error of the sextant + 2′ 30″, height of the eye 22 feet. Required the latitude and longitude of the ship when the second observation was taken, assuming latitudes 49° 10′ N. and 49° 50′ N.

	D.	H.	M.	S.
M.T.G. Nov.	2	22	40	3

Sun's obs. alt.	21°	51′	30″
I.E.	+	2	30
	21	54	0
Dip	—	4	36
	21	49	24
Semi-d.	+	16	10
	22	5	34
Corr.	—	2	13
T. alt.	22	3	21

Nov. 3rd., Sun's decl.	
(N.A. p. II.)	15° 8′ 54″·6 S.
46″·77 × 1·33	— 1 2·2
Corr. decl.	15 7 52·4 S.
N.P.D.	105 7 52

	M.	S.
Eq. T. (N.A. p. II.)	— 16	21

The sun's declination and the equation of time are here corrected for 1h. 20m. from noon of Nov. 3rd.

Formula—

$$\text{Hav. P.} = \frac{\cos.\ s \sin.\ (s - a)}{\cos.\ l \sin.\ p}$$

$$\text{L hav. P} = \text{L sec. } l + \text{L co-sec. } p + \text{L cos. } s + \text{L sin. } (s - a) - 30$$

where a = alt., l = latitude, p = polar dist., and $s = \dfrac{a + l + p}{2}$.

A.M. *Longitudes corresponding to Lat.* 49° 10′ *and* 49° 50′ N.

T. alt.	22°	3′	21″		
Lat. N.	49	10	0	Sec.	0·184515
N.P.D.	105	7	52	Co-sec.	0·015323
Sum	176	21	13		
½-sum	88	10	36	Cos.	8·502670
Rem.	66	7	15	Sin.	9·961137
H.A.	1h.39m. 11s.			Hav.	8·663645
A.T.S. 2d.	22	20	49		
Eq. T.	—	16	21		
M.T.S. „	22	4	28		
M.T.G. „	22	40	3		
Long. in time		35	35 = 8° 53′ 45″ W. (A)		

T. alt.	22°	3′	21″		
Lat. N.	49	50	0	Sec.	0·190431
N.P.D.	105	7	52	Co-sec.	0·015323
Sum	177	1	13		
½-sum	88	30	36	Cos.	8·415015
Rem.	66	27	15	Sin.	9·962247
H.A.	1h.30m.16s.			Hav.	8·583016
A.T.S. 2d.	22	29	44		
Eq. T.	—	16	21		
M.T.S. „	22	13	23		
M.T.G. „	22	40	3		
Long. in time		26	40 = 6° 40′ W. (B)		

Thus we have obtained through the first observation two positions, viz.—

(A) in lat. 49° 10′ N., long. 8° 53′ 45″ W.
(B) in lat. 49° 50′ N., long. 6° 40′ 0″ W.

	D. H. M. S.		M. S.
M.T.G. Nov.	3 3 8 5	Eq. T. (N.A.p.II.)	— 16 21

Nov. 3rd, Sun's decl. (N.A. p. II.)	15°	8′	54″·6 S.
46″·77 × 3·13	+	2	26·4
Corr. decl.	15	11	21 S.
N.P.D.	105	11	21

Sun's obs. alt.	14°	30′	5″
I.E.	+	2	30
	14	32	35
Dip	—	4	36
	14	27	59
Semi-d.	+	16	10
	14	44	9
Corr.	—	3	30
T. alt.	14	40	39

The circle N W S E	The Rational Horizon.
P	The Pole.
W Z E	The Prime Vertical.
p and p'	Polar Distances.
W Q E	Equinoctial
d d'	Parallel of Declination.
X	Position of sun at 1st obs.
X′	Position of sun at 2nd obs.
z' and z	Zenith distances.

The two circles cutting each other in Z are the circles of Position, and Z the observer's position. The circles of Position appear on a Mercator's Chart as straight lines. The first altitude has been corrected for run between the observations.

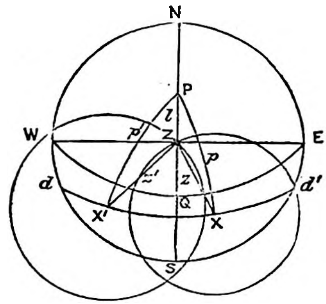


Fig. 1.

P.M. *Longitudes corresponding to Lat.* 49° 10′ *and* 49° 50′ N.

T. alt.	14°	40′	39″		
Lat. N.	49	10	0	Sec.	0·184515
N.P.D.	105	11	21	Co-sec.	0·015443
Sum	169	2	0		
½-sum	84	31	0	Cos.	8·980259
Rem.	69	50	21	Sin.	9·972540
A.T.S. 3d.	2h.	57m.	12s.	Hav.	9·152757
Eq. T.	—	16	21		
M.T.S. „	2	40	51		
M.T.G. „	3	8	5		
Long. in time		27	14 = 6° 48′ 30″ W. (C)		

T. alt.	14°	40′	39″		
Lat. N.	49	50	0	Sec.	0·190431
N.P.D.	105	11	21	Co-sec.	0·015443
Sum	169	42	0		
½-sum	84	51	50	Cos.	8·953100
Rem.	70	10	21	Sin.	9·973460
A.T.S. 3d.	2h.	52m.	53s.	Hav.	9·132434
Eq. T.	—	16	21		
M.T.S. „	2	36	32		
M.T.G. „	3	8	5		
Long. in time		31	33 = 7° 53′ 15″ W. (D)		

We have now, through the second observation, obtained two positions, viz.—

(C) in lat. 49° 10′ N., long. 6° 48′ 30″ W.
(D) in lat. 49° 50′ N., long. 7° 53′ 15″ W.

These constitute the whole of the computations ; now take up the chart.

Exactly as you would prick off a ship's position on the chart, by the aid of dividers and parallel rules, prick off the two positions A and B on the chart (*see* plate, " Sumner's Method ").

Having marked the positions A and B, draw a straight line to connect them, and passing beyond them, if necessary. It is supposed that the ship, at the first observation, is at some spot on this line, which, as it trends (in this case) N. 65° E., would, if the ship were put on that course, lead direct to the Scilly Islands.

Next, plot on the chart the two positions C and D; and also, as before, join them by a straight line.

The question tells you that the ship made 32 miles on a true east course, in the interval of the observations. From the graduated meridian, opposite the line extending from A to B, take off the distance 32 miles with the dividers; lay the dividers down, and taking the parallel rules, place them on the compass, in the centre of the chart, over east and west; then work the parallels (strictly preserving the direction) towards the line of the first position A to B; having reached that line, draw another line extending from it in the direction of the course, east, and on this last line lay off the 32 miles of distance, already taken in your dividers: let us call the extremity of this distance P.

Next, lay the edge of the parallel rules on the line A B, and work them (preserving the direction) to P; through this point (P) draw a new line to cut the line C D; this new line (which is parallel to the line A B) is taken to be that on some spot of which the ship would have been had the first observation been made where the second was taken; and the intersection of the lines is supposed to be the exact position of the ship when the second altitude was observed—in this case—

Lat. 49° 25½′ N., Long. 7° 13½′ W.

If the sun's azimuth is required at either of the times of observation, proceed as follows—

The lines A B and C D are the *lines of position*. The sun's azimuth at the time of the first observation is the direction of a line at right angles to the direction of A B to be reckoned *easterly*, because the observation was taken a.m.; the sun's azimuth at the time of the second observation is the direction of a line at right angles to the direction of C D to be reckoned *westerly*, because the second observation was taken p.m.

As regards this example, the *first line of position* trends N. 65° E. and S. 65° W.; and the sun's true bearing is S. 25° E. The *second line of position* trends N. 46° W. and S. 46° E.; and the sun's true bearing is S. 44° W.

Examples for Practice

Example 1.—At sea on February 28th a.m. at ship and uncertain of my position, when the chronometer (corrected) indicated Feb. 27d. 23h. 24m. 43s. Greenwich mean time, the observed altitude of sun's L.L. was 21° 0′ 40″; and again p.m. on the same day when the chronometer indicated Feb. 28d. 3h. 54m. 41s. G.M.T., the observed altitude of the sun's L.L. was 28° 5′ 20″; the ship having made 44 miles on a true N. 77° E. course in the interval of the observations; height of eye 24 feet. Required the line of position when the first altitude was observed, also the bearing of the sun by projection, and the position of the ship by Sumner's Method

when the second altitude was observed, the ship being supposed to be between the parallels of 49° 30′ and 50° 10′ N.

Ans. (A) in lat. 49° 30′ N., long. 31° 41′ 30″ W.
(B) in „ 50° 10′ N., „ 30° 43′ 0″ W.
(C) in „ 49° 30′ N., „ 29° 8′ 52″ W.
(D) in „ 50° 10′ N., „ 31° 1′ 30″ W.

First line of position trends N. 43° E. and S. 43° W., sun's bearing S. 47° E., and position of ship at second altitude, lat. 49° 53½′ N., long. 30° 14½′ W.

Example 2.—February 3rd a.m. at ship, when the chronometer (corrected) indicated Feb. 2d. 22h. 51m. 52s. G.M.T., the observed altitude of the sun's L.L. was 17° 35′ 10″; and again p.m. on the same day, when the chronometer indicated Feb. 3d. 4h. 15m. 23s. G.M.T., the observed altitude of the sun's L.L. was 9° 32′ 30″; the ship having made 41 miles on a true E. ¾ S. course in the interval of the observations; height of eye 23 feet. Required the line of position at the time each altitude was observed, and the sun's bearing by projection at each observation; also the position of the ship by Sumner's Method when the second altitude was observed, the ship being supposed to be between the parallels of 50° 30′ and 51° 10′ N.

Ans. (A) in lat. 50° 30′ N., long. 10° 24′ 0″ W
(B) in „ 51° 10′ N., „ 8° 35′ 30″ W.
(C) in „ 50° 30′ N., „ 9° 28′ 30″ W.
(D) in „ 51° 10′ N., „ 10° 24′ 30″ W.

First line of position trends S. 60° W. and N. 60° E., sun's bearing S. 30° E.;

Second line of position trends N. 42° W. and S. 42° E., sun's bearing S. 48° W.

The ship's position at the second altitude is lat. 50° 24′ N., long. 9° 20′ W., which also determines the position at the first altitude by carrying back the course and distance, and gives lat. 50° 30′ N., long. 10° 24′ W.; when the line of the first position was determined, if the ship had been put on a N. 60° E. course it would have led 11 miles southward of the Saltees lightship.

Example 3.—April 4th a.m. at ship, and uncertain of my position, when the chronometer (corrected) indicated Apr. 4d. 8h. 0m. 12s. Greenwich mean time, the observed altitude of the sun's L.L. was 23° 22′ 25″; and again a.m. on the same day, when the chronometer indicated Apr. 4d. 11h. 0m. 40s. G.M.T., the observed altitude of the sun's L.L. was 44° 26′ 20″; the ship having made 30 miles on a true N. 20° E. course in the interval of the observations; height of eye 26 feet. Required the line of position when each altitude was observed, and the bearing of the sun by projection at each observation; also the position of the ship at each observation by Sumner's Method, the ship being supposed to be between the parallels of 49° 6′ and 49° 46′ N.

Ans. (A) in lat. 49° 6′ N., long. 179° 46′ W.
(B) in „ 49° 46′ N., „ 179° 24′ W.
(C) in „ 49° 6′ N., „ 178° 51′ 30″ E.
(D) in „ 49° 46′ N., „ 178° 34′ 30″ W.

First line of position trends N. 20° E. and S. 20° W., sun's bearing S. 70° E.;

Second line of position trends N. 68° E. and S. 68° W., sun's bearing S. 22° E.;

Also the first line of position trending in the direction of the course, the intersection of the two lines of position gives the ship's true position at the second observation—viz., lat. 49° 31′ N., long. 179° 32½′ W.; but the first observation was probably made S. 20° W. 30 miles from the position just given—viz., in lat. 49° 3′ N., long. 179° 48′ W.

Example 4.—September 23rd p.m. at ship, when the chronometer (corrected) indicated Sept. 22d. 13h. 56m. 27s. G.M.T., the observed altitude of the sun's L.L. was 40° 17′ 30″; and again p.m. on the same day, when the chronometer (corrected) indicated Sept. 22d. 17h. 4m. 30s. G.M.T., the observed altitude of the sun's L.L. was 17° 24′; the ship having made 26 miles on a true north course in the interval of the observations; height of eye 27 feet. Required the line of position when the second altitude was observed, also the bearing of the sun by projection; and the position of the ship by Sumner's Method when the second altitude was observed, the ship being supposed to be between the parallels of 47° 5′ and 47° 45′ N.

Ans.	(A)	in lat.	47° 5′ N.,	long.	166° 40′ 30″ E.
	(B)	in „	47° 45′ N.,	„	164° 13′ 15″ E.
	(C)	in „	47° 5′ N.,	„	165° 39′ 45″ E.
	(D)	in „	47° 45′ N.,	„	165° 18′ 0″ E.

Second line of position trends N. 20° W. and S. 20° E., sun's bearing S. 70° W.; and position of ship at second altitude is lat. 47° 54½′ N., long. 165° 12½′ E.

The method given on pp. 439 to 442 is that originally proposed by Capt. Sumner. The publication of Azimuth Tables, and the short and easy way of finding Azimuths from Tables similar to the A, B, and C Azimuth Tables in Norie's Tables have modified the method. It has been stated that the azimuth is the direction at right angles to the direction of a *line of position*, hence a line of position can be determined from the azimuth. It is now therefore usual to proceed in the following manner—

Rule.—Find the longitude at the time of taking the first observation, using the latitude by account, and take from the tables the azimuth corresponding to the hour angle found in the computation. On the chart lay down the position of the point given by the latitude by account and the longitude found. From this point draw a line in the direction of the azimuth, then a line drawn through the point at right angles to the direction of the azimuth is the *first line of position.*

Find the longitude at the time of taking the second observation, using the latitude by account at the time, and take from the tables the azimuth corresponding to the hour angle found in the computation. On the chart lay down this latitude and longitude, and from the point thus determined draw a line in the direction of the azimuth, then a line drawn through the point at right angles to the direction of the azimuth is the *second line of position.* From the point found from the first observation draw a line in the direction of the course, and set off on it the distance sailed in the

interval between the observations, a line parallel to the *first line of position* drawn at the end of the distance will meet the *second line of position* in the ship's position at the time of taking the second observation.

It is thus seen that by this method the working is very much shortened, *two* longitudes only being required instead of *four*. But the simplest form of the problem is when simultaneous altitudes of two stars have been taken. The latitude by D.R. is used to calculate a longitude from each observation. The azimuths are found, and the *lines of position* laid down. The ship's place is therefore at the point in which the lines of position intersect each other.

Example 5.—December 6th, at about 6h. 28m. a.m. at ship; lat. by D.R. 49° 58′ N.; long. by D.R. 14° 30′ W.; when the chronometer (corrected for error for G.M.T.) indicated 7h. 30m. 20s., the following simultaneous altitudes were observed: *Regulus* (α Leonis) observed altitude was 48° 41′ 30″ W. of meridian, and *Arcturus* (α Bootis) observed altitude was 46° 32′ 30″ E. of meridian; height of eye 26 feet. Required the ship's position.

		H.	M.
Time at ship, Dec. 6th		6	28 a.m.
	5d.	18	28
Long. 14° 30′ W.		+	58
Approx. Gr. date	5	19	26

	H.	M.	S.
Sid. T. (N.A. p. II.)	16	56	56·9
Accel. for Gr. T.		3	12·3
Mean Sun's R.A.	17	0	9·2
Green. M.T.	19	30	20
Sid. T. at Gr.	12	30	29·2

	D.	H.	M.	S.
Gr. M.T. by Chron. Dec.	5	19	30	20

	H.	M.	S.		H.	M.	S.
Regulus' R.A.	10	2	33·1	Arcturus' R.A.	14	10	39·2
„ Decl.	12°	30′	5″ N.	„ Decl.	19°	45′	5″ N.
„ N.P.D.	77	29	55	„ N.P.D.	70	14	55

R = Position of Regulus.

A = Position of Arcturus.

The circles of Position are omitted.

The explanation of the fig. is the same as Fig. 1.

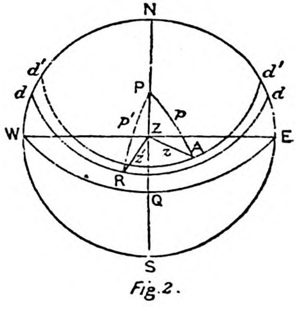


Fig. 2.

Formula—

$$\text{Hav. P} = \frac{\cos. s \sin. (s - a)}{\cos. l \sin. p}$$

where a is the alt., l is the lat., p the polar distance, and $s = \dfrac{a + l + p}{2}$

Regulus' obs. alt.	48° 41'·5			
Cor.	— 5·8			
T. alt.	48 35·7			
N. lat.	49 58	Sec.	0·191632	
N.P.D.	77 29·9	Co-sec.	0·010416	
Sum	176 3·6			
½-sum	88 1·8	Cos.	8·536258	
Rem.	39 26·1	Sin.	9·802912	
	H. M. S.			
Hour-ang. W.	1 25 58·5	Hav.	8·541218	
Regulus' R.A.	10 2 33·1			
Sid. T. ship	11 28 31·6			
Sid. T. Gr.	12 30 29·2			
Long. in time	1 1 57·6			
Long.	15° 29' 24" W. by Regulus			

Arcturus' obs. alt.	46° 32'·5		
Cor.	— 5·9		
T. alt.	46 26·6		
N. lat.	49 58	Sec.	0·191632
N.P.D.	70 14·9	Co-sec.	0·026325
Sum	166 39·5		
½-sum	83 19·7	Cos.	9·065132
Rem.	36 53·1	Sin.	9·778304
	H. M. S.		
Hour-ang. E.	2 38 43	Hav.	9·061394
	24		
,, W.	21 21 17		
Arcturus' R.A.	14 10 39·2		
Sid. T. ship	11 31 56·2		
Sid. T. Gr.	12 30 29·2		
Long. in time	0 58 33		
Long.	14° 38' 15" W. by Arcturus		

For T. Az., and Line of Position by Regulus.

Azimuth Tables (A) gives	—	3'·04	
,, ,, (B) ,,	+	0·61	
,, ,, (C) ,,	—	2·43	= S. 32°·7 W.

Since the T. Azimuth of *Regulus* is S. 32½° W. the line of the ship's position trends N. 57½° W. and S. 57½° E.

For T. Az., and Line of Position by Arcturus.

Azimuth Tables (A) gives	+	1'·44	
,, ,, (B) ,,	—	0·57	
,, ,, (C) ,,	+	0·87	= S. 60°·8 E.

Since the T. Azimuth of *Arcturus* is S. 61° E. the line of the ship's position trends N. 29° E., and S. 29° W.

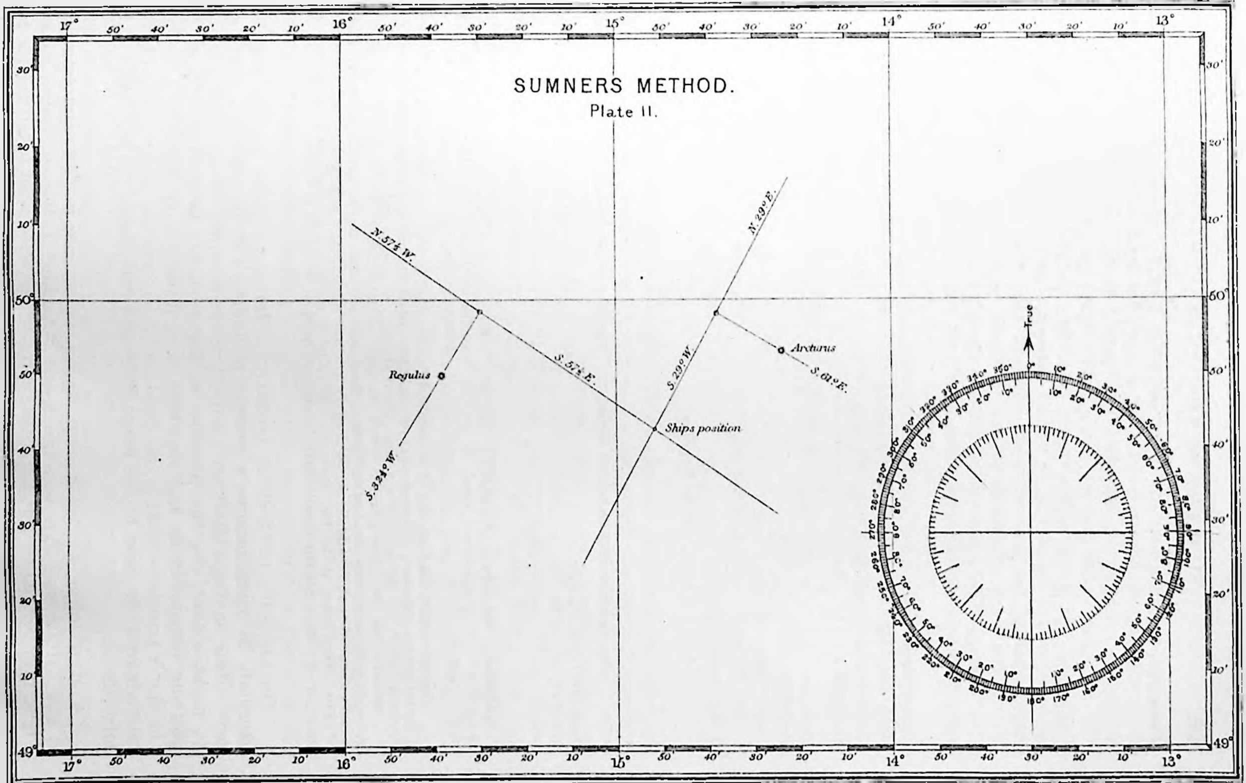
Take the Chart, and on the D.R. parallel (in this case lat. 49° 58' N.), mark the long. as determined by *Regulus* ; through the spot indicated draw the line of position due to the alt. of *Regulus*, which, as shown above, trends about N. 57½° W. and S. 57½° E.

Similarly, on the same parallel, mark the long. as determined by *Arcturus* ; and through the spot indicated draw the line of position due to the alt. of *Arcturus*, which, as shown above, trends about N. 29° E. and S. 29° W.

The intersection of the two lines of position gives the ship's true position lat. 49° 42' N., long. 14° 52' W. *See* the small chart "Sumner's Method, Plate II."

The foregoing methods by means of a Mercator's chart are the simplest methods of using Sumner's process of finding a ship's position, but they necessitate having a chart on a fairly large scale, unless the navigator is able to make a chart for the purpose, or has a supply of sectional paper for plotting down. If he has no chart or sectional paper, he must calculate the ship's position—that is, he must find the *corrections* to be applied to the latitude by D.R., and to the longitude found from using the erroneous D.R. latitude.

A very simple way of finding the *corrections* can be obtained from the A and B numbers taken from the A and B Azimuth Tables in Norie's Tables. These numbers when put together represent the change of longitude for *one*



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mile change of latitude. If the azimuths are in the same or opposite quadrants, the difference of these numbers is the total change of longitude for *one mile* change of latitude. If the azimuths are in adjacent quadrants, the sum of the numbers is the total change of longitude for *one mile* change of latitude. Hence if the whole difference of longitude between the longitudes found from the observations is divided by the total change of longitude for *one mile* change of latitude, the result is the *correction for latitude.* And consequently the *correction for longitude* will be found by multiplying the number used in getting an azimuth by the correction for latitude. The correction is to be applied to the longitude found from the same observation as the azimuth was found from.

In the example just given the numbers are 2·43 and 0·87, and the azimuths are in adjacent quadrants, the total change of longitude for *one mile* change of latitude is the sum 3·3. The total difference of longitude is 51′·1, and 51′·1 divided by 3·3 gives 15′·5, the *correction for latitude.* The *correction for longitude* is 0·87 multiplied by 15′·5, that is 13′·5.

Lat. by D.R.	49°	58′	N.	Long.	14°	38′	W.
Cor. for lat.		15·5	S.	Cor. for long.		13·5	W.
True lat.	49	42·5	N.	True long.	14	51·5	W.

Rule for applying the corrections.—When the azimuths are in adjacent quadrants, add the *correction for longitude* if the longitude used is the less longitude, but subtract if the longitude used is the greater longitude.

When the azimuths are in the same or opposite quadrants, mark the difference of longitude between the observations E. or W., reckoning from the point of less azimuth to that of greater azimuth. Then if the longitude used and difference of longitude have the same name, add the correction for longitude, but subtract if the longitude used and difference of longitude have different names.

The correction for latitude takes its name from the *line of position* drawn through the point ; this must be north-easterly and south-westerly or north-westerly and south-easterly. Having determined the direction of the *line of position* and knowing from the rule for applying the *correction for longitude* whether it is E. or W., the letter in the *line of position* connected with the letter of the *correction for longitude* is the name of the *correction* for latitude. Thus if the *line of position* is north-easterly and south-westerly and the correction for longitude is E., then the correction for latitude is N.

In the example the azimuths are in adjacent quadrants, the *correction for longitude* is W. because it is added to the less longitude, which is W. The *line of position* through the point used is north-easterly and south-westerly, and the correction for longitude is W., therefore the correction for latitude is S.

If two observations of the sun have been taken, and the course and distance in the interval noted, then the longitude must be determined from each observation, using the latitude by account at the time of the observation. The longitude found from the first observation must be corrected for the run in the interval, to reduce it to the longitude at the time of taking the second observation. The difference between the reduced longitude

and the longitude found from the second observation is the total difference of longitude to be divided by the change of longitude in *one mile* change of latitude found by the A and B numbers in the A and B Azimuth Tables in Norie's Tables.

The A and B numbers will be found combined in Norie's new Tables, Longitude Correction Table.

This problem has been made very familiar to navigators by Mr. A. C. Johnson, R.N., through the pamphlet entitled "On Finding the Latitude and Longitude in Cloudy Weather and at other times."

If in the case of the sun the second observation is taken near noon within the limits of the Reduction to the Meridian problem, finding the longitude by using the estimated latitude would be altogether useless. The estimated longitude would now be used to find the latitude.

Example.—December 12th, in lat. by account 30° 12′ N., long. 72° W.

Ship times nearly	Chron. times	Obs. alts. sun's L.L.
H. M	H. M. S.	
8 45 a.m.	1 26 59	19° 4′ 54″
11 30 a.m.	4 11 2	35 45 12

The true course and distance in the interval was north 30 miles. The chronometer was estimated to be 1m. slow for mean time at Greenwich. Height of the eye 20 feet, I.E.—2′ 10″. Find the ship's position at the time of taking the second observation.

	D. H. M. S.
G.M.T.	12 1 27 59

Sun's decl. (N.A. p. II.)	23° 6′ 40″ S.		Eq. T. (N.A. p. II.)	M. S. — 6 3·01
10″·95 × 1·5	+ 16		1·18s. × 1·5	1·77
Corr. decl.	23 6 56		Corr. Eq. T.	6 1·24
P.D.	113 6 56			

Obs. alt.	19° 4′ 54″	Tr. alt.	19° 12′ 2″				
I.E.	— 2 10	Lat.	30 12 0	Sec.	·063348		
	19 2 44	P.D.	113 6 56	Co-sec.	·036346		
Dip	— 4 23		162 30 58			A = + ·52	
	18 58 21	½-sum	81 15 29	Cos.	9·181799	B = + ·57	
S.D.	+ 16 17	Rem.	62 3 27	Sin.	9·946167	Sum = + 1·09	
	19 14 38		H. M. S.				
Sun's cor.	— 2 36	H.A.	3 14 8 E.	Hav.	9·227660	Az. S. 47° E.	
Tr. alt.	19 12 2		D. H. M. S.				
		A.T.S.	11 20 45 52				
		Eq. T.	— 6 1				
		M.T.S.	11 20 39 51				
		M.T.G.	12 1 27 59				
		Long. in time	4 48 8 = 72° 2′ W.				

	D.	H.	M.	S.
G.M.T.	12	4	12	2

Sun's decl. (N.A. II.)	23° 6′ 40″ S.	Eq. T. (N.A. p. II.)	+ 6 M. 3·01 S.
10″·95 × 4·2	+ 46	1·18s. × 4·2	— 4·96
Corr. decl.	23 7 26	Corr. eq. T.	5 58·05
Lat. by acct.	30° 12′ N.	Long.	72° 2′ W.
Diff. lat.	30 N.	Diff. long.	0
Est. lat. at 2nd obs.	30 42 N.	Est. long. at 2nd obs.	72 2 W.

	H.	M.	S.
G.M.T.	4	12	2
Eq. T.	+	5	58
S.M.T.	4	18	0
Long.	4	48	8
H.A.	0	30	8 E.

H.A.	0 30 8 E.	log. rising (+ 5)	7·93613
Lat.	30° 42′ 0″ N.	Cos.	9·934424
Decl.	23 7 26 S.	Cos.	9·963627
Sum	53 49 26	Co-sec.	10·093015
Red.	29′ 4″	Sine	7·927196

Obs. alt.	35°	45′	12″
I.E.	—	2	10
	35	43	2
Dip	—	4	23
	35	38	39
S.D.	+	16	17
	35	54	56
Sun's cor.	—	1	13
Tr. alt.	35	53	43
Red.	+	29	4
Mer. alt.	36	22	47
Zen. dist.	53	37	13 N.
Decl.	23	7	26 S.
Lat.	30	29	47 N.

The calculation of the ship's position is now very simple. Since by reference to the Table, p. 379, the hour-angle of the second observation is within the limits of the Reduction to the Meridian, it follows that the latitude found will be the *correct* latitude, unless the latitude and longitude used in the calculation are both very erroneous.

Lat. at 2nd obs.	30°	30′	N.
Run.		30	N.
Lat. at 1st obs.	30	0	N.

From this it follows that the first observation has been calculated with a latitude 12′ in error, and for this observation it has been found that the diff. long. for 1′ diff. lat. is 1·09, the total diff. long. is therefore 1·09 × 12 = 13′·08 and is W. because the true latitude is S. of the lat. by acct. and the *position line* trends south-westerly.

Long. calculated with erroneous lat.	72°	2′	W.
Diff. long.		13	W.
True long. at 1st obs.	72	15	W.
Course being N. „ „ 2nd „	72	15	W.

The position of the ship is therefore lat. 30° 30′ N., long. 72° 15′ W.

If the second observation had not been within the limits of the Table on p. 379 and the azimuth had been small, or if the estimated latitude and longitude had been very erroneous, neither the latitude nor the longitude could have been found with any degree of accuracy by the ordinary method.

COMPUTATION OF ALTITUDES

To Compute the Altitude of a Heavenly Body

All the necessary elements from the Nautical Almanac must be corrected for the Greenwich date.

If the sun is the object, the *apparent time from the nearest noon* is the sun's hour angle or meridian distance.

For the *meridian distance, in time*, or hour angle of a fixed star, a planet, or the moon.—To the mean time at place add the mean sun's right ascension, and from their sum (which is the right ascension of the meridian) subtract the right ascension of the object ; the result will be the object's *westerly* hour angle or *meridian distance*, which, *if greater than* 12h., take from 24h. for the *easterly* (or nearest) hour angle.

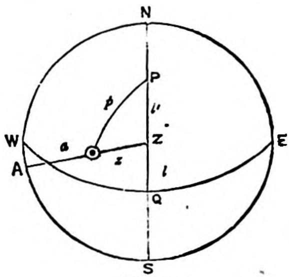


Fig. 1.

The circle represents the rational horizon, N Z S the meridian of the place, P the pole N. or S. according to the latitude (in this case N.), Z the zenith of the observer, W Q E the equinoctial, ⊙ the place of the object. Then P ⊙ (p) is the polar distance, P Z (l') the co-latitude, Z Q (l) the latitude, Z ⊙ (z) the zenith distance, A ⊙ (a) the true altitude, ∠ P the hour angle and ∠ Z the azimuth. The problem is to find A ⊙ (a) by means of the triangle P Z ⊙.

The method of finding altitudes by computation has been much facilitated by a table of Natural Haversines, which has been included in Norie's Tables and will be found adjacent to the log. haversine of the same arc or angle. The method is mathematically sound, and in consequence of the shortness of the calculation is recommended in preference to other methods, of which there are many.

To find the altitude of a celestial object, having given the hour angle, the polar distance, and the latitude—

For Arc I. add together the log. haversine of the hour angle, the log. sine of the co-latitude, and the log. sine of the polar distance; the sum, rejecting tens in the index, is the log. haversine of Arc I.

For the zenith distance add together the natural haversine of Arc I. and the natural haversine of the difference of the co-latitude and polar distance; the sum is the natural haversine of the zenith distance, which subtract from 90° for the true altitude.

Formula—

$$\text{L. hav. Arc I.} = \text{L. hav. P} + \text{L. sin. } l' + \text{L. sin. } p - 20.$$

$$\text{Nat. hav. } z = \text{nat. hav. Arc I.} + \text{nat. hav. } (l' \sim p)$$

where P = hour ∠, l' = co-latitude, p = polar dist., z = zenith dist.

Note.—Arc I. need not be taken out, but only the natural haversine of Arc I., which will be found adjacent to the log. haversine of Arc I., as will be seen in the following examples.

From the True Altitude to find the Apparent Altitude.—The corrections must be applied in reverse order and with contrary signs to those with which the true is derived from the *apparent* altitude.

For the Sun or for a Planet.—Subtract the parallax in altitude and add the refraction.

For a Star.—Add the refraction.

For the Moon.—By Computation.—To the log. *secant* of the moon's true altitude add the proportional log. of moon's reduced horizontal parallax; the result will be the proportional log. of the *approximate* parallax in altitude, which take out.

Subtract this *approximate* parallax in altitude from the moon's true altitude, and with the result proceed as before to get the *correct* parallax in altitude, which *subtract* from the true altitude; after which add the refraction, to get the moon's apparent central altitude.

NOTE.—On rare occasions the correction for the parallax in altitude may have to be taken out a third time.

The difference between the parallax in altitude and the refraction is the moon's correction of altitude, and can ordinarily be taken from Moon's Correction Table in Norie's Tables.

Example 1.—May 27th p.m. 3h. 2m. 49s. mean time at ship. Position by D.R. lat. 33° 54′ S., long. 108° E. Required the moon's true altitude.

	D.	H.	M.	S.
	27	3	2	49
Long.	—	7	12	00
M.T.G.	26	19	50	49

	H.	M.	S.
Sid. T. (N.A.)	4	16	1·68
Accel. for M.T.G.	+	3	15·15
Mean sun's R.A.	4	19	16·83

☾'s decl.	12°	26′	0·3″	
10″·99 × 12·03	+	2	12·2	
☾'s cor. decl.	12	28	12·5	N.
S.P.D.	102	28	12·5	

	H.	M.	S.
M.T.S.	3	2	49·2
Mean sun's R.A.	4	19	16·8
R.A. mer.	7	22	6·0
☾ R.A.	10	54	52·5
☾'s H.A.	3	32	46·5

	H.	M.	S.
☾ R.A	10	55	15·67
1·923s. × 12·03		—	23·13
☾ Cor. R.A.	10	54	52·54

Formula—

$$\text{L. hav. Arc I.} = \text{L. hav. P} + \text{L. sin. } l' + \text{L. sin. } p - 20$$

$$\text{Nat. hav. } z = \text{nat. hav. Arc I.} + \text{nat. hav. } (l \sim p)$$

where P = hour angle, l' co-lat., p polar dist., z zenith dist.

	H.	M.	S.			
H.A.P =	3	32	46·5	Hav. 9·30200		
l'	56°	6′	0″	Sin. 9·91908		
p	102	28	12·5	Sin. 9·98963		
	Arc I.			Hav. 9·21071	Nat. hav.	·16245
$l' \sim p$	46	22	12·5	. . . . .	Nat. hav.	·15501
z =	68°	35′	15″		Nat. hav.	·31746
	90	00	00			
True Alt. =	21	24	45			

Example 2.—June 25th p.m. at ship. Position by D.R. lat. 15° 42′ N., long. 63° 1′ 30″ E. Time by chronometer 5h. 47m. 54s., supposed to be correct. Required the altitude of Mars.

	D.	H.	M.	S.
M.T.G.	25	5	47	54
Long. in time		4	12	6
M.T. Ship		10	00	00
R.A.M. ⊙		6	15	57
R.A.M.		16	15	57
Mars R.A.		15	42	21
Mars H.A.			33	36

	H.	M.	S.
Sid. time	6	14	18·21
Corr. for Cor. time	+	1	38·30
Sid. time or R.A.M. ⊙	6	15	56·51
R.A. Mars	15	53	28·53
Cor.		—	7·60
Mar's Corr. R.A.	15	42	20·93

Formula—Same as Example I.

			″
Dec.	22°	40′	42·6 S.
		—	2·9
	22	40	39·7
	90	00	00
N.P.D.	112	40	39·7

	H.	M.	S.				
P	0	33	36	Hav.	7·72948		
l'	74°	18′	00″	Sin.	9·98349		
p	112	40	40	Sin.	9·96506		
			Arc I.	Hav.	7·67803	Nat. hav.	0·00477
$l' \sim p$	38°	22	40″		. . . . .	Nat. hav.	0·10803
			Zen. Dist.	39° 15′ 0″		Nat. hav.	0·11280
				90 00 00			
			True Alt.	50 45 0			

Example 3.—May 27th, 3h. 5m. 56s. p.m. A.T.S. Lat. 33° 54′ S., long. 108° 44′ E. Find the true altitude of the sun.

	D.	H.	M.	S.
A.T.S. May	27	3	5	56
Long. in time	—	7	14	56
A.T.G.		19	51	00

	°	′	″
Dec.	21	20	21 N.
Corr.	—	1	45
	21	18	36 N.
S.P.D.	111	18	36

	H.	M.	S.				
P	3	5	56	Hav.	9·19234		
l'	56°	6′	0″	Sin.	9·91909		
p	111	18	36	Sin.	9·96924		
			Arc I.	Hav.	9·08067	Nat. hav.	0·12041
$l' \sim p$	55°	12′	36″		. . . . .	Nat. hav.	0·21471
			Zen. Dist.	70° 44′ 45″		Nat. hav.	0·33512
				90 00 00″			
			True Alt.	19 15 15			

N.B.—The formula used in solving the above will be found the most expeditious for finding the third side when two sides and the included angle are given.

Method II.—To the log. *haversine* of the *supplement* of the hour angle add the log. *sine* of the co-latitude and the log. *sine* of the polar distance; the sum (rejecting 20 from the index) is the log. *haversine* of an angle, which call x. Then add together the co-latitude and polar distance and place x beneath this sum; find their sum and difference, and half sum, and half difference. Add together the log. *sine* of the half sum and the log. *sine* of the half difference (rejecting 10 from the index); the result is the log. *haversine* of the true zenith distance, which subtract from 90° for the true altitude.

Formulæ—

$$\text{Hav. } x = \text{hav. } (12\text{h.} - h) \times \sin. l' \times \sin. p$$

$$\text{Hav. } z = \sin. \frac{(l' + p) + x}{2} \times \sin. \frac{(l' + p) \sim x}{2}$$

where h is the hour angle, l' the co-latitude, p the polar distance, x an auxiliary angle, and z the zenith distance.

Blank Form for this Method

	H.	M.	S.		
12h. — H.A.	..	..	..	Hav.	....
Co-lat.	..°	..′	..″	Sin.	....
Pol. dist.	..	..	..	Sin.	....
Sum	..	..	..		
x	..	..	..	Hav.	....
Sum	..	..	..		
Diff.	..	..	..		
Half sum	..	..	..	Sin.	....
Half diff.	..	..	..	Sin.	....
Zenith dist.	..	..	..	Hav.	....
	90				
True alt.	..	..	..		

LATITUDE BY DOUBLE ALTITUDES

The latitude may be found with sufficient accuracy by two altitudes of the same object taken at any time of the day, the interval or elapsed time between the observations being measured by the chronometer or a good watch.

There are several methods of solving this problem, but we shall only give two, " IVORY'S METHOD," and what is known as the " DIRECT METHOD."

For IVORY'S METHOD proceed as follows—

1. Subtract the time of the first observation from the time of the second observation ; the result will be the mean interval.

2. Turn the mean interval into an apparent interval (*see* CONVERSION OF TIMES).

3. Divide the elapsed time, or apparent interval, by 2 for the half elapsed time (H.E.T.).

4. To the time by chronometer (corrected) at first observation add half the mean interval ; the result will be the middle mean time at Greenwich.

5. Correct the sun's declination (Nautical Almanac, p. II.) for middle mean time at Greenwich.

6. Correct the observed altitudes and get the true altitudes.

7. Reduce the first true altitude to what it would have been if it had been taken where the second altitude was taken. To do this the ship's course and distance between the observations is required (*see* CORRECTION OF ALTITUDES).

8. Find the half sum and also half difference of true altitudes.
   The above applies to the sun ; if a star be observed the mean interval between the observations would have to be converted into a sidereal interval (*see* CONVERSION OF TIMES).

When a planet is observed the mean interval will have to be converted into a planetary interval as follows : Obtain the sidereal interval as before ; then multiply the planet's variation in right ascension in one hour of longitude (Nautical Almanac) by the mean interval ; the product will be the correction to apply to the sidereal interval in order to find the planetary interval ; to be added when the planet's right ascension is decreasing and to be subtracted when the planet's right ascension is increasing.

Example.—Greenwich date July 4d. 13h., mean interval 3h. 6m.; find the planetary interval for Jupiter.

	H.	M.	S.
Mean interval	3	6	0
Arc for 3h. 6m.	+		30·55
Sid. interval	3	6	30·55
Correction	+		3·1
Planetary int.	3	6	33·65

Jupiter's var. in R.A. in 1 hour = 1s.

s. s.
1 × 3·1 = 3·1

When the moon is observed the mean interval must be converted into a lunar interval as follows: Obtain the sidereal interval; then multiply the moon's variation in right ascension in 1m. by the mean interval and subtract the product from the sidereal interval, and the result will be the lunar interval.

To Compute the Various Arcs

For Arc I.—Add together log. co-secant of half elapsed time and log. secant of declination; the sum will be log. co-secant of arc I., which take out.

For Arc II.—Add together log. co-secant of arc I., log. cosine of half sum of true altitudes, and log. sine of half difference of true altitudes; the sum will be log. sine of arc II, which take out.

For Arc III.—Add together log. secant of arc I., log. sine of half sum of true altitudes, log. cosine of half difference of true altitudes, and log. secant of arc II.; the sum will be log. cosine of arc III., which take out.

For Arc IV.—Add together log. sine of declination, and log. secant of arc I.; the sum will be log. cosine of an arc, which take out; it will be arc IV. if latitude and declination are of the same name; but if latitude and declination are of contrary names, subtract it from 180° for arc IV.

For Arc V.—Generally the difference of arcs III. and IV. will be arc V.; but if the latitude of the ship and the declination of the object are of the same name and latitude less than declination, the sum of arcs III. and IV. will be arc V.

For the Latitude.—Add together the log. secant of arc II. and the log. secant of arc V.; the sum will be the log. co-secant of the latitude, which take out. It will be of the same name as the latitude by dead reckoning.

If it is uncertain whether the latitude or the declination is the greater, arc V. will have two values; consequently each value of arc V. will give a different latitude. Having found the latitude for both values of arc V., take that as the correct one which is nearer to the latitude by dead reckoning.

When the altitudes are equal, compute arcs I. and IV. as above, but arc II. will be 0° 0′ 0″.

For Arc III.—Add together the log. secant of arc I. and the log. sine of the altitude; the sum will be the log. cosine of arc III., which take out.

For the Co-Latitude.—When the polar distance exceeds the co-latitude the difference of arcs III. and IV. will be the co-latitude; when the polar

distance is less than the co-latitude the sum of arcs III. and IV. will be the co-latitude, which subtract from 90° for the latitude.

When the declination is 0° 0′ 0″ the half elapsed time will be arc I., and arc IV. will be 90°. For the remaining arcs proceed as before.

The latitude just found is on the assumption that there has been no change in the declination, which is only true in the case of a star. When, however, the sun, planet, or moon is the object observed, a correction for the change of declination in the half elapsed time will have to be applied to the approximate latitude.

The computation of the latitude correction is as follows: Add together the log. of the change of declination in the half elapsed time, the log. secant of the approximate latitude, the log. co-secant of the half elapsed time, and the log. sine of arc II. (all to four places of decimals); the sum will be the log. of the correction in seconds of arc, which take out.

The correction is to be added to the approximate latitude when the second altitude is the greater, and the polar distance decreasing, or when the second altitude is the less and the polar distance increasing; otherwise it is subtractive.

The Direct Method

Students will find this method much shorter, and there is no ambiguity in finding any of the arcs or angles.

When the sun, planet, or moon is the object observed, find the apparent, planetary, or lunar interval in the manner explained above. For a star find the sidereal interval, also the half sidereal interval.

Correct the declination for the Greenwich time at the first and second observations and find the polar distance at each observation.

Find the true altitudes at the first and second observations, and also the true zenith distances, not forgetting to correct the first altitude for the course and distance run in the interval between the observations. These constitute all the data used in solving this problem by the Direct Method.

When the polar distance at each observation is the same, as in the case of a star, the calculation is considerably curtailed by the introduction of Napier's Rules, as will be seen in the two examples given.

We shall now give an example of Ivory's Method and several examples of the Direct Method, explaining each portion of the latter as we proceed.

Example 1.—February 28th; latitude by dead reckoning 50° N.; longitude 30° W.; the following observations of the sun were taken for latitude by double altitudes:—

Ship time nearly.			Time by Chron. (corrected).				Obs. Alts. of Sun's L.L.				
	H.	M.		H.	M.	S.		°	′	″	
1st obs.	9	24 a.m.	1st obs.	11	24	43	1st obs.	21	0	40	bearing S. 46° E.
2nd obs.	1	54 p.m.	2nd obs.	3	54	40·8	2nd obs.	28	5	20	west of Meridian,

height of eye 24 feet; ship's course and distance in the interval between the observations N. 77° E., 44 miles: find the latitude when the second observation was taken. For construction of problem see Fig. 1.

Ivory's Method

	D.	H.	M.	S.
1st obs. M.T.G. Feb.	27	23	24	43
2nd obs M.T.G. Feb.	28	3	54	40·8
Mean interval		4	29	57·8
Half interval		2	14	58·9
Middle M.T.G.	28	1	39	41·9

	H.	M.	S.
Mean Int.	4	29	57·8
Eq. Time 0·48s. × 4·5 h.	+		2·2
App. Int.	4	30	00
Half elapsed time	2	15	00
H.E.T. in arc	33°	45	00

	°	′	″
Decl. (N.A. p. II.)	7	51	53·3 S.
56·76″ × 1·66 h. =	—	1	31·2
Corr. decl.	7	50	19·1 S.

Sun's bearing	S. 46 E.
Ship's course	N. 77 E.
	123
	180
Angle between ship's course and sun's bearing	57°

57° as course and dist. 44m. gives 24′ in d. lat. column } in Traverse Table.

	°	′	″
1st obs. alt.	21	0	40
Corr.	+	9	3
True alt.	21	9	43
Corr. for run	+	24	00
True alt. corr. for run	21	33	43

	°	′	″
2nd obs. alt.	28	5	20
Corr.	+	9	43
2nd true alt.	28	15	3
1st true alt.	21	33	43
Sum	49	48	46
Diff.	6	41	20
Half sum	24	54	23
Half diff.	3	20	40

For Arc I.

	°	′	″		
H.E.T.	33	45	00	L. co-sec.	10·255261
Decl.	7	50	19	L. sec.	10·004077
Arc I.	33	23	35	L. co-sec.	10·259338

For Arc II.

	°	′	″		
Arc I.	33	23	35	L. co-sec.	10·259338
½ sum alts.	24	54	23	L. cos.	9·957606
½ diff. alts.	3	20	40	L. sin.	8·765955
Arc II.	5	31	1	L. sin.	8·982899

For Arc III.

	°	′	″		
Arc I.	33	23	35	L. sec.	10·078358
½ sum tr. alt.	24	54	23	L. sin.	9·624423
½ diff. tr. alt.	3	20	40	L. cos.	9·999260
Arc II.	5	31	1	L. sec.	10·002016
Arc III.	59	36	34	L. cos.	9·704057

For Arc IV.

	°	′	″		
Decl.	7	50	19	L. sin.	9·134762
Arc I.	33	23	35	L. sec.	10·078358
	80	35	55	L. cos.	9·213120
	180	00	00		
Arc IV.	99	24	5		

For Arc V.

	°	′	″
Arc IV.	99	24	5
Arc III.	59	36	34
Arc V.	39	47	31

For the Latitude.

	°	′	″		
Arc II.	5	31	1	L. sec.	10·002016
Arc V.	39	47	31	L. sec.	10·114428
Approx. lat.	49	53	26 N.	L. co-sec.	10·116444
Corr.	+		34		
Lat. in	49	54	00 N.		

Latitude correction for change of declination in half elapsed time.

56″·76 × 2·25h = 127″·7		Log.	2·1062
Approx. lat.	49° 53½′	L. sec.	10·1909
H.E.T. (in arc)	33 45	L. co-sec.	10·2553
Arc II.	5 31	L. sin.	8·9829
Lat. correction	34″·3	Log.	1·5353

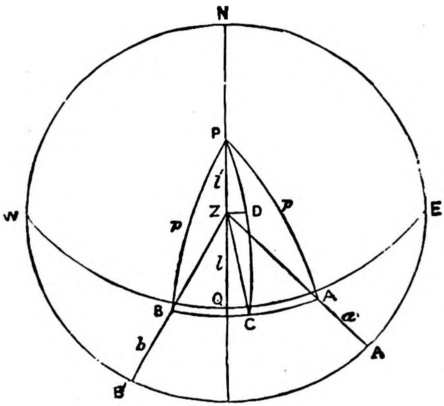


Fig. I.

Explanation of the figure.

N W S E	Rational horizon.
W Q E	Equinoctial.
W Z E	Prime vertical.
N Z S	Observer's mer.
p. p.	Polar dist. at middle time.
P	The pole.
P C	Perpendicular bisecting angle P and side A C.
A B	Great circle joining A B.
A	Sun's position, 1st obs.
B	Sun's position, 2nd obs.
Z A	Zenith dist., 1st obs
Z B	Zenith dist., 2nd obs.
a or A' A	1st true alt.
b or B' B	2nd true alt.
l' or P Z	Co-lat.
l or Q Z	Latitude.
A C is Arc I.	
Z D is Arc II. and is perpendicular to P C.	
C D is Arc III.	
P C is Arc IV.	
P D is Arc V.	

Direct Method

The following is the direct method of finding the latitude by double altitudes. The same data as used in Example 1 are used in this example.

	D.	H.	M.	S.	Eq. time
M.T.G. 1st obs.	27	23	24	43	0·48s
M.T.G. 2nd obs.	28	3	54	40·8	4·5
Mean interval		4	29	57·8	240
Eq. time		+		2·2	192
App. interval		4	30	00	2·160

	°	′	″	Var.
Dec.	7	51	53 S.	56·76″
Corr.	+		31	·6
	7	52	27	34·056
	90	00	00	
P. dis. 1st obs.	97	52	97	

	°	′	″	Var
Dec.	7	51	53·3 S.	56·76″
Corr.	—	3	41·4	3·9
	7	48	11·9	51084
	90	00	00	17026
P. dist. 2nd obs.	97	48	12	221·364
				3 41·4

	°	′	″
1st true alt.	21	33	43
	90	00	00
1st true zenith dist.	68	26	17

	°	′	″
2nd true alt.	28	15	3
	90	00	00
2nd true zenith dist.	61	44	57

In the spherical triangle P A B, Fig. 2; given ∠ P, the apparent interval, P A the first polar distance, and P B the second polar distance, to find A B.

Formula—

$$\text{Hav. } \theta = \text{hav. } \angle \text{ P} \times \text{sin. P A} \times \text{sin. P B}$$

$$\text{Nat. hav. A B} = \text{nat. hav. } \theta + \text{nat. hav. (P A} \sim \text{P B)}$$

N.B.—When the difference between P A and P B is less than 15 minutes of arc, θ is equal to A B.

	H.	M.	S.		
∠ P	4	30	0	L. Hav.	9·48948
	°	′	″		
P A	97	52	27	L. Sin.	9·99589
P B	97	48	12	L. Sin.	9·99596
θ	66	47	10	L. Hav.	9·48133

(1)

P A ∼ P B 0° 4′ 15″

θ = A B = 66° 47′ 10″ because the difference between P A and P B is less than 15 minutes of arc.

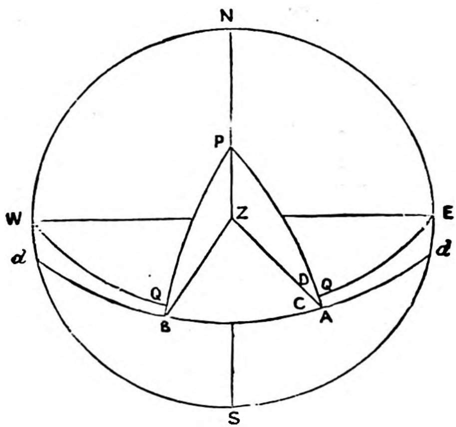


Fig. 2.

Explanation of the figure.

N W S E	Rational horizon.	Z	The zenith.
W Q E	Equinoctial.	Z A	1st zenith dist.
W Z E	Prime vertical.	Z B	2nd zenith dist.
N Z S	Observer's mer.	P	The pole.
d d	Parallel of dec.	P A	Polar dist., 1st obs.
A B	Arc of great circle joining A and B.	P B	Polar dist., 2nd. obs.
A	Sun's position, 1st obs.	P Z	Co-lat.
B	Sun's position, 2nd obs.		

In the same triangle, given the sides P A, P B, and A B to find ∠ A.

Formula—

$$\text{Cos.}\ \frac{A}{2} = \sqrt{\frac{\text{sin. S} \times \text{sin. (S} - \text{P B)}}{\text{sin. PA} \times \text{sin. AB}}} \quad \text{where S} = \frac{\text{P A} + \text{P B} + \text{A B}}{2}$$

P B	97°	48′	12″			
P A	97	52	27	L. Co-sec.	10·004114	
A B	66	47	10	L. Co-sec.	10·036720	
	262	27	49			
S	131	13	55	L. Sin.	9·876245	(2)
S — P B	33	25	43	L. Sin.	9·741071	
	°	′	″		2)19·658150	
$\frac{A}{2}$ =	47	34	24	L. Cos.	9·829075	
			2			
∠ A =	95	8	48			

In the spherical triangle Z A B, Fig. 2. ; given the sides Z A, the first zenith distance, Z B, the second zenith distance, and A B to find ∠ C.

Formula—Same as (2).

Z B	61°	44′	57″			
Z A	68	26	17	L. Co-sec.	10·031500	
A B	66	47	10	L. Co-sec.	10·036720	
	2)196	58	24			
S	98	29	12	L. Sin.	9·995218	(3)
S — Z B	36	44	15	L. Sin.	9·776810	
	°	′	″		2)19·840248	
$\frac{C}{2}$	33	41	42	L. Cos.	9·920124	
			2			
∠ C	67	23	24			

∠ D = ∠ A — ∠ C

	°	′	″
A	95	8	50
C	67	23	24
D	27	45	26

In the spherical triangle P Z A, Fig. 2 ; given ∠ D and sides Z A and P A, to find P Z, the co-latitude.

Formula—Same as (1).

	°	′	″					
D	27	45	26	L. Hav.	8·75993			(4)
Z A	68	26	17	L. Sin.	9·96849			
P A	97	52	27	L. Sin.	9·99589			
				θ L. Hav.	8·72431	Nat. hav.	·05300	
Z A ~ P A	29°	26′	10″	. . . .	. . .	Nat. hav.	·06454	
P Z or co-latitude	40°	6′	00″			Nat. hav.	·11754	
	90	00	00					
Latitude	49	54	00 N.					

Example 2.—December 5th, 1890; latitude by dead reckoning 2° 20′ S.; the following observations of the sun were made for latitude by double altitudes:—

M.T.G. by chron. (corrected).	D.	H.	M.	S.	Obs. alts. sun's L.L.	°	′	″	
1st obs. Dec.	4	21	47	11	1st obs.	28	47	25	bearing S. 65¼° E.
2nd obs. Dec.	5	2	47	16	2nd obs.	65	25	00	east of meridian

height of eye 20 feet; ship's course and distance in the interval between the observations N. 34 miles; find the latitude when the second observation was made.

	D.	H.	M.	S.	Eq. time
M.T.G. 1st obs.	4	21	47	11	var.
M.T.G. 2nd obs.	5	2	47	16	1·0s. × 5 hrs.
Mean interval		5	0	5	5s.
Eq. of time		—		5	5·0s.
App. interval		5	0	0	

	°	′	″	Var.		°	′	″	Var.
Dec.	22	24	55·8 S.	15·8″		22	24	56·3 S.	18·9″
Corr.	—		41·4	2·2	Corr.	+		52	2·8
	22	24	14·4	41·36		22	25	48·4	52·64
	90	00	00			90	00	00	
P.D. 1st obs.	67	35	45·6		P.D. 2nd obs.	67	34	11·6	

	°	′	″
1st obs. alt.	28	47	25
Corr.	+	10	17
	28	57	42
Corr. for run	—	14	6
1st tr. alt.	28	43	36
	90	00	00
1st tr. Z. dist.	61	16	24

Sun's bearing	S. 65¼° E.
Ship's course	N. 0°
Angle between course and bearing	114¾°
	180
	65¼°

Course 65¼° and dist. 34 miles gives 14·1° in d. lat. col.

	°	′	″
2nd obs. alt.	65	25	0
Corr.	+	11	30
2nd tr. alt.	65	36	30
	90	00	00
2nd tr. Z. dist.	24	23	30

In the spherical triangle A P B, Fig. 3; given ∠ P, the apparent interval, P A, the first polar distance, and P B, the second polar distance, to find the side A B.

Formula— $\text{Hav. } \theta = \text{hav. } \angle P \times \sin. PA \times \sin. PB$ (1)

$\text{Nat. hav. } AB = \text{nat. hav. } \theta + \text{nat. hav. } (PA \sim PB)$

	H. / °	M. / ′	S. / ″		
∠P	5	0	0	L. Hav.	9·56889
P A	67	35	46	L. Sin.	9·96592
P B	67	34	12	L. Sin.	9·96583
θ	68	29	37	Hav.	9·50064

The difference between P A and P B being less than 15′ of arc, A B and θ are equal.

A B = 68° 29′ 37″

In the spherical triangle P A B, Fig. 3; given the three sides P A, P B, and A B, to find the angle C.

Formula—

$$\text{Cos. } \frac{C}{2} = \sqrt{\frac{\text{sin. S} \times \text{sin. (S} - \text{P B)}}{\text{sin. P A} \times \text{sin. A B}}} \quad \text{where S} = \frac{\text{P B} + \text{P A} + \text{A B}}{2}$$

	°	′	″		
P B	67	34	12		
P A	67	35	46	L. Co-sec.	10·034084
A B	68	29	37	L. Co-sec.	10·031341
	2)203	39	35		
S	101	49	47	L. Sin.	9·990676
S — P B	34	15	35	L. Sin.	9·750466
					2)19·806567
$\frac{C}{2}$	36	50	10	L. Cos.	9·903283
			2		
∠ C	73	40	20		

(2)

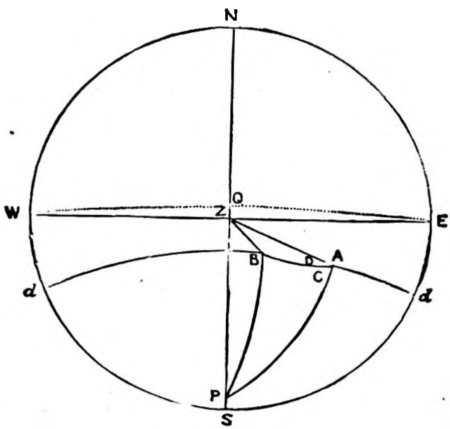


Fig. 3.

Explanation of the figure

N W S E	Rational horizon.	Z B	2nd zenith dist.
W Q E	Equinoctial.	P	The pole.
W Z E	Prime vertical.	P A	1st polar dist.
N Z S	Observer's mer.	P B	2nd polar dist.
d d	Parallel of dec.	A	Sun's position, 1st obs.
A B	Arc of great circle joining A and B	B	Sun's position, 2nd obs.
Z	The zenith.		
Z A	1st zenith dist.		

In the spherical triangle Z A B, Fig. 3; given Z A, the first zenith distance; Z B, the second zenith distance, and the side A B, to find ∠ D.

Formula—Same as (2).

	°	′	″			
Z B	24	23	30			
Z A	61	16	24	L. Co-sec.	10·057039	
A B	68	29	37	L. Co-sec.	10·031341	
	2)154	9	31			
S	77	4	45	L. Sin.	9·988862	
S ∼ Z B	52	41	15	L. Sin.	9·900553	(3)
					2)19·977795	
$\frac{D}{2}$	12	54	2	L. Cos.	9·988897	
			2			
∠ D	25	48	4			

∠ A = ∠ C + ∠ D

	°	′	″
∠ C	73	40	20
∠ D	25	48	04
∠ A	99	28	24

In the spherical triangle Z A P, Fig. 3; given ∠ A and the sides P A and Z A, to find P Z, the co-latitude.

Formula—Same as (1).

(4)

	°	′	″				
∠ A	99	28	24	L. Hav.	9·76514		
P A	67	35	46	L. Sin.	9·96592		
Z A	61	16	24	L. Sin.	9·94296		
				θ Hav.	9·67402	Nat. hav.	·47209
P A ∼ Z A	6	19	22	. . .	. . .	Nat. hav.	·00305
P Z or co-latitude	87°	9′	0″			Nat. hav.	·47514
	90	0	0				
Latitude	2	51	00 S.				

Example 3.—February 10th and 11th, 1890; latitude by dead reckoning 39° 20′ N., longitude 174° 30′ W.; the following observations of Procyon (α Canis Minoris) were taken for latitude by double altitudes:—

Ship time nearly.	H.	M.	Time by chron.	H.	M.	S.	Obs. altitudes.	°	′	″	
1st obs.	8	27 p.m.	1st obs.	8	6	46	1st obs.	49	35	35	bearing S. 40° E.
2nd obs.	1	00 a.m.	2nd obs.	0	38	44	2nd obs.	38	26	35	west of meridian

height of eye 19 feet; ship's course and distance in the interval between the observations, East, 35 miles. Required the latitude by double altitudes when the second altitude was taken.

	D.	H.	M.	S.
M.T.G. 1st obs.	10	20	6	46
M.T.G. 2nd obs.	11	0	38	44
Mean interval		4	31	58
Acceleration		+		44·68
Sidereal interval P		4	32	42·68
$\frac{P}{2}$		2	16	21·34
$\frac{P}{2}$		34°	5′	20″

	°	′	″
Dec.	5	30	18 N.
	90	00	00
P. dist.	84	29	42
1st obs. alt.	49	35	35
Corr.	—	5	5
	49	30	30
Corr. for run	+	22	30
1st tr. alt.	49	53	00
	90	00	00
1st tr. Z. dist.	40	7	00

Star's bearing	S. 40° E.
Ship's course	S. 90 E.
Angle between course and star's bearing	50°

Course 50° and dist. 35 miles gives 22·5′ in d. lat. col.

	°	′	″
2nd obs. alt.	38	26	35
Corr.	—	5	27
2nd tr. alt.	38	21	8
	90	00	00
2nd tr Z dist.	51	38	52

In the isosceles triangle A P B, Fig. 4, given angle P and the sides P A and P B, to find the side A B. From P drop a perpendicular on A B, bisecting the angle P and the side A B; then in the right-angled spherical triangle A P C, right-angled at C, given $\frac{P}{2}$, the half sidereal interval, and P A, the polar distance, to find A C equal to half A B.

N.B.—Pay strict attention to the plus and minus signs when solving problems by Napier's Analogies.

Formula— $$\text{Sin. A C} = \text{sin. P A} \times \text{sin.} \frac{P}{2} \qquad (1)$$

	°	′	″		
P A	84	29	42	L. Sin.	9·997992
$\frac{P}{2}$	34	5	20	L. Sin.	9·748557
A C or $\frac{A B}{2}$	33	54	36	L. Sin.	9·746549
			2		
A B	67	49	12		

In the same triangle given P A and A C, to find angle A.

Formula— Cos. A = tan. A C × cot. P A

	°	′	″			
A C	33	54	36	L. Tan.	9·827515	(2)
P A	84	29	42	L. Cot.	8·983975	
A	86	17	7	L. Cos.	8·811490	

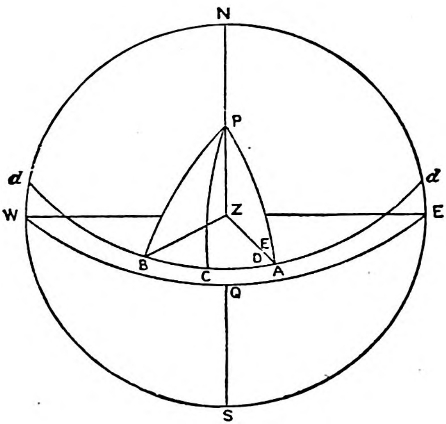


Fig. 4.

Approximate H.A. for constructing the figure.

	H.	M.	
Time at ship	8	27	
Sid. time	21	23	
R.A.M.	5	50	
Procyon's R.A.	7	34	
H.A. 1st obs.	1	44	E.

	H.	M.	
Time at ship	13	0	
Sid. time	21	23	
R.A.M.	10	23	
Procyon's R.A.	7	34	
H.A. 2nd obs.	2	49	W.

Explanation of the figure.

N W S E	Rational horizon.
W Q E	Equinoctial.
W Z E	Prime vertical.
N Z S	Observer's mer.
d d	Parallel of dec.
A B	Arc of great circle joining A and B.
Z A	1st zenith dist.
Z B	2nd zenith dist.
P A	Polar dist., 1st obs.
P B	Polar dist., 2nd obs.
P C	Perpendicular bisecting angle P and side A B.
P	The pole.
Z	The zenith.
P Z	Co-lat.
A	Position of star, 1st obs.
B	Position of star, 2nd obs.

In the spherical triangle Z A B, Fig. 4, given Z A, the first zenith distance, and Z B, the second zenith distance, and A B, to find angle D.

Formula—

$$\text{Cos. } \frac{D}{2} = \sqrt{\frac{\text{sin. S} \times \text{sin. (S} - \text{Z B)}}{\text{sin. Z A} \times \text{sin. A B}}} \quad \text{where S} = \frac{\text{Z A} + \text{Z A} + \text{A B}}{2}.$$

	°	′	″			
Z B	51	38	52			
Z A	40	07	00	L. Co-sec.	10·190881	
A B	67	49	12	L. Co-sec.	10·033388	
	2)159	35	4			(3)
S	79	47	32	L. Sin.	9·993071	
S — Z B	28	8	40	L. Sin.	9·673662	
					2)19·891002	
$\frac{D}{2}$	28°	6′	27″	L. Cos.	9·945501	
			2			
∠ D	56	12	54			

$$\angle E = \angle A - \angle D$$

	°	′	″
∠ A	86	17	7
∠ D	56	12	54
∠ E	30	4	13

In the spherical triangle P Z A, Fig. 4, given angle E and the sides P A and Z A, to find P Z, the co-latitude

Formula— Hav. θ = hav. ∠ E × sin. P A × sin. Z A

Nat. hav. P Z = nat. hav. θ + nat. hav. (P A ~ Z A) (4)

	°	′	″				
∠ E	30	4	13	L. Hav.	8·82799		
P A	84	29	42	L. Sin.	9·997992		
Z A	40	7	00	L. Sin.	9·809119		
			θ	L. Hav.	8·635101	Nat. hav.	·04316
(P A ~ Z A)	44°	22′	42″	. .	. . . .	Nat. hav.	·14264
Co-latitude or P Z	51°	4′	00″			Nat. hav.	·18580
	90	00	00				
Latitude	38	56	00 N.				

Example 4.—March 3rd, 1890; a.m. at ship; latitude by account 9° 45′ S.; longitude 85° E.; the following observations of α Centauri were taken for latitude by double altitudes:—

Ship time nearly.		Time by chron.				Obs. alts. α^2 Centauri.				
	H. M.		H.	M.	S.		°	′	″	
1st obs.	1 25 a.m.	1st obs.	7	59	20	1st obs. alt.	34	37	30	bearing S. 19° E.
2nd obs.	4 45 a.m.	2nd obs.	11	19	26	2nd obs. alt.	38	7	45	west of Meridian

Height of eye 20 feet; ship's course and distance in the interval between the observations, North, 17 miles: find the latitude when the second observation was taken.

	D.	H.	M.	S.
M.T.G. 1st obs.	2	7	59	20
M.T.G. 2nd obs.	2	11	19	26
Mean interval		3	20	6
Acceleration		+		32·9
Sidereal interval		3	20	38·9
Half sid. interval		1	40	19·5
$\frac{P}{2}$		25°	4′	52″

	°	′	″
Dec.	60	22	31 S.
	90	00	00
P. dist.	29	37	29

	°	′	″
1st obs. alt.	34	37	30
Corr.	—	5	46
	34	31	44
Corr. for run	—	16	6
1st tr. alt.	34	15	38
	90	00	00
1st tr. Z. dist.	55	44	22

	°	′	″
2nd obs. alt.	38	7	35
Corr.	—	5	35
2nd tr. alt.	38	2	0
	90	0	0
2nd tr. Z. dist.	51	58	0

Angle between star's bearing and ship's course = 161° 180° − 161° = 19°. 19° as course and dist. 17′ gives 16·1′ in d. lat. col.

In the isosceles triangle A P B, Fig. 5; given ∠ P, the sidereal interval and the sides P A and P B, to find the side A B. From P let fall a perpendicular on A B, bisecting the ∠ P and the side A B; then in the right-angled spherical triangle A P C, right angled at C, given $\frac{P}{2}$ and the side P A to find A C.

Formula—

$$\text{Sin. A C} = \sin. \frac{P}{2} \times \sin. \text{P A}$$

	°	′	″		
$\frac{P}{2}$	25	4	52	L. Sin.	9·627264
P A	29	37	29	L. Sin.	9·694005
$\frac{AB}{2}$ or A C	12	5	44	L. Sin.	9·321269
			2		
A B	24	11	28		

(1)

In the same triangle given P A and A C to find ∠ D.

Formula—

$$\text{Cos.} \angle D = \tan. \text{A C} \times \cot. \text{P A}$$

(2)

	°	′	″		
A C	12	5	44	L. Tan.	9·331019
P A	29	37	29	L. Cot.	10·245155
∠ D	67	51	40	L. Cos.	9·576174

Approximate H.A. for constructing the figure.

	H.	M.
M.T. ship	13	25
Sid. time	22	41
R.A.M.	12	06
Centauri's R.A.	14	32
H.A. 1st obs.	2	26 E.

	H.	M.
M.T. ship	10	45
Sid. time	22	41
R.A.M.	15	26
Centauri's R.A.	14	32
H.A. 2nd obs.	0	54 W.

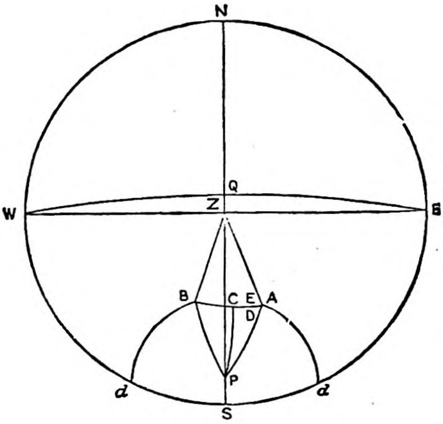


Fig. 5.

Explanation of the figure

N W S E	Rational horizon.
W Q E	Equinoctial.
W Z E	Prime vertical.
N Z S	Observer's mer.
d d	Parallel of declination.
A B	Arc of great circle joining A B.
Z A	Zenith dist., 1st obs.
Z B	Zenith dist., 2nd obs.
P B and P A	Polar dist.
P C	Perpendicular bisecting angle P and side A B.
P	The pole.
Z	The zenith.
P Z	Co-latitude.
A	Position of star, 1st obs.
B	Position of star, 2nd obs.

In the spherical triangle Z A B, Fig. 5; given the three sides Z A, Z B and A B, to find ∠ E.

Formula—

$$\text{Cos.}\frac{E}{2} = \sqrt{\frac{\sin. S \times \sin. (S - Z\,B)}{\sin. Z\,A \times \sin. A\,B}} \text{ where } S = \frac{Z\,A + Z\,B + A\,B}{2}$$

	°	′	″			
Z B	51	58	00			
Z A	55	44	22	L. Co-sec.	10·082765	
A B	24	11	28	L. Co-sec.	10·387447	(3)
	2)131	53	50			
S	65	56	55	L. Sin.	9·960556	
S — Z B	13	58	55	L. Sin.	9·383126	
					2)19·813894	
	°	′	″			
E/2	36	10	59	L. Cos.	9·906947	
			2			
∠ E	72	21	58			

$$\angle A = \angle E + \angle D$$

	°	′	″
E	72	21	58
D	67	51	40
∠ A	140	13	38

In the spherical triangle Z P A, Fig. 5; given P A, Z A, and ∠ A, to find P Z, the co-latitude.

Formula— Hav. θ = hav. A × sin. P A × sin. Z A

Nat. hav. P Z = nat. hav. θ + nat. hav. (P A ~ Z A)

	°	′	″					
A	140	13	38	L. Hav.	9·94659			(4)
Z A	55	44	22	L. Sin.	9·91724			
P A	29	37	29	L. Sin.	9·69401			
				θ L Hav.	9·55784	Nat. hav.	·36127	
Z A ~ P A	26	6	53	. .	. . .	Nat. hav.	·05105	
						Nat. hav.	·41232	

	°	′	″
P Z or co-latitude	79	54	00
	90	00	00
Latitude	10	6	00 S.

Examples for Practice in finding a Ship's position from two observations

N.B.—To correct the altitude for run the azimuth is required; this can be taken from Norie's A and B Tables.

Example 1.—December 3rd, 1890, at ship, the following observations of the sun were taken:—

Approx. ship time. H. M.	Chron. time. H. M. S.	Alt. sun's L.L.
9 0 a.m.	9 31 2	48° 26′ 35″ bearing N. 84° E.
1 10 p.m.	1 41 2	69 0 0

height of eye 17 feet. At the time of taking the first observation the ship by dead reckoning was in latitude 36° S.; longitude 10° W. The true course in the interval S. 27° E., distance 32 miles. The chronometer was supposed to be correct for Greenwich mean time. Required the latitude at the time of taking the second observation.

Ans. Lat. 36° 13′ 55″ S.

Example 2.—November 3rd, 1890, at ship, the following observations of the sun were taken :—

Ship time nearly.		Chron. time.			Alt. sun's L.L.			
H.	M.	H.	M.	S.				
7	50 a.m.	2	56	59	19°	26′	25″	bearing S. 66° E.
10	50 a.m.	5	56	45	51	41	23	

height of eye 26 feet. When the first observation was taken the ship was in latitude by account 19° N.; longitude by account 65° E. The true course in the interval was S. 24° W., distance 32 miles. The chronometer was supposed to be 15m. 40s. slow for Greenwich mean time. Required the latitude when the second observation was taken.

Ans. Lat. 18° 47′ 57″ N.

Example 3.—January 10th, 1890, at ship, the following observations of the sun were taken :—

Ship time nearly.		Chron. time.			Alt. sun's L.L.			
H.	M.	H.	M.	S.				
9	53 a.m.	1	30	54	15°	12′	46″	bearing S. 30° E.
0	55 p.m.	4	32	1	20	3	29	

Index error of the sextant 2′ to be added, height of the eye 26 feet. When the first observation was taken the latitude by account was 47° N.; longitude by account 52° 30′ W. The true course in the interval was S.W., distance 31 miles. The chronometer was supposed to be correct for mean time at Greenwich. Required the latitude of the ship when the second observation was taken.

Ans. Lat. 46° 48′ N.

Example 4.—June 14th, 1890, at ship, the following observations of the sun were taken :—

Approx. ship time.		Chron. time.			Alt. sun's L.L.			
H.	M.	H.	M.	S.				
7	40 a.m.	10	20	49	23°	56′	6″	bearing N. 66° E.
10	55 a.m.	1	35	39	63	54	57	

height of eye 19 feet. At the time of taking the first observation the ship was by dead reckoning in latitude 3° N.; longitude 40° 30′ W. The true course in the interval was S. 66° W., distance 20 miles. The chronometer was supposed to be correct for Greenwich mean time. Required the latitude at the time of taking the second observation.

Ans. Lat. 3° 2′ 54″ N.

Example 5.—January 26th, 1890, at about 10h. 57m. p.m. ship time, and again at about 4h. 5m. a.m. January 27th, the following observations of Regulus (α Leonis) were taken :—

Chron. time.			Star's alt.			
H.	M.	S.				
11	55	0	41°	44′	40″	bearing S. 52° E.
5	2	40	40	16	0	

height of the eye 20 feet. When the first observation was taken the ship

was in latitude by account 49° 50′ N.; longitude by account 11° W. The true course in the interval E. 8° N., distance 36 miles. The chronometer was supposed to be correct for mean time at Greenwich. Required the latitude of the ship when the second observation was taken.

Ans. Lat. 49° 50′ N.

Example 6.—January 10th, 1890, at about 8h. 5m. p.m. ship time, and again January 11th at about 0h. 20m. a.m. ship time, the following observations of Sirius (α Canis Majoris) were taken :—

Chron. time.			Star's alt.			
H.	M.	S.				
10	14	26	12°	15′	20″	bearing S. 65° E.
2	31	26	21	34	16	

height of eye 19 feet. When the first observation was taken the position of the ship was supposed to be latitude 50° N., longitude 30° W. The true course in the interval was west, distance 38 miles. The chronometer was supposed to be 2m. 10s. fast for mean time at Greenwich. Required the latitude when the second observation was taken.

Ans. Lat. 50° 10′ N.

Example 7.—August 12th, 1890, at about 11h. 44m. p.m. ship time in lat. by dead reckoning 51° 30′ N., long. by dead reckoning 6° W., when the chronometer (corrected) indicated 0h. 4m. 50s. the following simultaneous observations were taken :—Capella (α Aurigæ) observed altitude was 19° 36′ 30″ east of the meridian and Benetnasch (η Ursæ Majoris) observed altitude was 26° 55′ 40″ west of the meridian, height of the eye 24 feet. Required the ship's position.

Ans. Lat. 50° 58′ N.; Long. 6° 12′ W.

Example 8.—March 2nd, 1890, at about 7h. 22m. p.m. ship time in latitude by account 46° 43′ N., longitude by account 52° 30′ W., the following observations were taken when the chronometer indicated 10h. 42m. 20s. Procyon (α Canis Minoris) observed altitude 44° 17′ 30″ east of meridian and Aldebaran (α Tauri) observed altitude 53° 46′ 20″ west of meridian, height of the eye 24 feet. The chronometer was supposed to be 6m. 36s. slow on mean time at Greenwich. Required the latitude and longitude by observation.

Ans. Lat. 47° 0′ N.; Long. 52° 4′ W.

Elements from the Nautical Almanac, see page 585.

EQUATION OF EQUAL ALTITUDES

FOR FINDING THE ERROR OF THE CHRONOMETER

In the case of a star the easterly and westerly hour-angles corresponding to the equal altitudes will be the same, and consequently half the sum of the times by chronometer when the observations were made is the exact time of the meridian passage of the star; but for the sun and planets the easterly and westerly hour-angles are unequal, in consequence of the change of declination in the interval. Half the sum of the times by chronometer when the observations were made is not, therefore, the correct time of meridian passage. The correction to be applied to the half-sum of the chronometer times to give the exact time of the meridian passage is called the *Equation of Equal Altitudes.*

The Observation by the Sun.—On shore, at a place whose longitude is *accurately* known, and whose latitude is *approximately* known, observe with an artificial horizon the same altitude both in the morning and in the afternoon, as near the prime vertical as convenient after the altitude is more than 10°, noting the times by a chronometer. In low latitude, however, the method of *equal altitudes* will often give very accurate results, even when the observations are quite near the meridian.

In general, a sufficiently accurate result may be obtained if the observations are taken when the sun's change of altitude is not less than 10″ in half a second of time, or when the change in the altitude taken with the artificial horizon is not less than 20″ in half a second of time.

It is most convenient, as well as conducive to accuracy, to take the observation in the following manner. In the morning bring the lower limb of the sun, reflected from the sextant mirrors, and the upper limb of that reflected from the mercury, into approximate contact; move the 0 (zero) of the vernier forward (say from 10′ to 20′), and set it on a division of the limb; the images will be *over-lapped* and will be *separating;* wait for the instant of contact, note it by chronometer, and immediately set the vernier on the next division of the limb, that is 10′ in advance; notice the instant of contact again, and proceed in the same manner for as many observations as are thought necessary. If the sun rises too rapidly let the intervals on the limb be 20′.

Find (roughly) the time when the sun will be at the same altitude in the afternoon, and just before that time set the vernier on the last altitude noted in the morning (of course using the same sextant); the images of the sun will be *separated*, but will be *approaching;* wait for the instant of contact, note it by chronometer, set the vernier *back* to the next division of the limb (10′ or 20′ as the case may be), note the contact again, and so proceed until all the a.m. altitudes have been again noted as p.m. altitudes.

RULE.—*For the Greenwich Date.*—To the date at place (noon) 0h. 0m. 0s. apply the longitude in time, additive if W., subtractive if E.; the result will be the Greenwich *apparent* date.

From the Nautical Almanac, p. I., take out the *sun's declination* and the *equation of time*, and correct them for the Greenwich apparent date.

For the Interval and the *Middle Time by Chronometer.*—Find the *mean* of the *first times* of observation shown by *chronometer*, and also the *mean* of the *second times by chronometer;* take the first *mean* from the second *mean* for the *interval* between the observations. Then take the *half* of the interval, and add it to the *mean* of the first times by chronometer; the result will be the *middle time by chronometer.*

For the Change of Declination in Method I.—From Nautical Almanac write down the sun's declination for the day preceding and the day following the date of observation; take their *difference* and reduce the result to seconds ("), for the *change of declination in two days.*

NOTE.—In March and September when the declination has changed its name, the sum of the declinations may be their difference.

For the Change of Declination in Method II.—Multiply the "Var. in 1 hour" by the half-elapsed time.

Method I.

RULE.—1. For the *first part* of the *equation of equal altitudes*, add together—

Log. A of the interval (Table for Computing the Equation of Equal Altitudes in Norie's Tables).

Log. of the seconds in the change of declination in two days, and

Log. *tangent* of the latitude of observer.

The sum will be the log. of the *first* part, and take out the number corresponding thereto.

2. For the *second part* of the *equation of equal altitudes*, add together—

Log. B of the interval (Table for Computing the Equation of Equal Altitudes in Norie's Tables).

Log. of the seconds in the change of declination in two days, and

Log. *tangent* of the declination.

The sum will be the log. of the *second* part and take out the number corresponding thereto.

Method II.

RULE.—1. For the *first part* of the *equation of equal altitudes*, add together—

Log. of the seconds in the change of declination in half-elapsed time.

Log. *co-secant* of half-elapsed time.

Log. *tangent* of the latitude.

The sum, rejecting 20 from the index, will be the logarithm of the *first part*, and take out the number corresponding thereto.

2. For the *second part* of the *equation of equal altitudes*, add together—

Log. of the seconds in the change of declination in half-elapsed time.

Log. *co-tangent* of half-elapsed time.

Log. *tangent* of the declination.

The sum, rejecting 20 from the index, will be the logarithm of the *second* part, and take out the number corresponding thereto.

Formula—

$$e = c \times \text{co-sec. } h \times \tan. l - c \times \cot. h \times \tan. d$$

where e is the equation of equal altitudes, c is the change of declination in half-elapsed time, h is the half-elapsed time, l is the latitude, and d the declination.

For both Methods.—Mark the *first part* + when the polar distance is increasing, and — when it is decreasing.

Mark the *second part* + when the declination is increasing, and — when it is decreasing.

When the *parts* have the same sign their sum is the *equation of equal altitudes*, of the same sign as the parts.

When the *parts* have different signs their difference is the *equation of equal altitudes*, of the same sign as the greater.

Note to Method II.—The *equation of equal altitudes* is in seconds of arc, it must therefore be divided by 15 to give seconds of time.

The application of the equation of equal altitudes to the middle time by chronometer gives the *corrected middle time* by chronometer when the sun was on the meridian of the place of observation.

For the Error of the Chronometer on Mean Time at Place.—To the date at place d. 0h. 0m. 0s. apply the equation of time according to precept, Nautical Almanac, p. I., the result will be mean time at place, the difference between which and the *corrected middle* time by chronometer will be the *error of Chronometer on mean time at place* of observation.

For the Error of the Chronometer on Greenwich Mean Time.—To the mean time at place apply the longitude in time, additive if W., but subtractive if E.; the result will be mean time at Greenwich, the difference between which and the *corrected middle time* by chronometer will be the *error of the chronometer* on Greenwich mean time on the given date.

Example.—March 2nd: lat. 32° 2′ N., long. 81° 3′ W.: the following times by chronometer were noted when the sun had equal altitudes.

At the a.m. sights. H.	M.	S.	At the p.m. sights. H.	M.	S.
2	38	12	8	17	30
3	15	9	7	40	22
3	52	12	7	3	11

Required the equation of equal altitudes, also the error of the chronometer on mean time at place, and on mean time at Greenwich.

	D.	H.	M.	S.								M.	S.
March	2	0	0	0	Decl. (N.A. p. I.)	7°	6′	3″·5 S.	Eq. T. (N.A. p. I.)			12	18·57
Long. W.		5	24	12	57″·3 × 5·4 —		5	9 ·4	·522 × 5·4	—			2·82
App.T.Gr.		5	24	12	Cor. decl.	7	0	54 ·1 S.		+		12	15·75

1st times by chron.

	H.	M.	S.
	2	38	12
	3	15	9
	3	52	12
3)	9	45	33
1st mean	3	15	11

2nd times by chron.

	H.	M.	S.
	8	17	30
	7	40	22
	7	3	11
3)	23	1	3
2nd mean	7	40	21
1st mean	3	15	11
Interval	4	25	10
½-interval	2	12	35
1st mean	3	15	11
Middle T. by chron. Mar. 2	5	27	46

March 1st decl. (N.A. p. I.)	7°	28′	55″·6 S.
,, 3rd ,, ,,	6	43	5·6 S.
		45	50
			60
Change of decl. in 2 days			2750″

	H. M. S.				
Interval	4 25 10	Log. A	7·7492	Log. B	7·6721
Change of decl.	2750	Log.	3·4393		3·4393
N. lat.	32° 2′	Tan.	9·7964	S. decl. 7° 1′ tan.	9·0902
1st part —	9·66 s	Log.	0·9849	2nd part 1·59 s Log.	0·2016
2nd part —	1·59				

Sum — 11·25 because the signs are alike.

	H.	M.	S.
Middle T. by chron., March 2	5	27	46
Equat. of equal alt.		—	11·2
Corrected middle time by chron. at app. noon	5	27	34·8

	D.	H.	M.	S.
App. T. ship, March 2		0	0	0
Eq. T.		+	12	15·8
Mean time at ship		0	12	15·8
Corrected mid. T. by chron.		5	27	34·8
Chron. fast on M.T. ship		5	15	19

	H.	M.	S.
. . . .	0	12	15·8
Long. in time	5	24	12
M.T.G.	5	36	27·8
Cor. mid. T. by chron.	5	27	34·8
Chron. slow on M.T.G.		8	53

Example.—June 16th: lat. 10° 26′ N., long. 45° 1′ E.: the following times by chronometer were noted when the sun had equal altitudes—

At the a.m. sight.			At the p.m. sight.		
H.	M.	S.	H.	M.	S.
6	58	58	1	57	48

Required the equation of equal altitudes, also the error of the chronometer on mean time at place, and on mean time at Greenwich.

	D	H.	M.	S.
June	16	0	0	0
Long E.		3	0	4
A.T.G.	15	20	59	56

16th decl. (N.A. p. I.)	23°	22′	3″·5 N.
5″·17 × 3		—	15·5
	23	21	48 N.

	M.	S.
Eq. T. (N. A. p. I.)	0	23·6
·54 × 3	—	1·6
	+ 0	22

	H.	M.	S.
1st time by chron.	6	58	58
2nd ,, ,,	1	57	48
Interval	6	58	50
½-interval	3	29	25
1st time	6	58	58
Middle T. by chron.	10	28	23 or 22 H. 28 M. 23 S. for Gr. date.

Var. in 1 hour	5″·17
H.E.T.	3·5
	2585
	1551
Change of decl.	18·095

Change of decl. 18″·09	Log.	1·2574	. . . .	Log.	1·2574
H.E.T. 3h. 29m. 25s.	Co-sec.	10·1014	. . . .	Cot.	9·8873
Lat. 10° 26′	Tan.	9·2651	. . . Decl. 23° 22′	tan.	9·6355
1st part — 4ˢ·20	Log.	0·6239	. . . 2nd part 6ˢ·03	log.	0·7802
2nd part + 6·03					
15)1·83 Diff. because the signs are unlike.					
+ 0·12					

	H.	M.	S.
Middle T. by chron.	10	28	23
Eq. equal alt.		+	·1
Corrected mid. T. by chron. at app. noon	10	28	23·1
Or, for Gr. date	22	28	23·1

	D.	H.	M.	S.
App. T. ship, June	16	0	0	0
Eq. T.			+	22
M.T. ship ,,	16	0	0	22
Cor. mid. T. by chron.		10	28	23·1
Chron. slow on M.T. at ship		1	31	58·9

	D.	H.	M.	S.
. . . .	16	0	0	22
Long. E.		3	0	4
M.T. at Gr.		21	0	18
Cor. mid. T. by chron.		22	28	23·1
Chron. fast on M.T. Gr.		1	28	5·1

EQUAL ALTITUDES OF A FIXED STAR. *The Observation.*—Set the sextant and wait for the coincidences of the two images of the star, as in the case of the sun's limb, noting the times by chronometer.

The Computation.—Take the mean of the times before the meridian passage, also the mean of the times after the meridian passage.

The mean of the two sets of times is the *chronometer time of the star's transit*. This time, if the chronometer is right, will agree with the true mean time of the star's transit, which is found as follows—

To the star's right ascension apply the longitude (in time) of the place, adding in W., subtracting in E.; the result is the *Greenwich sidereal time of star's transit;* from this subtract the sidereal time at the *previous* mean noon at Greenwich (Nautical Almanac, p. II.); the remainder is the *sidereal interval* since mean noon. From the Table for Reducing Sidereal to Mean Time, "Retardation," with the argument sidereal interval take out the correction, which subtract from the sidereal interval; the remainder is the *Greenwich mean time of star's transit.* The time by chronometer will be more or less than this, according as the chronometer is fast or slow.

Example.—July 12th: lat. 33° 56′ S., long. 18° 29′ E.; equal altitudes of *Antares* (α Scorpii) were observed as follows. Required the error of the chronometer on mean time at Greenwich.

	Chron. times, object E.		
	H.	M.	S.
	5	32	14
	5	33	50
	5	36	47
	3)16	42	51
E. chron. times	5	34	17
W. " "	9	40	7
	2)15	14	24
Chron. T. of *'s transit	7	37	12
G.T. " "	7	46	9
Chron. slow on M.T.G.		8	57

	Chron. times, object W.		
	H.	M.	S.
	9	42	0
	9	40	40
	9	37	41
	3)29	0	21
	9	40	7
	H.	M.	S.
(N.A.) Antares R.A.	16	22	41·6
18° 29′ long. E.	1	13	56
Gr. sid. T.	15	8	45·6
12th, Gr. sid. T. (N.A. p. II.)	7	21	19·9
· Sidereal int.	7	47	25·7
Retardation	—	1	16·6
Gr. M.T. of *'s transit	7	46	9·1

Examples for Practice

Example 1.—August 27th: in lat. 32° 3′ S., long. 115° 46′ E.; the following times by chronometer were noted when the sun had equal altitudes—

At the a.m. sights.			At the p.m. sights.		
H.	M.	S.	H.	M.	S.
2	0	0	6	31	20
2	8	20	6	23	0
2	14	31	6	16	49

Required the equation of equal altitudes; also the error of the chronometer on mean time at place, and on mean time at Greenwich.

Ans. Equation of equal altitudes—10·99s. Chronometer fast on M.T. at place 4h. 14m. 2s. Chronometer slow on M.T. Gr. 2m. 54s.

Example 2.—January 12th: in lat. 20° 10′ S., long. 57° 32′ E.; the following times by chronometer were noted when the sun had equal altitudes—

At the a.m. sights.			At the p.m. sights.		
H.	M.	S.	H.	M.	S.
6	8	2	10	11	4
6	38	25	9	40	50
7	7	41	9	11	51

Required the equation of equal altitudes; also the error of the chronometer on mean time at place, and on mean time at Greenwich.

Ans. No equation of equal altitudes. Chronometer slow on M.T. at place 3h. 58m. 55s. Chronometer slow on M.T. Gr. 8m. 47s.

Example 3.—April 13th : in lat. 30° 25′ N., long. 81° 25′ W. ; the following times by chronometer were noted when the sun had equal altitudes—

At the a.m. sights.			At the p.m. sights.		
H.	M.	S.	H.	M.	S.
2	42	50	8	18	50
2	44	20	8	17	0
2	46	47	8	15	49

Required the equation of equal altitudes ; also the error of the chronometer on mean time at place, and on mean time at Greenwich.

Ans. Equation of equal altitudes—7s. Chronometer fast on M.T. at place 5h. 30m. 23s. Chronometer fast on M.T. Gr. 4m. 43s.

NOTE.—Owing to the facility with which Greenwich mean time is obtained in the principal ports of the world, the method of determining the error of the chronometer by equal altitudes is almost obsolete in the Merchant Service ; but it is still an essential problem in a surveying vessel.

GREAT CIRCLE SAILING

On the sphere the shortest distance between two places is the arc of a great circle intercepted between them, as may be readily tested experimentally by stretching a thread evenly between any two places selected at random on the surface of a terrestrial globe. Hence *Great Circle Sailing* is the method of finding what places a ship must go through, and what courses she must steer, that her track may be on the arc of a great circle (or nearly so) passing through the place sailed from and that bound to. This was well understood by the old navigators, who continually practised this method, and especially before the introduction of Mercator's Chart. Thus John Davis at the end of the sixteenth century, in "The Secrets of the Sea," says that "Great Circle navigation is the chiefest of all the sailings, in which all the others are contained, and by them this kind of sailing is performed, continuing a course by the shortest distance between places, not limited to any one course, but by it those courses are ordered to the full perfection of this rare practice, whose benefits in long voyages are to great purpose, disposing all traverses to a perfect conclusion." And again, the pilot "shall by this kind of sailing find a better and shorter course, ... so that without this knowledge I see not how courses may be ordered to their best advantage."

The fundamental theorem of what the old navigators usually called "globular" sailing is therefore this,—that the *arc of a great circle* joining two points on the surface of a sphere *is the shortest distance* between them; and on no other than on a great circle course does the ship steer for her port, heading towards it as if it were in sight.

Steamers be ng to a certain extent independent of winds and currents can take a great circle route which is impossible to sailing ships, but the latter may often shorten the distance, when adverse winds are encountered, by taking a course anywhere between the Great Circle and the Rhumb line. When the places are widely separated, as in high southern latitudes, a great circle course is impossible to steamer and sailing ship alike, but advantage may be taken of a *composite route*, formed by sailing partly on a great circle and partly on a parallel.

Windward Great Circle Sailing.—On this subject Towson's observations are trite and to the point.

"When a ship cannot (on account of adverse winds) sail directly to her port, she obviously ought to be put on that tack by which she nears her port by the greatest proportion of the distance sailed. It is also evident that she must do this when her track deviates by the least amount from the direct line which connects her with her destination, or, in other words, when she is put on that tack which deviates less from the true course than the other tack. In adopting this rule, it must, however, be especially borne in mind that the *true course* alone can serve as a guide in choosing the tack; and that the Great Circle, and not the Rhumb, is the *true course*. But, since the mariner is more conversant with the Rhumb than the Great Circle, too much attention cannot be directed to the importance of making this

distinction between these two courses in connection with windward sailing. In crossing the Pacific, the Rhumb course frequently deviates four points from the true course; under such circumstances it is impossible that the mariner can navigate his vessel with advantage, if he fail to make himself acquainted with the Great Circle course.

"The term 'Windward Great Circle Sailing,' is employed with special reference to these facts. This new form of describing the application of the *true course* is rendered necessary on account of the prevalent erroneous opinions—that 'to a sailing vessel, Great Circle Sailing is of comparatively little value;' and that 'steamers, being in a measure independent of the winds, could, more readily than sailing vessels, avail themselves of the advantages of Great Circle Sailing.' The reverse is the fact: to a sailing vessel the advantage of being guided by the true course, when contending with adverse winds, is fourfold as great as that which is conferred on a steamer. Thus, for example, the increase of distance arising from the direct track being diverted two points is only one mile in 12; but, if a ship that sails six points from the wind deviate two points further from the angle of the true position of her port on account of the wrong course being employed, she cannot in the least degree near her port, whilst, under the same circumstances, the knowledge of the true course would enable the mariner so to choose his track as to make good 8½ miles by a run of 12 miles.

"The rule for Windward Great Circle Sailing is as follows:—Ascertain the Great Circle course, and put the ship on that tack which is the nearer to the Great Circle course."

Composite Great Circle Sailing.—When the track shows that the Great Circle would take the ship into too high a latitude, a *maximum* latitude is selected, and two great circles are drawn touching the parallel of the *maximum* latitude chosen, the one through the point of departure, the other through the point of destination. The ship first sails on the great circle from the point of departure until she reaches the selected parallel of *maximum* latitude, then she sails along this parallel until she reaches the point where the second great circle drawn from the point of destination touches the *maximum* latitude, and finally along the second great circle to their destination.

All the computations in Great Circle Sailing are effected through the methods of spherical trigonometry. Into these it is presently intended to enter, but it may be well to indicate a few of the principal terms connected therewith. The equator, which is a great circle, bisects every other great circle on the earth's surface, and there must necessarily be two points in every such circle equidistant from the equator and at the same time farthest removed from it; in Great Circle Sailing each of these points is called the *Vertex;* and the *Latitude of Vertex*, which is the highest latitude attained in sailing on a great circle, is the nearest approach to the elevated pole. The meridian cutting the Great Circle at the vertex is the *Meridian of Vertex;* and the *Longitude from Vertex* is the arc of the equator intercepted between the meridian of any place and the meridian of vertex.

The Great Circle and the Rhumb line differ most widely from each other in high latitudes and between places on nearly the same parallel. When the two places are on opposite sides of the equator, the Great Circle and the Rhumb line intersect each other, and the difference between them is not so perceptible.

Bear in mind that every point of a Great Circle course lies in a higher latitude than any point having the same longitude on the Rhumb line; and a course taken anywhere between the Great Circle Course and the Rhumb line will be attended with some saving of distance as compared with the Rhumb.

Mercator's Sailing answers every purpose for short voyages, or within the tropical regions; but in the present day of long voyages and great competition, much time and distance can be saved by resorting to the *Great Circle* track, or to a compromise between it and the Rhumb track, called a *Composite* track.

The great obstacle which once existed against the practice of Great Circle Sailing, viz., the determination of the longitude, a necessary element in the calculations, no longer exists. Again, the great labour of determining various points and pricking them off on a Mercator's Chart, drawing through them a freehand curve, is obviated by the use of charts upon which all great circles are represented as straight lines; hence, as with a Rhumb line on the Mercator's Chart, the entire track may be seen and obstacles thereby avoided.

Hence, there are two advantages in favour of the Great Circle track over the Rhumb track: the *difference of distance* and the great ratio in which it increases in high latitudes and between places separated by many degrees of longitude; and the *difference of time* saved on a voyage by a proper application of the principles of the great circle.

Definitions connected with Great Circle Sailing

I. *Circles* on the sphere or globe are of two kinds, great and small.

II. A great circle divides the sphere into two equal parts or hemispheres, its plane passing through the centre of the earth. The equator and meridians are great circles, but an infinite number of such circles can be projected on the sphere.

III. A small circle is one which divides the sphere into two unequal parts, its plane not passing through the centre of the earth, such as a parallel of latitude.

IV. A *spherical arc* is any portion of the circumference of a great circle.

V. A *spherical angle* is formed by the intersection of two great circles, and is measured by the plane angle which measures the inclination of the planes of the containing arcs.

VI. A *spherical triangle* is a portion of the sphere's surface included by three arcs of different great circles; and each of these arcs is less than a semi-circumference, or semi-circle.

The *data required* in the computation are the latitudes and longitudes of the two places, and the selection of one of the poles of the earth; preferentially, and for convenience, take the pole nearest to the place in the higher latitude. The spherical triangle is formed by projecting the meridians passing through the two places, and then joining the two places by a great circle: thus the co-latitudes (or two polar distances) form the two sides, and the difference of longitude the included (or polar) angle.

VII. Any two points upon the surface of a sphere must be situated upon the arc of a great circle; and this arc is the shortest distance between the two places.

The shortest distance between two places would therefore not be that

represented upon a Mercator's chart by the rhumb, or straight line, but rather by that represented on a globe by a fine line of silk thread stretched from point to point upon its surface.

Or, more correctly, by means of a terrestrial globe, especially one of 12 inches diameter, the shortest distance, and at the same time all the other elements of the great circle track would be found by bringing both places to coincide with the upper edge of the wooden horizon, which itself represents a great circle of the sphere.

The elements which would then appear would be—

1. The *distance* between the two places in degrees and minutes of arc.
2. The *angles of position* at the two places.
3. The *latitude* of the highest point of the circle, called the *vertex.*
4. The *longitude* of the meridian *of vertex.*
5. A succession of points on the arc, few or many, according to the choice of the navigator.
6. The course and distance from point to point, successively.

VIII. The *angle of position* is the angle *at the place* included between the plane of the great circle and the plane of the meridian, and shows the angular position of that place from any other place through which the great circle passes. Two such angles are found, and they are equivalent to the *initial and terminal courses* at the two places.

IX. The *distance* is the arc of the great circle between the two places; and being the *shortest* distance is the track on which the ship should be steered so as to head directly towards her port.

X. The *vertex* is that point in a great circle which is farthest from the equator. There are two such *vertices* in every great circle, one in the northern and one in the southern hemisphere; they mark in each hemisphere the points of greatest separation from the equator, which bisects every other great circle on the earth's surface. The arc intercepted between the vertex and the equator is called the *latitude of vertex;* the meridian that passes through the vertex is the *meridian of vertex;* and the arc of the equator contained between the *meridian of vertex* and the meridian of any place on the great circle is named the *longitude from vertex.*

The vertex may or may not fall between the two places; but if the two places are on a parallel of latitude, the vertex will be midway between them.

The meridian of the vertex always intersects the great circle at right angles, and, with the equator, divides a great circle into quadrants; and in each of these quadrants the elements are the same; that is, the latitudes, courses, and distances, corresponding to each degree of longitude from the vertex in one quadrant, truly represent those for the corresponding degree in each quadrant belonging to the same great circle.

XI. The angle at which the great circle crosses successive meridians is constantly altering, therefore, it becomes necessary to calculate, at recurring intervals, the approximate course and distance from point to point along the great circle.

There are several methods of determining the various parts of a spherical triangle, in which the fundamental *data* are the two sides and the included angle; in the method here given the following order is adopted as most direct—arising out of the two co-latitudes and difference of longitude.

1. The angles of position at the two places, or the initial and terminal courses;
2. The distance between the two places;
3. The position of the vertex, in latitude and longitude; and
4. A succession of points on the arc of the great circle, with the course and distance from point to point.

The following diagrams will illustrate the various elements and definitions.—In Fig. 1, let A P B be a spherical triangle in the *northern* hemisphere, the latitudes of A and B are known, A being in the higher latitude; the longitudes of A and B are also known. Let P A be the arc of the meridian passing through A, then P A is the polar distance, or co-latitude of A (or 90° — Latitude of A); and if P B be the arc of the meridian passing through B, then P B is the polar distance, or co-latitude of B (or 90° — Latitude of B); these are the two sides of the triangle; and the angle A P B, equal to the difference of longitude between A and B, is the angle *included* between the two sides P A and P B, and sometimes called the *polar* angle.

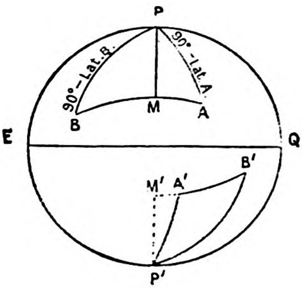


Fig. 1.

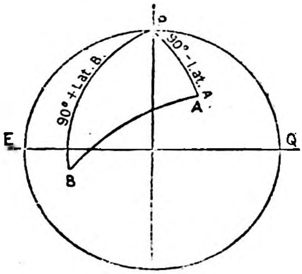


Fig. 2.

The third side of the triangle, which has to be found, is A B; it is the arc of the great circle passing through A and B, and is also the *distance*.

P A B is the angle of position at A, and initial course from A to B.
P B A is the angle of position at B, and initial course from B to A.
M is the *vertex*, or highest latitude on the great circle of which A B is an arc.
P M is the arc of the meridian extending through M, and is hence the *meridian of vertex*, reckoned from Greenwich—in this case falling between A and B.
A P M is the difference of longitude between A and M, and is hence the *longitude* of A *from vertex*.

In the lower part of Fig. 1 all the elements and definitions just noted are given, with the same letters *dashed*, but M′ is outside the arc of the great circle A′ and B′.

In Fig. 2, A being a place in N. latitude, and B in S. latitude, A's

co-latitude will be 90° — latitude of A, and B's co-latitude = 90° + latitude of B, reckoned from the pole of higher latitude.

When the two places lie on the same meridian their difference of latitude will be the arc of the meridian between them, and the position from one place to the other will be directly north or south.

When the two places are on the equator the distance between them is equal to their difference of longitude, and the position (or course) of one from the other will be due east or west.

General Rule to find the Angles of Position, or Course from A *to* B *and from* B *to* A

Reckon both co-latitudes from the pole NEARER *to* A.—Find half the sum, and half the difference of the co-latitudes; also find half the difference of longitude between A and B.

To find half sum of the angles at A and B.—Add together the L co-tangent of half the difference of longitude, the L secant of half the sum of co-latitudes, and the L cosine of half the difference of co-latitudes; the sum of the three logarithms (rejecting index 20) will be the L tangent of *half the sum of the angles* A *and* B.

To find half the difference of the angles A and B.—Add together the L co-tangent of half the difference of longitude, the L co-secant of half the sum of co-latitudes, and the L sine of half the difference of co-latitudes; the sum of the three logarithms (rejecting index 20) will be the L tangent of *half the difference of the angles* A *and* B.

The co-latitude of B being greater than that of A, the *sum* of the two angles $\frac{A+B}{2}$ and $\frac{A \sim B}{2}$ will be the first great circle course from A towards B, and their *difference* the first great circle course from B towards A.

The course is reckoned from N. if A is in N. latitude, but from S. if in S. latitude; also if the sum of $\frac{A+B}{2}$ and $\frac{A \sim B}{2}$ exceeds 90°, subtract it from 180°, changing N. to S. or S. to N.

Formulæ—(see Fig. 1)

$$\text{Tan.}\ \frac{A+B}{2} = \text{cot.}\ \frac{P}{2} \times \text{sec.}\ \frac{BP+AP}{2} \times \text{cos.}\ \frac{BP-AP}{2}$$

$$\text{Tan.}\ \frac{A \sim B}{2} = \text{cot.}\ \frac{P}{2} \times \text{co-sec.}\ \frac{BP+AP}{2} \times \text{sin.}\ \frac{BP-AP}{2}$$

To find the Distance

Add together the L cosine of half the sum of co-latitudes, the L secant of ($\frac{1}{2}$ sum of angles A and B), and the L sine of half the difference of longitude; the sum of the three logarithms (rejecting index 20) will be the L cosine of *half* the distance in arc, which take out, multiply by 2, and convert into nautical miles.

Formulæ— $$\text{Cos.}\ \frac{AB}{2} = \text{cos.}\ \frac{BP+AP}{2} \times \text{sec.}\ \frac{A+B}{2} \times \text{sin.}\ \frac{P}{2}$$

To know whether the Vertex falls within or without the triangle

If the angles A and B are both greater or both less than 90°, the meridian of vertex falls within the triangle, but if one angle is greater and the other less than 90° the meridian of vertex falls without the triangle.

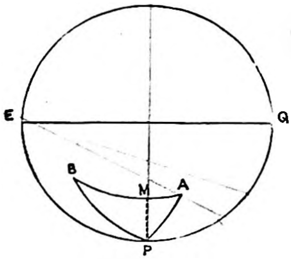


Fig. 3.

Example.—Find by Great Circle Sailing the first course from lat. 37° 31′ S. long. 178° 1′ E. to lat. 55° 59′ S. long. 67° 16′ W.; find also the distance, the latitude and longitude of the vertex, and of a succession of points on the great circle differing 10° in longitude commencing with the 180th meridian.

Lat. B 37° 31′ S.	co-lat.	52° 29′	(B P)	Long.	178° 1′ E.	
Lat. A 55 59 S.	„	34 1	(A P)	„	67 16 W	
	Sum	86 30			245 17	
	Diff.	18 28			114 43	Diff. long. $= P$

$\frac{BP + AP}{2}$ or ½ sum = 43° 15′ 57° 21½′ = ½ d. long. $= \frac{P}{2}$

$\frac{BP - AP}{2}$ or ½ diff. = 9° 14′

For the First Course

$$\text{Tan.}\ \frac{A+B}{2} = \text{cot.}\ \frac{P}{2} \times \text{sec.}\ \frac{BP+AP}{2} \times \text{cos.}\ \frac{BP-AP}{2}$$

$$\text{Tan.}\ \frac{A \sim B}{2} = \text{cot.}\ \frac{P}{2} \times \text{cosec.}\ \frac{BP+AP}{2} \times \text{sin.}\ \frac{BP+AP}{2}$$

$\frac{P}{2}$	57° 21½′	Cot.	9·806554	Cot.	9·806554
$\frac{BP+AP}{2}$	43° 15′	Sec.	10·137647	Cosec.	10·164193
$\frac{BP-AP}{2}$	9° 14′	Cos.	9·994336	Sin.	9·205354
$\frac{A+B}{2}$	40° 57′ 33″	Tan.	9·938537 $\frac{A \sim B}{2} = 8° 31′ 51″$	Tan.	9·176101
$\frac{A \sim B}{2}$	8 31 51				
∠B, or 1st Course	S. 32 25 42 E.				

The diff. is taken because AP *is less than* BP.

For the Distance

$$\text{Cos.}\ \frac{AB}{2} = \cos.\ \frac{BP + AP}{2} \times \sec.\ \frac{A+B}{2} \times \sin.\ \frac{P}{2}$$

$\frac{BP+AP}{2}$	43°	15′	0″	Cos.	9·862353
$\frac{A+B}{2}$	40	57	33	Sec.	10·121951
$\frac{P}{2}$	57	21	30	Sin.	9·925343
$\frac{AB}{2} =$	35	41	30	Cos.	9·909647
			2		
A B =	71	23	0		
	60				
Dist.	4283 miles.				

The remaining parts of the "Great Circle" problem are solved by means of Napier's rules for "Circular parts" as follows—

To find Latitude of Vertex

In the spherical triangle B P M right-angled at M (Fig. 3) given ∠B and side B P to find P M, and its complement equal to the lat. of M—

$$\text{Sin. } PM = \sin.\ \angle B \times \sin.\ PB$$

∠B	32°	25′	42″	sin	9·729363
P B	52	29	00	sin	9·899370
P M =	25	10	19	sin	9·628733
Latitude of M =	64°	49′	41″ S.		

To find Longitude of Vertex from B

In the same triangle, given B P, and P M, to find ∠P

$$\text{Cos. } \angle P = \tan.\ PM \times \cot.\ BP$$

P M	25°	10′	19″	tan.	9·672067
B P	52	29	00	cot.	9·885242
∠P	68	50	54	cos.	9·557309

Long. B + ∠P = Long. vertex.

Long. B	178°	1′	0″ E.
∠P	68	50	54
Long. M	246	51	54 E.
	360	00	00
Long. of vertex M	113	8	6 W.

To find the Successive Latitudes at which the Great Circle intersects the Meridians, whose Difference of Longitude is a Given Quantity

Find the angles at the pole (difference of longitude) between the meridian of vertex (the perpendicular) and each successive meridian. Any number of degrees may be selected as the difference of longitude, or difference of each polar angle; it is usual to take 5° or 10°.

To find the latitude of the points *a*, *b*, *c*, *d*, *e*, *f*, etc., on the great circle for every 10° of long. commencing at the 180th meridian, see Fig. 4, which is an enlargement of the triangle in Fig. 3.

The angle B P M = 113° 8′ 6″ and the meridian P*a* is the 180th meridian. therefore the angle *a* P M = (180° — 113° 8′) = 66° 52′, and the angles in succession will be 56° 52′, 46° 52′, etc., until arriving at M, when the angles will be 3° 8′, 13° 8′, 23° 8′, etc., until arriving at A.

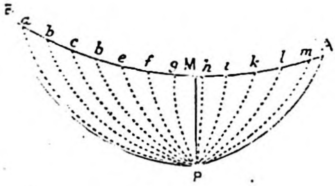


Fig. 4.

In the right-angled spherical triangle *a* P M, right-angled at M. see Fig. 4, given ∠P and side P M, to find P *a* or its complement, the latitude.

$$\text{Cos. P} = \text{tan. PM} \times \text{cot P}a$$

$$\therefore \text{Cot. P}a = \frac{\text{cos. P}}{\text{tan. PM}} \quad \text{or} \quad \text{Tan. lat. }(a) = \text{cos. P} \times \text{cot. P M.}$$

N.B.—P M is a constant for all the points to be found.

P M 25° 10′ Cot. 10·3280	Constant 10·3280	Constant 10·3280
P 66 52 Cos. 9·5943	56° 52′ Cos. 9·7377	46° 52′ Cos. 9·8349
Lat. (*a*) 39° 54′ Tan. 9·9223	Lat. (*b*) 49° 19′ Tan. 10·0657	Lat. (*c*) 55° 30′ Tan. 10·1629
Constant 10·3280	Constant 10·3280	Constant 10·3280
P 36° 52′ Cos. 9·9031	26° 52′ Cos. 9·9504	16° 52′ Cos. 9·9810
Lat. (*d*) 59° 34′ Tan. 10·2311	Lat. (*e*) 62° 13′ Tan. 10·2784	Lat. (*f*) 63° 50′ Tan. 10·3090
Constant 10·3280	Constant 10·3280	Constant 10·3280
6° 52′ Cos. 9·9969	3° 8′ Cos. 9·9994	13° 8′ Cos. 9·9885
Lat. (*g*) 64° 40½′ Tan. 10·3249	Lat. (*h*) 64° 48′ Tan. 10·3274	Lat. (*i*) 64° 14¼′ Tan. 10·3165
Constant 10·3280	Constant 10·3280	Constant 10·3280
23° 8′ Cos. 9·9636	33° 8′ Cos. 9·9229	43° 8′ Cos. 9·8632
Lat. (*k*) 62° 56′ Tan. 10·2916	Lat. (*l*) 60° 42′ Tan. 10·2509	Lat. (*m*) 57° 13½′ Tan. 10·1912

For the Positions on the Great Circle

Also the Courses and Distances from point to point, by Mercator's Sailing.

Successive Latitudes.	Successive Longitudes.	By Mercator's Sailing. Courses.	Distance
			Mile .
B 37° 31′ S.	B 178° 1′ E.		
a 39 54	*a* 180 0	B to *a* S. 33° 2′ E.	170·6
b 49 19	*b* 170 0 W.	*a* to *b* S. 37 0½ E.	707·5
c 55 30	*c* 160 0	*b* to *c* S. 44 31½ E.	520·4
d 59 34	*d* 150 0	*c* to *d* S. 52 49½ E.	403·8
e 62 13	*e* 140 0	*d* to *e* S. 61 24½ E.	332·2
f 63 51	*f* 130 0	*e* to *f* S. 70 12 E.	289·3
g 64 40	*g* 120 0	*f* to *g* S. 79 20 E.	264·7
h 64 48	*h* 110 0	*g* to *h* S. 88 11 E.	252·4
i 64 14½	*i* 100 0	*h* to *i* N. 82 35½ E.	259·8
k 62 56	*k* 90 0	*i* to *k* N. 73 34 E.	277·5
l 60 42	*l* 80 0	*k* to *l* N. 64 40 E.	313·2
m 57 13½	*m* 70 0	*l* to *m* N. 55 59 E.	373·6
A 55 59	A 67 16	*m* to A N. 50 32½ E.	116·4
By Mercator's Sailing from point to point, Distance ..			4281·4

Course and Dist. by Mer. Sailing from B to A = S. 76° 36′ E. 4781 miles.
Distance by Mer. Sailing from point to point .... 4281·4 „
Distance by Great Circle Sailing .... 4283 „

Gain by Great Circle Sailing over Mercator's Sailing 498 miles ; the loss by the point to point system is only 1·6 miles.

NOTE.—*This example is placed on the small Great Circle Chart,* p. 497.

Example.—A point (A) is in lat. 49° 50′ N., long. 5° 12′ W., a point (B) is in lat. 13° 6′ N., long. 59° 20′ W. Find by Great Circle Sailing the first course from A to B, the first course from B to A, the distance between the two points, the latitude and longitude of vertex, and a succession of points differing 10° in longitude on the great circle. Commencing at A—

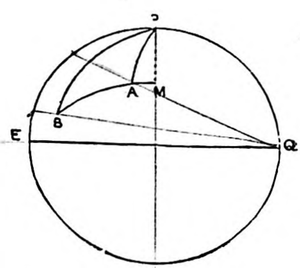


Fig. 5.

(A) Lat. 49° 50′ N. Co-lat. 40° 10′ (A P) Long. 5° 12′ W.
(B) ,, 13 6 N. ,, 76 54 (B P) Long. 59 20 W.

Sum 117 4 — 54 8 Diff. long. = P

Diff. 36 44 — 27 4 = ½d. long. = $\frac{P}{2}$

$\frac{BP + AP}{2}$ or ½ sum 58 32

$\frac{BP \sim AP}{2}$ or ½ diff. 18 22

For the Angles of Position or Courses

$$\frac{A + B}{2} \text{ tan.} = \text{cot.} \frac{P}{2} \times \text{sec.} \frac{BP + AP}{2} \times \text{cos.} \frac{BP - AP}{2}$$

$$\frac{A \sim B}{2} \text{ tan.} = \text{cot.} \frac{P}{2} \times \text{cosec.} \frac{BP + AP}{2} \times \text{sin.} \frac{BP - AP}{2}$$

$$\angle A = \frac{A + B}{2} + \frac{A \sim B}{2}$$

$$\angle B = \frac{A + B}{2} - \frac{A \sim B}{2}$$

$\frac{P}{2}$	27° 4′	Cot. 10·291586	Cot. 10·291586
$\frac{BP + AP}{2}$	58 32	Sec. 10·282327	Cosec. 10·069079
$\frac{BP - AP}{2}$	18 22	Cos. 9·977293	Sin. 9·498444
$\frac{A + B}{2}$	74° 18′ 6″	Tan. 10·551206	$\frac{A \sim B}{2}$ 35° 51′ 54″ .. Tan. 9·859109
$\frac{A - B}{2}$	35° 51′ 54″		

∠ A 110° 10′ ∴ 1st Course from A to B = N. 110° 10′ W. or S. 69° 50′ W.
∠ B 38° 26′ 12″ ∴ 1st Course from B to A = N. 38° 26′ 12″ E.

For the Distance

$$\text{Cos.} \frac{AB}{2} = \text{cos.} \frac{BP + AP}{2} \times \text{sec.} \frac{A + B}{2} \times \text{sin.} \frac{P}{2}$$

$\frac{BP + AP}{2}$	58° 32′ 0″	Cos. 9·717673
$\frac{A + B}{2}$	74 18 6	Sec. 10·567716
$\frac{P}{2}$	27 4 0	Sin. 9·658037
$\frac{AB}{2}$	28 36 52	Cos. 9·943426
	2	
AB =	57 13 44	
	60	

Distance 3433·7 in nautical miles.

To find the Latitude of M, the Vertex

In the spherical triangle P B M right-angled at M, Fig. 5, given ∠ B and side P B, to find P M and its complement the latitude of M.

Sin. P M and cos. lat. M = sin. P B × sin. B

∠ B 38° 26′ 12″	............sin.	9·793546
P B 76 54 0	............sin.	9·988548
Sin. P M and cos. lat. M		9·782094

P M 37° 15′ 44″; Lat. M 52° 44′ 16″ N.

To find the Longitude of Vertex

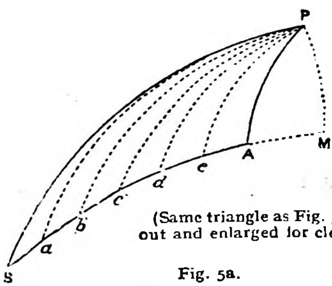
In the same triangle, given P B and P M, to find ∠ P.

Cos. P = tan. P M × cot. P B

P M 37° 15′ 44″	............tan.	9·881244
P B 76 54 00	............cot.	9·366810
∠ P = 79 48 11	............cos.	9·248054

Long. of B	59° 20′ W.
∠ P = D. long. between B and vertex	79 48 E.
Long. of vertex	20 28 E.

To find the Latitudes of the Points e, d, c, b, a, on the Great Circle for every 10° of Longitude reckoned from A



(Same triangle as Fig. 5, taken out and enlarged for clearness.)

Fig. 5a.

To find the angle at P—

Long. M = 20° 28′ E.	20° 28′ E.	20° 28′ E.	20° 28′ E.	20° 28′ E.
Long. (*e*) = 15 12 W.	(*d*) 25 12 W.	(*c*) 35 12 W.	(*b*) 45 12 W.	(*a*) 55 12 W.
P = 35 40	P = 45 40	P = 55 40	P = 65 40	P = 75 40

In the right-angled spherical triangle *e* P M right-angled at M, Fig. 5a, given ∠ P and side P M to find P *e*, or its complement the latitude.

$$\text{Cos. P} = \text{tan. P M} \times \text{cot. P}e$$

$$\therefore \text{Cot. P}e = \frac{\text{cos. P}}{\text{tan. P M}}, \text{ or}$$

$$\text{Tan. lat. }(e) = \text{cos. P} \times \text{cot. P M.}$$

N.B.—P M is a constant as before.

P M 37° 16′	cot. 10·1187		cot. 10·1187		cot. 10·1187
P 35 40	cos. 9·9098	45° 40′	cos. 9·8444	55° 40′	cos. 9·7513
Lat. (e) 46 53 N.	tan. 10·0285	(d) 42 34 N.	tan. 9·9631	(c) 36 33 N.	tan. 9·8700

P M 37° 16′	cot. 10·1187		cot. 10·1187
65° 40′	cos. 9·6147	75° 40′	cos. 9·3937
Lat. (b) 28 26 N.	tan. 9·7336	(a) 18 2 N.	tan. 9·5124

In the selection of certain points on the great circle, the smaller the alterations in longitude are taken, the nearer will this method approach the truth; but it is sufficient at all times to compute to every 5° of longitude, the length of an arc of 5° differing but little from its chord or tangent.

The Positions on the Great Circle

Also the Courses and Distances from point to point, by Mercator's Sailing.

Polar Angle from Vertex.	Successive Latitudes.	Successive Longitudes.	By Mercator's Sailing. Courses.	By Mercator's Sailing. Distances. Miles.
25° 40′	(A) 49° 50′	(A) 5° 12′		
35 40	(e) 46 52	(e) 15 12	(A to e) S. 66° 1′ W.	436·5
45 40	(d) 42 34	(d) 25 12	(e to d) 58 46	498·4
55 40	(c) 36 33	(c) 35 12	(d to c) 51 59	586·2
65 40	(b) 28 26	(b) 45 12	(c to b) 46 5	702·0
75 40	(a) 18 2	(a) 55 12	(b to a) 41 21	832·5
79 48	(B) 13 6	(B) 59 20	(a to B) S. 39 2 W.	379·8
By Mercator's Sailing, from point to point, Distance....3435·4				

Course and Dist. by Mer. Sailing from A to B = S. 50° 37′ W. 3474 miles.
Distance by Mer. Sailing, from point to point.............. 3435·4
Distance by Great Circle Sailing........................... 3433·7

Gain by Great Circle over Mercator's Sailing 40·3 miles; the loss by the point to point system is only 1·7 miles.

If we wish to know only the Great Circle distance between two places, this can be found without first obtaining the *angles of position*.

To find the Distance.—Find the two co-latitudes and the difference of longitude in the usual way and then proceed as in the following example using the data in Example 1.

Formula—

Log. hav. θ = log. hav. P + log. sin. B P + log. sin. A P — 20
where P = diff. long., B P = co-lat. B, and A P = co-lat. A
· Nat. hav. dist. = nat. hav. θ + nat. hav. (B P ∼ A P)
where P = D. long. and B P and A P the two co-latitudes

P 114° 43′	L hav.	9·85069		
B P 52 29	L sin.	9·89937		
A P 34 1	L sin.	9·74775		
	hav.	9·49781	nat. hav.	·31464
B P ∼ A P 18° 28′	. . . .		nat. hav.	·02575
71° 23′	. . . .		nat. hav.	·34039
60				
Distance 4283 miles				

N.B.—The Natural Haversines have been included in Norie's Tables, as by their use many calculations are much shortened ; especially is this the case when two sides and the included angle are given to find the third side. θ need not be taken out, but only the nat. hav. corresponding to the log. hav. θ, as shown above.

COMPOSITE GREAT CIRCLE SAILING

Find the initial course on a composite track from A in lat. 40° S., long. 20° E., to B in lat. 35° S. and long. 135° E., the ship not to sail in a higher latitude than 45° S. Find also the distance from A to B on this track and the longitudes at which the ship arrives at and leaves the parallel of 45° S.

To find the Initial Course

In the right-angled spherical triangle P V A, right-angled at V Fig. 6, given P A and P V, to find angle A.

$$\text{Sin. P V} = \text{sin. A} \times \text{sin. P A} \;\therefore\; \text{sin. A} = \frac{\text{sin. P V}}{\text{sin. P A}}$$

P V	45° 0′ 0″	sin. 9·849485		
P A	50 0 0	sin. 9·884254	Initial course S. 67° 22′ 40″ E.	
A	67 22 40	sin. 9·965231		

9·849485
10·115746
9·965231

To find the Longitude of V

In the same triangle, given P A and P V, to find ∠ P.

Cos. P = cot. P A × tan. P V, where P A = 90°— lat. A.

P A	50° 0′ 0″	cot.	9·923814
P V	45 0 0	tan.	10·000000
P	32 57 17	cos.	9·923814

Long. V = Long. B + ∠ P

Long. B	20°	0′	0″	E.
∠ P	32	57	17	
Long. V	52	57	17	E.

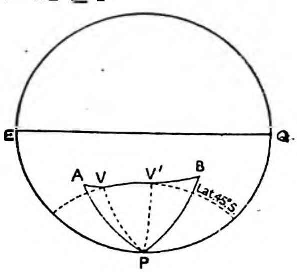


Fig. 6.

To find the Longitude of V'

In the right-angled triangle P V' B right-angled at V' Fig. 6, given P V' and P B, to find ∠ P.

Cos. P = tan. P V' × cot. P B, where P B = 90° — lat. B.

P V	45°	0'	0"	tan.	10·000000
P B	55	0	0	cot.	9·845227
∠ P	45	33	30	cos.	9·845227

Longitude V' = longitude B — ∠ P.

Long. B	135°	0'	0"	E.
∠ P	45	33	30	
Long. V'	89	26	30	E.

To find the Distance on first Great Circle

In the spherical triangle A P V (Fig. 6), given ∠ P and side P V, to find V A.

$$\text{Sin. } PV = \tan. VA \times \cot. \angle P \therefore \tan. VA = \frac{\sin. PV}{\cot. \angle P}$$

P V	45°	0'	0"	sin.	9·849485
∠ P	32	57	17	cot.	10·188292
V A =	24	37	27	tan.	9·661193
	60				
	1477·45				

Distance on first Great Circle, 1477·45 miles.

To find the Distance run on Parallel of Maximum Latitude, given the Difference of Longitude between V and V'

Dist. = d. long. × cos. max. lat.

Long. V	52°	57'	17"
Long. V'	89	26	30
	36	29	13
	60		

D. long.	2189·2	Log.	3·340289
Max. lat.	45°	Cos.	9·849485
		Log.	3·189774

Distance on parallel, 1548 miles.

To find the Distance on second Great Circle

In the spherical triangle P V' B (Fig. 6), given ∠P and side P V', to find B V'.

$$\text{Sin. } PV' = \tan. BV' \times \cot. \angle P \therefore \tan. BV' = \frac{\sin. PV'}{\cot. \angle P}$$

P V'	45°	0'	0"	sin.	9·849485
∠ P	45	33	30	cot.	9·991535
B V'	35	47	30	tan.	9·857950
	60				

B V' = 2147·5 miles.

To find the whole distance add the three distances just found together—

Distance on first Great Circle	=	1477·45	miles
" " max. latitude	=	1548	"
" " second Great Circle	=	2147·5	"
Total distance	=	5172·95	"

Examples for Practice

Required the initial and final courses, the distance, and position of the vertex on the great circle between the following places; also the distance on the rhumb line. Examples 1, 2 and 3.

1. Lizard Lights, in lat. 49° 58′ N., long. 5° 12′ W., and Cape Frio, in lat. 23° 1′ S., long. 41° 58′ W.

Ans. Initial course S. 34° 1′ W.; final course S. 23° 1′ W.; distance 4,796 miles. Lat. of vertex 68° 54′ N.; long. vertex 57° 28′ E.; rhumb-line distance 4,804 miles.

2. Cape Frio, in lat. 23° 1′ S., long. 41° 58′ W., and Cape of Good Hope, in lat. 34° 21′ S., long. 18° 30′ E.

Ans. Initial course S. 63° 22′ E.; final course N. 85° 14′ E.; distance 3,208 miles; lat. vertex 34° 38′ S.; long. vertex 10° 5′ E.; rhumb-line distance 3,248 miles.

3. Otago, New Zealand, in lat. 45° 47′ S., long. 170° 45′ E., and Callao, in lat. 12° 4′ S., long. 77° 14′ W.

Ans. Initial course S. 65° 44′ E.; final course N. 40° 34′ E.; distance 5,764 miles; lat. vertex 50° 31′ S.; long. vertex 157° 5′ W.; rhumb-line distance 6,088 miles.

4. Lat. 51° 0′ N., long. 160° 0′ E., and lat. 51° 0′ N., long. 129° 0′ W.

Ans. Initial course N. 61° E.; final course S. 61° E.; distance 2,572 miles; lat. vertex 56° 36′ N.; long. vertex 164° 30′ W.; distance on rhumb-line 2,681 miles.

5. Find the first course and the distance on a composite track from A in lat. 40° S., long. 15° W. to B in lat. 42° S., long. 140° E., the maximum parallel of latitude to be 50° S.

Ans. Initial course S. 57° 2½′ E.; distance 6,380 miles.

6. Find the distance on a composite track from lat. 37° 31′ S., long. 178° 1′ E. to lat. 55° 59′ S., long. 67° 16′ W., the maximum lat. being 60° S.

Ans. Distance 4,325 miles.

Towson's "Tables to Facilitate the Practice of Great Circle Sailing" obviate the necessity of computation; and are equally useful, with or without the index chart which accompanies the Tables.

Godfray's "Chart" on the *gnomic projection* "to facilitate the practice of Great Circle Sailing," shows the Great Circle course as a straight line, and the points for transfer to Mercator's Chart can readily be taken off it. It is also accompanied by a course and distance diagram, which will give by inspection the various courses and the distance to be run on each, in such manner as to keep within an eighth of a point of the constantly-changing course of the great circle.

Where neither Towson's Tables nor Godfray's Chart is at hand the navigator can speedily determine the practicability of the Great Circle route by Sir G. B. Airy's *Method for sweeping an arc of a circle* on Mercator's Chart, which approaches very nearly to the correct projection of a Great Circle on one side of the equator; the sweep of the arc is accomplished by attending to the following precepts adapted to the Table which accompanies them.

Rule 1.—Join the two places (on the chart) by a straight line. Find its middle. Draw thence a perpendicular to that line on the side next the equator, and, if necessary, continue it beyond the equator.

2. With the middle latitude (between the two places) enter the following table, and take out the corresponding parallel.

3. The centre of the required sweep will be the intersection of this parallel with the perpendicular.

Airy's Table for delineating the Arc of a Great Circle

Middle Lat.		Corresponding Parallel.		Middle Lat.		Corresponding Parallel.	
20°		81°	13′	52°		11°	33′
22		78	16	54		6	24
24		74	59	56		1	13
26		71	26				
28	Opposit Name	67	38	58		4	0
30		63	37	60		9	15
32		59	25	62		14	32
34		55	5	64	Same Name	19	50
36		50	36	66		25	9
38		46	0	68		30	30
40		41	18	70		35	52
42		36	31	72		41	14
44		31	38	74		46	37
46		26	42	76		52	1
48		21	42	78		57	25
50		16	39	80		62	51

From the nature of the problem a tentative method does not apply in all cases, nevertheless facility in delineating the Great Circle on the chart has this advantage: when, from adverse winds or other causes, a vessel has to deviate from the track, the Great Circle may at any moment be struck off anew, such that it shall pass through the *actual* position and the port to which the vessel is bound.

On p. 497 a chart on the Gnomonic projection is shown. In this projection we suppose the eye to be at the centre of the earth, and all the circles drawn on the earth are projected on a tangent plane which touches the earth's surface at the pole. The pole will therefore appear as the centre of the projection, and all the meridians as straight lines from the pole, making the same angles with each other as they do on the earth. All other great circles will also appear as straight lines. The parallels of latitude will appear as concentric circles, the pole being their common centre. As in Mercator's projection the pole cannot be represented, so in the gnomonic projection the equator cannot be included. The outlines of the land will therefore be very much distorted in low latitudes, and hence, when projecting the parallels of latitude, it is not advisable to use a lower latitude than 15° or 20°. This will not matter in practice, for there is very little advantage in Great Circle Sailing within the tropics.

The construction of the chart is exceedingly simple, and, when constructed. latitudes and longitudes can be laid down with the same ease as on a Mercator's chart. Then if a straight line be drawn from any one position to any other position, this line will show the Great Circle track from one to the other. Every important detail of the track can thus be ascertained at sight. It will show whether the track would take the ship into a latitude too high for practical navigation, or whether any land or other obstruction was in the way. It will show the latitude and longitude of the vertex, and the latitudes of all points in which it cuts the different meridians, and hence the various courses and distances can be quickly ascertained. Should one of the positions be in N. latitude and the other in S. latitude, the whole of the Great Circle track cannot be drawn. In this case the longitude of the point you would like to cross the lowest parallel of latitude should be joined to the point of departure ; this will give the Great Circle track in the one hemisphere (say N.) Then choose the longitude you would like to cross the same parallel in the other hemisphere (that is S.) and join this point to the point of destination ; this will be the Great Circle track in the other hemisphere. (*Caution.*—Remember that the chart represents N. latitudes at one time and S. latitudes at the other time.) The course from the one point on the lowest parallel to the other point will be on the *Rhumb line* between them.

The chart is very useful in Composite Great Circle Sailing. When the track shows that the Great Circle would take the ship into too high a latitude, a *maximum* latitude is selected, and two great circles are drawn touching the parallel of the *maximum* latitude chosen, the one through the point of departure, the other through the point of destination. The ship first sails on the Great Circle through the point of departure until she reaches the selected parallel of *maximum* latitude ; then she sails along this parallel until she reaches the Great Circle through the point of destination, and finally along the last-mentioned Great Circle, until she is at her point of destination. On the chart this is easily constructed. From each position draw a straight line touching the *maximum* parallel of latitude chosen. A glance will show the longitudes on reaching and leaving the parallel, the important points in Composite Great Circle Sailing.

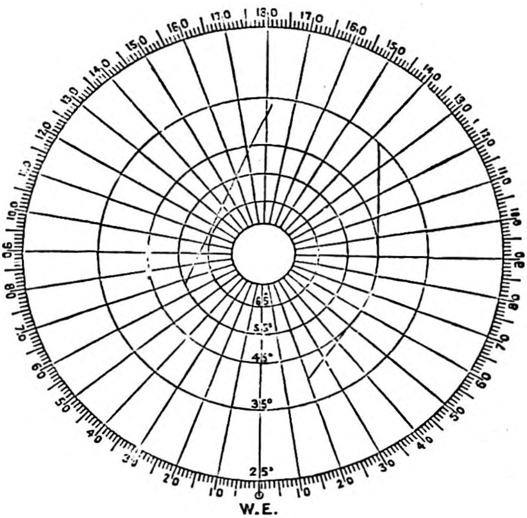
We will now show how anyone can construct a chart for himself. The only formula required is that for the radii of the circles representing the parallels of latitude. This is — $r \times$ cot. lat., where r is any constant multiplier.

The parallels are now projected in the following manner : Fix on the parallel of lowest latitude, and enter the accompanying table with this latitude and take out the corresponding number. Then choose the value of r so that when it is multiplied by the number from the table the result shall be such that a circle described with it as radius will be easily included in the paper selected for the chart. The centre of this circle will be the pole, and all circles must be described with this point as centre. To draw any other parallel of latitude : take from the table the number corresponding to the degree of latitude and multiply it by the same value of r that was used for the first circle drawn ; this product is the value of the radius of the parallel. In the same way as many parallels as may be considered necessary can be drawn. Any straight line drawn through the poles so as to make a diameter of the various circles may be considered as the

meridians of 0° and 180°, and if the semicircular arcs of the largest circle on each side of this meridian be divided into 18 equal parts, and lines be drawn from the centre to the points of division, these lines will represent meridians 10° apart, and the longitudes E. and W. 10°, 20°, 30°, etc., can be marked on this outer circle. With a sufficiently large circle these divisions may easily be divided into single degrees.

The following table gives the co-tangent for every 5° from 10° to 80°. *The co-tangents are *natural co-tangents* and may be found by taking the L co-tangent from Table of Log. Sines, etc. (rejecting 10 from the index) and finding from Log. table the *natural number* corresponding to it.

cot. 10° = 5·671	cot. 30° = 1·732	cot. 50° = ·839	cot. 70° = ·364
cot. 15° = 3·732	cot. 35° = 1·428	cot. 55° = ·7	cot. 75° = ·268
cot. 20° = 2·747	cot. 40° = 1·192	cot. 60° = ·577	cot. 80° = ·176
cot. 25° = 2·145	cot. 45° = 1	cot. 65° = ·466	



On the chart here given radius equals ·75 inch.

To exemplify this :—Suppose the paper to be cut square and a side to measure *one* foot, then by choosing $r = 1$ inch, the radius of the parallel of 10° will be 5·671 inches, which can be included in the paper. The radius of the parallel of 20° will be 2·747 inches, and so on. But if 20° be considered the lowest latitude, then make $r = 2$ inches, and the radius of the parallel of 20° will be $2{\cdot}747 \times 2 = 5{\cdot}494$ inches ; the radius of the parallel of 30° will be 3·464 inches, etc., etc.

* Norie's Tables contains a Table of Natural Cotangents.

NEW NAVIGATION

It is claimed for the New Navigation that it is the best method of finding a ship's position, and is equally accurate whether the observations are taken near the prime vertical or near the meridian; and in all cases it gives a more accurate *line of position* than is given by our ordinary methods when the dead reckoning differs from the true position. The method is due to Capt. Marcq St. Hilaire, of the French Navy, and is based on the difference between the altitude observed and the altitude calculated from the dead reckoning position of the ship. The computation, therefore, consists in finding the altitude by one of the methods given. The last method given has been specially proposed, as it finds not only the altitude but also the azimuth in the same computation.

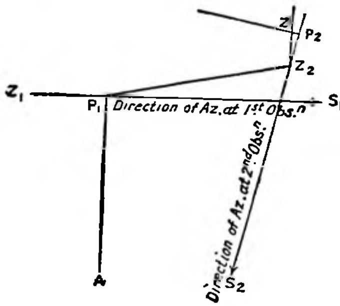
Computation and Projection Combined.—If the sun is the object observed, take from the Nautical Almanac the declination and equation of time and correct them for the Greenwich time in the usual way. If a star is the object observed take out the right ascension, the declination, and the mean sun's right ascension, correcting the last mentioned. Then compute the altitude of the object, using the latitude by dead reckoning, the declination and the hour-angle.

Find also the true altitude from the observed altitude. Take the difference between the computed altitude and the true altitude from the observation, and mark it + when the true altitude from the observation is greater than the computed altitude, but mark it — when the true altitude from the observation is less than the computed altitude. Next find the azimuth from A, B and C Azimuth Tables in Norie's Tables. Enter the Traverse Table with the azimuth as course and the difference of altitudes in the distance column and take out the difference of latitude and departure and convert the departure into difference of longitude. Apply this difference of latitude and difference of longitude to the latitude and longitude by dead reckoning in the direction of the azimuth if the difference of altitudes is marked +, but in the opposite direction if the difference of altitudes is marked —, and the result is a new point from which the course and distance in the interval must be reckoned to give the estimated position at the time of taking the second observation.

Again, take the required data from the Nautical Almanac and correct them for the Greenwich time of the second observation. Compute the altitude, using the latitude and longitude just found. Find also the true altitude from the observed altitude and take the difference between the computed true altitude and the true altitude from the observation. Mark it + when the true altitude from observation is greater than the computed true altitude, but mark it — when the true altitude from observation is less than the computed true altitude. Next find the azimuth by Burdwood's or Davis's Time Azimuth Tables; but if the object used is not within the limits of those tables, the azimuth can be found by inspection from the A, B and C Azimuth Tables in Norie's Tables.

On the chart lay down the position given by the dead reckoning at the time of taking the first observation, and from it draw a line in the direction of the azimuth. Set off on this line from the dead reckoning position a distance equal to the difference of altitudes, in the direction of the azimuth if the difference of altitudes is marked +, but in the opposite direction if marked —, and through the point thus obtained draw a line at right angles to the direction of the azimuth; this line is the *first line of position*. Next, from this latter point (not the dead reckoning position) set off the true course and the distance made good in the interval between the observations, thus giving an approximate position at the time of the second observation. From this point set off the azimuth and the difference of altitudes at the second observation, in the direction of the azimuth if it is marked +, but in the opposite direction if it is marked —, and through the point thus obtained draw a line at right angles to the direction of the azimuth; this is the *second line of position*. Lastly through the approximate position at the time of the second observation draw a line parallel to the *first line of position*; the intersection of this line with the *second line of position* gives the true position of the ship.

The accompanying sketch will show this plainly. Let Z_1 be the dead reckoning position at the first observation, $Z_1 S_1$ the direction of the azimuth. Suppose the difference of altitudes to be + 20′. Set off 20′ from Z_1 in the direction of S_1 because the sign is +; let this be $Z_1 P_1$. From P_1 draw $P_1 A$ at right angles to $Z_1 S_1$, $P_1 A$ is the *first line of position*. From P_1 draw $P_1 Z_2$ to represent the course and distance in the interval, thus placing the ship at Z_2 when the second observation was taken. From Z_2 draw $Z_2 S_2$ the direction of the azimuth. Suppose the difference of altitudes to be — 10′. Set off 10′ from Z_2 in the opposite direction to S_2 because the sign is —, that is on $S_2 Z_2$ produced, let this be $Z_2 P_2$. From P_2 draw $P_2 Z$ at right angles to $Z_2 S_2$, $P_2 Z$ is the *second line of position*. Lastly through Z_2 draw $Z_2 Z$ parallel to $P_1 A$, *the first line of position*, and its intersection with the *second line of position* at Z is the true position of the ship.



By calculation.—To obtain the position by calculation, it is necessary to reduce the altitude taken at one position to what it would have been if it had been taken at the other position, by the rule on pp. 259-61. In this problem it will be more convenient to *reduce the altitude taken at the second observation*, and thus determine the ship's position at the time of taking the first observation; the position at the second observation being then found by using the course and distance in the ordinary way. By doing this, all the calculations belonging to the first observation can be made during the interval, and a *line of position* found if considered necessary.

Take from the Nautical Almanac the required data and correct them for the Greenwich time at the first observation. For the sun, take the declination and equation of time; for a star, the right ascension, the declination, and the mean sun's right ascension. Compute the altitude of the object, using the latitude and longitude by dead reckoning. Find also the true altitude from the observed altitude, and take the difference between the computed altitude and the true altitude from the observation. Mark it + when the true altitude from the observation is greater than the computed altitude, but mark it — when the true altitude from the observation is less than the computed altitude. Next find the azimuth from the A, B and C Azimuth Tables.

For simplicity call the difference of altitudes d_1.

Again take the required data from the Nautical Almanac and correct them for the Greenwich time at the second observation. Compute the altitude of the object, *using the latitude and longitude by dead reckoning at the first position.* Find the azimuth from A, B and C Azimuth Tables. Find the true altitude from the observed altitude and apply the correction to reduce it to what it would have been if taken at the first position. Take the difference between the reduced true altitude and the computed altitude, and mark it + when the reduced true altitude is greater than the computed altitude, but mark it — when the reduced true altitude is less than the computed altitude.

For simplicity call the difference of altitudes d_2.

Enter the Traverse Table with the angle between the directions on which d_1 and d_2 are measured, that is, the angle between the azimuths (remembering that a — sign represents a direction opposite to the azimuth), as course and d_2 in the departure column, take out the distance and call it a. Enter again with d_1 in the departure column, and take out the difference of latitude and call it b. If the angle is less than 90° subtract b from a, which is $(a - b)$; but if the angle is more than 90° add b to a, which is $(a + b)$.

Again enter the Traverse Table with the azimuth on which d_1 is measured as course and d_2 in the distance column, take out the difference of latitude and departure and mark them according to the direction of the line. Enter also with the complement of the azimuth as course and with $(a - b)$ or $(a + b)$ in the distance column, and take out the difference of latitude and departure, placing them underneath those first taken out, and name them with the names at right angles to the azimuth used. (The line at right angles to the azimuth trends both northerly and southerly; it is necessary to discover which of the directions must be taken. If $(a - b)$ is positive, that is, b can be subtracted from a in the ordinary way, that direction must be taken which would make an acute angle with the direction of d_2. But if $(a - b)$ is negative, that is, a has to be subtracted from b, then that direction must be taken which would make an obtuse angle with the direction of d_2; $(a + b)$ is always positive. Add the differences of latitude together if of

the same name, but subtract them if of different names, marking the sum or difference with the name of the greater. Also add or subtract the departures by the same rule. This gives the total difference of latitude and departure. Convert the departure into difference of longitude and apply the difference of latitude and difference of longitude to the latitude and longitude by dead reckoning; the result is the position of the ship at the first observation.

To exemplify this, take the Example on p.445.

Alt. of Regulus computed with D.R.	48° 14′ 38″	Alt. of Arcturus computed with D.R.	46° 31′ 9″
True alt.	48 35 42	True alt.	46 26 36
Diff.	21 4	Diff.	4 33
or d_1 =	+ 21′·07	or d_2 =	− 4′·55
	Az. S. 33° W.		Az. S. 61° E.

Then from the rule d_1 will be reckoned in the direction S. 33° W., but d_2 will be reckoned in the opposite direction to the azimuth, that is, N. 61° W.; the angle between these is 86°. Enter the Traverse Table with 86° as course and d_2 4·55 in the departure column; the distance will be 4·56, call this a. Enter again with d_1 21·07 in the departure column; the difference of latitude will be 1·47, call this b; then the angle being less than 90°, $(a - b)$ is 3·09.

Now enter the table with S. 33° W. as course, that is the azimuth on which d_1 is measured, and with d_1 21·07 in the distance column, the difference of latitude is 17·67 S. and the departure 11·47 W. Also with 57° (the complement of 33°) as course and $(a - b)$ 3·09 in the distance column, the difference of latitude is 1·68 N. and the departure 2·59 W. (Since d_2 is measured in a direction N. 61° W. and $(a - b)$ is positive, the line at right angles to S. 33° W. must be N. and W. to make an acute angle with the direction of d_2.)

These results are—

D. Lat.	Dep.
17·67 S.	11·47 W.
1·68 N.	2·59 W.
15·98 S.	14·06 W. = 22′ diff. long.

Lat. by D.R.	49° 58′ N.	Long. by D.R.	14° 30′ W
Diff. lat.	16 S.	Diff. long.	22 W.
True lat.	49 42 N.	True long.	14 52 W.

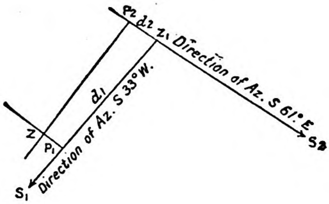
The formulæ are—

$$\left.\begin{aligned} a &= d_2 \times \text{cosec. A} \\ b &= d_1 \times \text{cot. A} \end{aligned}\right\} \text{A = angle between the lines on which } d_1 \text{ and } d_2 \text{ are measured.}$$

$$\left.\begin{aligned} &\text{1st d. lat.} = d_1 \times \text{cos. az.} && \text{1st dep.} = d_1 \times \text{sin. az.} \\ &\text{2nd ,,} = (a - b) \times \text{sin. az.} && \text{2nd ,,} = (a - b) \times \text{cos. az.} \end{aligned}\right\} \text{The azimuth being that on which } d_1 \text{ is measured.}$$

When the second altitude is reduced to the first position the graphic method is slightly modified, and becomes similar to that of simultaneous

altitudes of two objects. A sketch of the example just worked will show this.



Let Z_1 be the dead reckoning position to which the second altitude is reduced. Draw $Z_1 S_1$ in the direction of the azimuth S. 33° W. and $Z_1 S_2$ in the direction of the azimuth S. 61° E.; d_1 is $+ 21'{\cdot}07$ and is therefore represented by $Z_1 P_1$, d_2 is $- 4'{\cdot}55$ and is therefore represented by $Z_1 P_2$. Draw $P_1 Z$ at right angles to $Z_1 S_1$ and $P_2 Z$ at right angles to $Z_1 S_2$; their intersection Z is the position of the ship at the first observation.

In the diagram $P_1 Z$ is $(a - b)$, and the final portion of the computation is finding the difference of latitude and departure from Z_1 to P_1 and then from P_1 to Z; thus giving the total difference of latitude and departure between Z_1 and Z.

The following example shows the full working of the method, the sun's azimuth being small at both observations.

December 1st, in lat. by account 49° 50′ N long. 15° 50′ W., the following observations were taken to find the ship's position—

Ship Times nearly	Chron. Times	Obs. Alts. Sun's L.L.
H. M.	H. M. S.	
10 0 a.m.	10 53 12·6	13° 16′ 30″
1 0 p.m.	1 50 10·1	16 35 4

The true course in the interval was N. 68° E., distance 32 miles. The chronometer was correct for Greenwich mean time. Height of the eye 18 feet, index error of the sextant — 1′ 10″.

	D.	H.	M.	S.			D.	H.	M.	S.
G.M.T. Nov.	30	22	53	12·6		G.M.T. Dec.	1	1	50	10·1
Sun's decl. Dec. 1st	21°	51′	25″ S.				21°	51′	25″ S.	
23″·08 × 1·1			25			23″·08 × 1·8			42	
Corr. decl.	21	51	0				21	52	7	

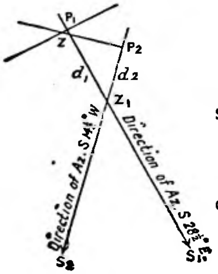
	M.	S.		M.	S.
Eq. T. +	10	46·4		10	46·4
·944s × 1·1		1·0	·944 × 1·8		1·7
Corr. Eq. T.	10	47·4		10	44·7

1st Obs.

	H. M. S.		
G.M.T.	22 53 12·6		
Eq. T.	+ 10 47·4		
G.A.T.	23 4 0	Az. S. 28½° E.	
Long.	1 3 20 W.		
S.A.T.	22 0 40		
H.A.	1 59 20	log. Ris.	5·12229
Lat.	49° 50′ N	Cos.	9·809569
Dec.	21 51 S.	Cos.	9·967624
Sum	71 41	Log.	4·899483
		N.N.	79338
		Vers.	685731
Zen. dist.	76° 24′ 45″	Vers.	765069
Tr. alt.	13 35 15		

2nd Obs.

	H. M. S.		
G.M.T.	1 50 10·1		
Eq. T.	10 44·7		
G.A.T.	2 0 54·8	Az. S. 14½° W.	
Long.	1 3 20 W.		
S.A.T.	0 57 34·8		
H.A.	0 57 34·8	log. Ris.	4·49686
Lat.	49° 50′ N.	Cos.	9·809569
Dec.	21 52 7″ S.	Cos.	9·967567
Sum	71 42 7	Log.	4·273996
		N.N.	18793
		Vers.	686039
Zen. dist.	72° 49′ 57″	Vers.	704832
Tr. alt.	17 10 3		



	1st Obs.		2nd Obs.	
Obs. alt.	13° 16′ 30″	Obs. alt.	16° 35′ 4″	
I.E.	— 1 10	I.E.	— 1 10	
	13 15 20		16 33 54	Angle between azimuth and Course = 53½°
Dip.	— 4 9	Dip.	— 4 9	
	13 11 11		16 29 45	
Sun's cor.	— 3 51	Sun's cor.	— 3 2	
	13 7 20		16 26 43	Co. 53½° dist. 32′
S.D.	+ 16 16	S.D.	+ 16 16	d. lat. = 19′
Tr. alt.	13 23 36	Tr. alt.	16 42 59	
Comp. alt.	13 35 15	Cor. for run	+ 19 0	
Diff.	11 49 or $d_1 = -11'{\cdot}8$	Red. tr. alt.	17 1 59	
		Computed ,,	17 10 3	
		Diff.	8 4 or $d_2 = -8'{\cdot}07$	

The angle between the directions of d_1 and d_2 is 43°, d_1 being in the direction N. 28½° W. and d_2 in the direction N. 14½° E.

The same Example solved by the Nat. Hav. method (see p. 502), using five-figure logs, is as follows—

	H.	M.	S.				
H.A.	1	59	20	Hav.	8·82126		
Co-lat.	40°	10′	00″	Sin.	9·80957		
Polar dist.	111	51	00	Sin.	9·96762		
				θ Hav.	8·59845	Nat. hav.	·03967
Co-lat. ~ Polar dist.	71°	41′	00″	. . . . . .		Nat. hav.	·34287
Zenith dist.	76°	24′	45″			Nat. hav.	·38254
	90	00	00				
Computed true altitude	13	35	15				

	H.	M.	S.				
H.A.	0	57	34·8	Hav.	8·19585		
Co-lat.	40°	10′	00″	Sin.	9·80957		
Polar dist.	111	52	7	Sin.	9·96757		
				θ Hav.	7·97299	Nat. hav.	·00940
Co-lat. ~ Polar dist.	71°	42′	7″	. . . . . .		Nat. hav.	·34302
Zenith dist.	72°	50′	00″			Nat. hav.	·35242
	90	00	00				
Computed true altitude	17	10	00				

Co. 43°, d_2 in dep. gives dist. or $a = 11{\cdot}83$
Co. 43°, d_1 in dep. gives d. lat. or $b = 12{\cdot}65$
Diff. $= -{\cdot}82$

Co. $28\frac{1}{2}$°, d_1 in dist. gives	d. lat.	10·37 N.	dep.	5·63 W.
Co. $61\frac{1}{2}$° $(b-a)$ in dist. gives	,,	·38 S.*	,,	·72 W.*
		9·99 N.		6·35 W.

6′·35 dep. = 10′ diff. long.

* Since d_2 is measured in a direction N. $14\frac{1}{2}$° E. and $(a-b)$ is negative, the line at right angles to S. $28\frac{1}{2}$ E. must be S. and W. to make an obtuse angle with the direction of d_2.

Lat. by D.R.	49° 50′ N.	Long. by D.R.	15° 50′ W.
Cor.	10 N.	Cor.	10 W.
Lat. at 1st Obs.	50 0 N.	Long. at 1st Obs.	16 0 W.
Run	12 N.	Run	46·3 E.
Lat. at 2nd Obs.	50 12 N.	Long. at 2nd Obs.	15 13·7 W.

If one of the observations is taken when the object is near the prime vertical, the old and the new methods can be combined. The longitude method can be used with the observation near the prime vertical, and the altitude method with the observation in small azimuth.

Rule.—Calculate the longitude by the observation near the prime vertical, using the dead reckoning latitude. Then calculate the altitude for the time of the other observation, using the dead reckoning latitude and the longitude found from the first observation. As before, take the difference between the computed and true altitudes, call it d and mark it + or —. Take out the azimuths. Enter the Traverse Table with the angle between the azimuths as a course and d in the departure column take out the distance. Enter again with the complement of the azimuth of the observation near the prime vertical as a course and the distance just found in the distance column, take out the difference of latitude and departure, convert the departure into difference of longitude. Mark them in the direction at right angles to the azimuth used. (In this case the direction must be taken to make an acute angle with the direction of d.) Apply the difference of latitude to the dead reckoning latitude and the difference of longitude to the computed longitude; the result is the ship's true position.

Example.—September 28th, at about 11h. 40m. p.m. at ship, in lat. by dead reckoning 46° 20′ N., long. 29° 45′ W., the following observations were taken to find the ship's position—

Chron. Time.	Obs. Alt. Aldebaran.	Obs. Alt. α Ursæ Majoris.
H. M. S.		
1 37 40	29° 5′ 54″ E.	19° 26′ 46″ E.

The chronometer was 1m. 21s. slow of Greenwich mean time. Height

of the eye 20 feet. The star Aldebaran was the star of greater azimuth.

	H.	M.	S.		H.	M.	S.
Aldebaran R.A.	4	29	38·49	α Ursæ Majoris R.A.	10	56	55·42
,, decl.	16°	17′	24″ N.	,, decl.	62°	20′	31″ N.

	D.	H.	M.	S.
Chron. T.	28	13	37	40
Error			1	21 slow
G.M.T.	28	13	39	1
M.S. R.A.		12	28	51·16
Acc. for 13h.			2	8·13
,, 39m.				6·41
G. Sid. T.		2	10	6·70

Obs. alt. Aldebaran	29°	5′	51″
Dip	—	4	23
	29	1	31
Ref.	—	1	42
Tr. alt.	28	59	49

Tr. alt.	28°	59′	49″		
Lat.	46	20	0	Sec.	10·160860
P.D.	73	42	36	Co-sec.	10·017794
2)	149	2	25		
½-sum	74	31	12	Cos.	9·426351
Rem.	45	31	23	Sin.	9·853414
H.A.	4h.	19m.	19·5s.	Hav.	9·458419
					Az. S. 84° E.

	H.	M.	S.
H.A.	4	19	19·5 E.
,,	19	40	40·5 W.
R.A.	4	29	38·5
S. Sid. T.	0	10	19
G. ,,	2	10	7
Long.	1	59	48
,,		29°	57′ W.

NOTE.—Observe the method of calculating the alt. and azimuth together.

	H.	M.	S.				
G. Sid. T.	2	10	7				
Long.	1	59	48 W.				
S. Sid. T.	0	10	19				
Star's R.A.	10	56	55				
H.A.	10	46	36	Cos.	9·977335	Tan.	9·520728
Decl.	62°	20′	31″ N.	Cot.	9·719396		
Arc. I.	26°	26′	49″ S.	Tan.	9·696731	Sin.	9·648719
Lat.	46	20	00 N.				
Arc. II.	19	53	11 N.	Tan.	9·558380	Sec.	10·026701
Arc. III. Az.	8	55	40	Cos.	9·994707	Tan.	9·196148
Arc. IV. Alt.	19°	39′	51″	Tan.	9·553087	Arc. III. Az.	N. 8° 55′ 40″ E.

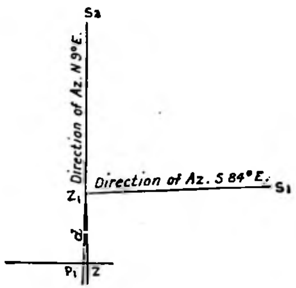
Obs alt.	19°	26′	46″
Dip	—	4	23
	19	22	23
Ref.	—	2	41
Tr. alt.	19	19	42
Computed ,,	19	39	51
Diff.		20	9 or $d = -20'{\cdot}15$

The angle between the azimuths is 87°.

The Natural Haversine method gives the following result—

	H.	M.	S.				
Star's H.A.	10	46	36	Hav. 9·98882			
Co-latitude	43°	40′	00″	Sin. 9·83914			
Polar dist.	27	39	29	Sin. 9·66670			
			θ	Hav. 9·49466	Nat. hav.	·31237	
Co-lat. ~ Polar dist.	16°	00′	31″	. . . .	Nat. hav.	·01939	
Zenith dist.	70°	20′	15″		Nat. hav.	·33176	
	90	00	00				
Computed true altitude	19	39	45				

The azimuth, found by inspection from the A, B and C Azimuth Tables, is N. 9° E.



Co. 87°, d 20′·1 in dep. gives 20·1 in dist.

Co. 6°, dist. 20·1 gives d. lat. 19·9 S.* and dep. 2·1 W.* or diff. long. 3′ W.

	°	′			°	′	
Lat. by D.R.	46	20	N.	Computed Long.	29	57	W.
Diff. lat.		19·9	S.	Diff. long.		3	W.
True lat.	46	0·1	N.	True long.	30	0	W.

NOTE.—In all problems a rough sketch will be a very great aid to name the d. lat. and dep.

* Since d is measured in a direction S. 9° W., the line at right angles to S. 84° E. must be S. and W. to make an acute angle with the direction of d, making the corrections for Latitude and Longitude S. and W. respectively.

THE COMPASS ADJUSTMENTS

If a small steel bar, the half of a knitting-needle, were truly balanced and then suspended by a thread or a fibre without twist, it would come to rest horizontally, pointing in no particular direction.

If the same bar were magnetised and suspended as before, it would, if the experiment were tried in London, take up a position in the magnetic meridian with its N. end dipping at an angle of 67° below the horizon ; the line through the axis of the needle marks out the line of dip.

A freely-suspended magnet will always try to point directly *at* the nearest pole, ignoring the curvature of the earth. The dip varies from 0 degrees at the magnetic equator to 90 degrees at the magnetic poles. In London at the present time (1915) the dip is 67 degrees.

It will be evident that, except on the magnetic equator, there must be a loss of horizontal directive force, which is the only part of use to the navigator. This is shown as follows—

A magnet freely suspended at London would come to rest in a position indicated by the line N S, which dips below the horizon, H O, equal to the angle H S N = 67 degrees.

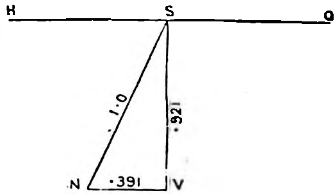


Fig. 1.

S N equals the total force 1·0.

S V equals the vertical force ·921.

N V equals the horizontal force ·391.

A compass is composed of several small magnets attached to a circular card, which is mounted on a pivot in the centre of a copper bowl. The plane of the card is below the point of suspension, to prevent dipping and to place the card in a position of stable equilibrium ; the horizontal gravitational force of the card and magnets is greater than the dipping force of the magnets, compelling the card to remain horizontal and not dip. The card is constructed and mounted so as to utilise the earth's horizontal force, which force compels the compass needle to become parallel to the magnetic meridian in any part of the world if under the earth's magnetic influence only.

All magnets have two poles with a neutral zone between.

Cracks or flaws will cause sets of poles called " consequent poles." For convenience in discussion the north-seeking end is called the Red end, while the south-seeking end of a magnet is called the Blue end.

(1) Poles or ends of the same name repel one another.

(2) Poles of opposite names attract one another.

(3) The attraction between two magnetic points varies inversely as the square of the distance between them.

(4) The attraction between two magnets varies as the cube of the distance between them.

(5) The greatest power to deflect occurs when one magnet is at right angles to the other ; the least when they are parallel to each other.

The harder a metal is the more difficult it is to magnetise, but the magnetism is retained with equal tenacity.

The softer a metal is the more easily it is magnetised, but it parts with its magnetism with equal ease once the magnetising influence is altered in position or removed.

The earth is a magnet and is governed by the foregoing laws affecting magnets. There are two poles and a neutral zone called a magnetic equator. The north magnetic pole is in lat. 70° N., long 96° 46′ W.; the south pole is in lat. 73½° south, and long. 155° (about) E. There are also consequent poles or secondary poles of little power.

About midway between the poles an uneven line traces the magnetic equator. It is not an exact circle. Because the true and magnetic poles are not coincident the true and magnetic meridians cut one another; the angle between them is called the variation. The variation is affected by the before-mentioned secondary poles.

The magnetic poles are not stationary. In the course of many years the magnetic poles alter their positions and bring about an increase or a decrease in the variation. In 1657 there was no variation at London: by 1815 it had become 24° 27′ W.; at the present time (1915) it has again decreased to less than 15° W. and is still decreasing about 6′ annually. The dip is also decreasing in a lesser degree.

Because the north-seeking end of a freely suspended magnet is called the red end, the north magnetic pole of the earth must be called the blue end, in obedience to the law "opposite names attract"; and for the same reason the south magnetic pole is called the red pole, as it attracts the south end of the magnet.

When discussing the magnetism of an iron or steel vessel certain technical terms are used; it has been suggested that these terms "could, with advantage, be simplified."

The terms in use and a proposed simplification are placed in parallel columns thus—

In Use	*Proposed*
Hard iron.	Hammered iron.
Soft iron.	Unhammered iron.
Sub-permanent magnetism.	Permanent magnetism.
Transient induced magnetism.	Non-permanent magnetism.
Horizontal induction.	Magnetism in horizontal iron.
Vertical induction.	Magnetism in vertical iron.

Explanation of terms on both sides

Hard iron.—Permanent or sub-permanent magnetism is supposed to exist in the hard, but more correctly speaking hammered, iron. The hammering seems to capture and enclose the magnetism in the metal worked upon.

Soft iron.—Transient induced magnetism is supposed to exist in soft, or more correctly speaking unhammered, iron. It is non-permanent

because such iron is magnetised by the earth without exerted force or concussion. It reaches its maximum disturbing power as the particular piece of iron under discussion becomes parallel with the mag. meridian or with the line of dip, and is at a minimum as it becomes at right angles to the mag. meridian or the line of dip.

All iron and steel structures are magnetised by the earth.

An iron or steel vessel in the course of construction becomes a mass of magnets which disturb the compass in many ways. If the vessel were steered by a gyro compass instead of a magnet there would be no use for this chapter to be written.

The line of dip at any place is the direction a freely suspended magnetised needle will assume.

In the Northern Hemisphere it will point to the north point of the horizon, and downwards to the north magnetic pole of the earth.

The line of dip represents the total force. *See* Fig. 1

A vessel as a whole becomes magnetised as though she were a solid block of iron, the position of the poles depending upon the angle at which the keel cuts the line of dip.

If through the centre of a vessel, while building, the line of dip is drawn from deck to keel, and this line is bisected by a plane at right angles to it, then, in the Northern Hemisphere, the lower side, that is, the side towards the north pole, will be coloured red, and the upper side will be coloured blue. This rule applies to all the iron in the vessel while on the stocks.

Let P. M. represent permanent magnetism, and let N. P. M. represent non-permanent magnetism. It will be found that N. P. M. in horizontal iron varies in two ways: viz., horizontally and vertically. In horizontal iron the N. P. M. reaches its maximum when it is parallel with the magnetic meridian in any magnetic latitude; this is equally true whether the iron is fore and aft, athwartships, or at any angle between; and it is least when at right angles to the meridian. This applies to both hemispheres, that is, all over the world. Strictly speaking, soft iron is never really free from magnetism, as the earth's magnetic force is always acting on it.

In vertical iron, by which is meant iron not horizontal, the upper ends containing N. P. M. are always coloured the same as the nearest magnetic pole, blue in the Northern Hemisphere, and red in the Southern.

The greatest intensity is reached when the vertical iron is parallel with the line of dip, and least when at right angles to it.

The permanent magnetism retains the same colour always and everywhere. Iron that is magnetised non-permanently varies in intensity and in colour, both depending upon its "present" position with reference to the line of dip or magnetic meridian.

This may be represented as follows—

In Fig 2 let W X Y Z represent a rectangular block of hard steel in a fixed position in London. If it is subjected to severe hammering all over it will become permanently magnetised, the colours appearing as at R and B. If the ends are reversed so that the end that was north is placed south no change in the magnetism will take place (*see* Fig. 2a).

But if a block of soft iron of the same size and shape which had not been hammered lay in the same position as the block of steel the colour towards the mag. north would always be the same, that is, red, no matter how it is turned about.

Herein lies the great difference between a permanently magnetised bar and a non-permanently magnetised bar (*see* Figs. 3 and 3a). If for a block of iron or steel we substituted a vessel building on the stocks, the iron or steel in her construction will be magnetised as the block of steel and the block of iron are magnetised.

This may be further illustrated. Consider a point at the exact middle of the vessel on the stocks at London. Let the eye follow along the line of dip, which will be north magnetic, and forming an angle of 67° with the horizon; this line will indicate the direction of the north magnetic pole.

Imagine a powerful bull's-eye lamp throwing rays of light (red) upward along the line of dip from the direction of the pole towards the observer; half the vessel would be illuminated red, the other half would be in the shade (blue). These colours would be divided by a line through the centre of the vessel at right angles to the line of dip. The points of greatest intensity (the poles of the vessel) would lie upon the line of dip, the red pole at a point nearest the magnetic pole, and the blue pole at a point equidistant in the opposite direction. As the vessel was being constructed so would the colours red and blue be developed and permanently fixed in all riveted parts as shown in the block of hard steel (Figs. 2 and 2a), but in the unhammered steel or iron there would be no permanency; the part facing towards the north magnetic pole would always remain the same colour, red, and the other half always blue, as shown in the block of soft iron (Figs. 3 and 3a).

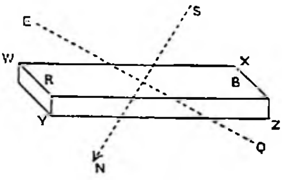


Fig. 2

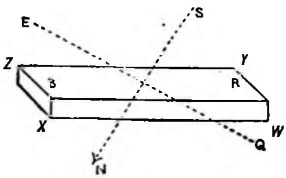


Fig. 2a.

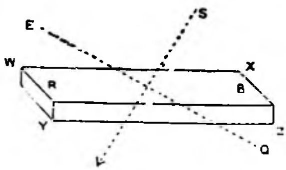


Fig. 3.

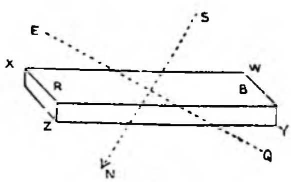


Fig. 3a.

Figs. 4 and 4a show two vessels built at London, one N., parallel with the mag. meridian, the other E., at right angles to the mag. meridian.

The student should draw figures showing vessels built in places of various degrees of dip and at different azimuths.

In a structure like a vessel the iron and steel of which she is built lies in every conceivable direction. She is a great magnet made up of many lesser ones, yet in the midst of these magnetic disturbing forces a compass must be placed which is to point north accurately at all times and in all places.

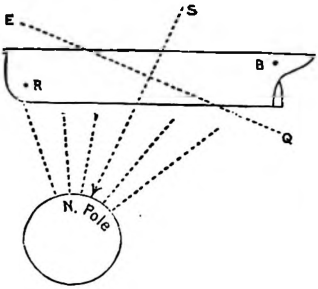


Fig. 4.

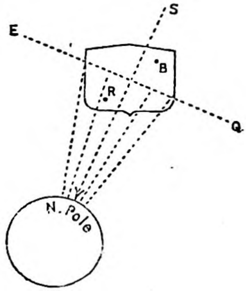


Fig. 4a.

15
20"

A "best possible" place should be found somewhere along the middle fore and aft line of a vessel. It should be as far as possible from any vertical iron, especially movable iron, such as ventilators, davits, derrick heads, bulkheads, and the upper parts of iron deck houses which cause disturbances very difficult to handle.

A dynamo should be at least 50 feet away, and all lights should be double wired if passing near a compass, and clipped together in order to neutralise their effect on the compass needle. The selected place should be tested with the vibrating needle for horizontal force and with the dipping needle for vertical force.

The horizontal vibrating needle is a small, strongly magnetised needle. It is flat, pointed at both ends, 3 inches long and half an inch broad, fitted with a cap like the compass card, and works on a pivot of its own for land observations. On board the same pivot must be used when the compass card and pivot have been removed from the bowl.

The vibrations are observed as follows—

With a spare magnet give the horizontal vibrating needle a good deflection, then when the needle is swinging through an arc of about 40 degrees count the vibrations, noting the instant when the north-seeking end has reached the extreme deflection to the right, subsequently noting the instant of every tenth vibration until the needle is nearly at rest.

The proportion of the earth's horizontal force between any two places is known to be inversely as the square of the number of seconds occupied by the same number of vibrations at each place. If therefore the time of making 10 vibrations on shore is found to be 20 seconds and the time of 10 vibrations on board at the place of the compass is 26 seconds, then the horizontal force at the compass is $\left(\frac{20}{26}\right)^2 = \frac{400}{676}$ or 0·59, the horizontal force on shore being represented by 1; or if the vibrations on board had taken 16 seconds instead of 26 the horizontal force on board would be $\left(\frac{20}{16}\right)^2 = \frac{400}{256}$ or 1·5. In

the first case the ship's force is acting against, and in the second with the earth's force. It is evident that the most favourable position is where the needle will vibrate at the same rate as on shore on *all* points of the compass. The horizontal force then found at the compass is the combined force of earth and ship P. M. and N. P. M., for the direction in which the needle points.

To reduce this force to the magnetic meridian it should be multiplied by the cos. of the deviation. The mean value thus found for a round turn of the compass is called λ (lamba). It represents the power of the compass for that particular position on board.

The P. M. should always produce unity, but the N. P. M. nearly always reduces the force below unity, because it introduces a red pole between the red end of the compass needle and the blue pole of the earth, in the Northern Hemisphere, and the reverse in the Southern Hemisphere. This red pole reduces the effect of the earth's directive force.

Vertical force is measured by the dipping needle. It consists of a needle the shape and size of the horizontal vibrating needle, but is mounted on a horizontal axis to swing vertically. The earth s vertical force is overcome by a small sliding weight at the other end of the needle. It is adjusted so as to make it horizontal on shore, then taken on board the vessel, which should lie at right angles to the mag. meridian. It is placed in the compass bowl on the pivot socket and parallel to the magnetic meridian. If the needle remain horizontal there is no vertical force ; a blue vertical force would draw the north-seeking end downward and a red vertical force would repel the N. end upwards.

The dipping needle is used to correct the heeling error without heeling the vessel.

Although every care may be taken it is almost impossible to find a practicable position in which to place the compass so that it shall be free from local attraction. The compass adjuster must therefore find—

In what direction the local attraction lies.

What is the cause of the disturbance.

Is it P. M. or N. P. M., or both ?

It has been said previously that the iron and steel of which the vessel is built lies in every conceivable direction, but it has been found possible to resolve them into four general directions, namely—

Fore and aft horizontally.

Athwartships horizontally

Diagonally horizontally.

Vertically.

This is done by calculating the co-efficients.

The Co-efficients.—Five letters, A, B, C, D, and E, are used to represent the various disturbances or forces.

A represents faulty fittings in the compass card, or accessories, or in the placing of the compass on board, the result of really bad workmanship.

B represents fore and aft P. M. forces.

C represents athwartships P. M. forces.

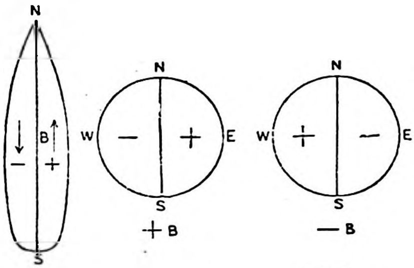
D fore and aft and athwartship N. P. M. forces.

E diagonal N. P. M.

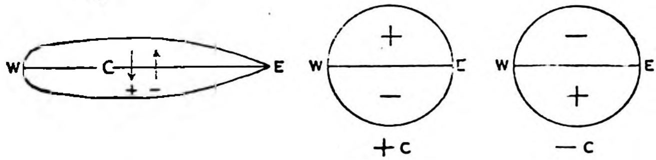
They are further divided into plus and minus deflections, or east and west deviations, and when the amount and cause are discovered they are

counteracted by similar but opposite P. M. or N. P. M. forces so placed as to produce opposite effects to the disturbing forces. The following illustrations show in detail the general order and sequence of the deflections or deviations as represented by the co-efficients.

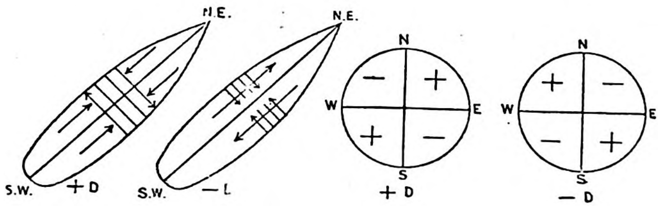
A needs no illustration as it is constant in amount on all points and in all places, plus if the deflection is to the east, minus if to the west.



The outline vessel shows the direction of force; the circles represent the deflection of the needle as the vessel is turned in azimuth from N. through E.S. and W. to N. again. It is zero at N. and S. and reaches its maximum at E. and W.

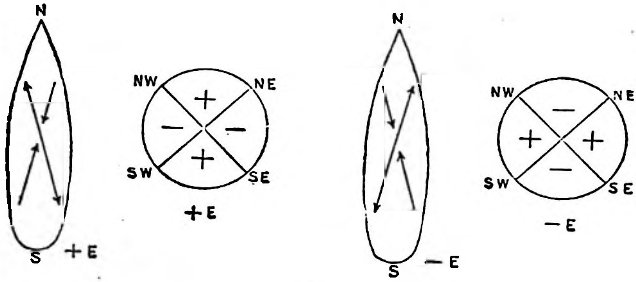


As before, the outline shows the direction of force, the circles the deflection of the needle in a round turn. The deflection is zero at E. and W. and reaches maximum at N. and S.



In these two figures the effect of the athwartship beams and fore and aft tie plates, etc., is shown. It may be seen that beams right across the vessel and tie plates, etc., that stop on opposite sides of the compass, have similar effects. The diagrams show how they regulate the deviations, one arrangement producing what is described as + D, the other — D. A minus D is very rare. In both cases the deviations reach their maximum on the quadrantal points and are zero on the cardinal points.

In the Figures + and — E the arrangement of iron is shown; it is similar to D, except in direction, the iron lying oblique instead of fore and aft or transversely. The deviations produced are shown in the two circles. The maximum is reached on the cardinal points and is zero at the quadrantal points.



The deviation $= A + B \sin. S' + C \cos. S' + D \sin. 2 S' + E \cos. 2 S'$.
(S′ being the direction of ship's head by compass.)

The method of calculating the co-efficients is shown on page 557.

The value of the co-efficients is threefold—

(*a*) The compass can be corrected more correctly and intelligibly.

(*b*) A full Table of deviations can be constructed from them.

(*c*) By means of the chart of Horizontal Force published by the Admiralty, the Table constructed above can be corrected with close approximation for any latitude or longitude.

Changes in the deviations caused by iron in the cargo, collisions, stranding, lying up in dock for long periods, or through extensive repairs, can be determined with accuracy by a recalculation of the co-efficients.

All that is required is a set of deviations on eight equidistant points before adjustment and another set after adjustment. The second set should be a strict modification of the first set. If these are tabulated, then a third set as advised above would at once indicate the nature and amount of the change.

By whatever method the deviations are found there is only one way of counteracting them, that is, by the use of magnets and soft iron correctors so placed as to produce opposite effects.

There are three methods of adjustment—

(*a*) The calculated co-efficient method.

(*b*) By deflector, when the forces are measured.

(*c*) By trial tentatively.

The Co-efficient method:—The amount and direction of the deviation being known, the magnets and correctors can be placed approximately in position by measurement, without swinging the vessel.

By Deflector.—The deflector is an arrangement of magnets worked by an index screw which measures the resistance offered by the compass needles

to the force exerted to deflect the needles a given number of degrees out of the magnetic meridian.

When employed, it is placed upon the glass cover over the centre of the card. The resistance, or force, when the ship's head is at N. mag., and again at S. magnetic, is to be read on the index ; the mean force is then set on the deflector, and the card will deviate a little from the point agreed upon above. It is made to come back again by means of a corrector magnet moved to or from the compass until the required result is produced.

The operation is repeated on E. and W. mag. and corrected in the same way. Generally the compass offers less resistance with the vessel's head at E. or W. than it does at N. or S., because the beams introduce a red pole between the pole of the earth and the red poles of the compass needles when her head is at right angles to the mag. mer.

The mean of the resistance at N. and S. and E. and W. is set on the deflector ; the correction is made with the soft iron sphere correctors placed athwartships on each side of the compass.

The third or tentative method is the most familiar to the navigator ; it is also the method used in the examination for a Master's certificate. A detailed account follows.

In each of the foregoing methods of adjustment B is described as a single fore and aft force. This is not always so, as it nearly always consists of two forces, a P. M. force and a varying quantity found in vertical iron either abaft or before the compass.

The greater vertical force is generally abaft the compass, and its effect depends upon two things : whether the upper or lower end is nearest the compass, and the hemisphere in which the vessel is. The magnetism in vertical iron is governed by the dip ; it reaches its maximum disturbing force at the magnetic poles and its least force at the magnetic equator, where the disturbing force should be zero.

A change of hemisphere produces a change in the colour of the poles.

The P. M. in B is constant in all latitudes ; the N. P. M. in vertical iron varies with the dip and is therefore inconstant.

Let the N. P. M. force in vertical iron be called β to distinguish it from B due to P. M. ; both B and β act in the fore and aft line. B must be separated from β for each to be properly compensated. There are two tentative ways of doing this, one when the vessel is on the magnetic equator, the other when the vessel makes a large change in the "dip" but does not reach the magnetic equator. There is also a method by calculation, but it is not used in the merchant service.

On the Magnetic Equator.—On the magnetic equator β is zero ; it is therefore assumed that all the fore and aft disturbance is due to B only, and the correction required is made with the fore and aft magnet. Should any disturbance appear after a considerable change of latitude, it is assumed to be caused by the fore and aft force represented by β and is corrected by a soft iron bar placed vertically before or abaft the compass so as to counteract the disturbance.

Not reaching the Equator.—When a vessel does not reach the equator, on her nearest approach thereto the B force predominates and is corrected by the fore and aft magnet. On her nearest approach to the magnetic pole the β force predominates, and is corrected by the vertical soft iron bar.

Tentative Method of Adjustment.—In detail the following procedure is

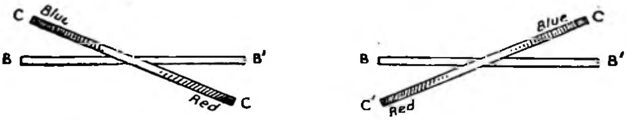
most commonly adopted. The vessel's head is placed E. or W. magnetic. The disturbing magnetism (β) in vertical iron, either before or abaft the compass in the middle line of the vessel, is sought for, its position is noted, and, as near as possible, the amount of this disturbance is estimated. The correction is made by placing a soft iron bar (Flinder's bar) in a position to counteract the disturbance. Most commonly the disturbing force is abaft the compass, therefore, the Flinder's bar is placed on the fore side, on the principle that a small piece of iron close to will have the effect of a large piece at a distance (*see* law of inverse squares). In the Kelvin Compass the Flinder's bar consists of pieces of soft iron of various lengths built up with pieces of wood, when necessary, all of which are placed in a brass case. The uppermost piece of iron should be in the same plane as the compass card. When a vessel is built with her head east or west the fore and aft disturbing force will be due to β only and is corrected by a Flinder's bar only. When built on any other point a P.M. force will be developed in the fore and aft direction and must be counteracted by a permanent magnet also placed fore and aft with its poles in the opposite direction to the disturbing poles, and moved to or from the compass until the card points as desired. Care should be taken that the poles of the corrector magnet are equidistant from the centre of the compass card and not nearer to it than twice the length of the corrector magnet used. When placed nearer than the above rule they give a deflection to the compass needles instead of counteracting the magnetism of the vessel. This deflection is most noticeable on the quadrantal points.

The correction for B and β is only an approximation for any place but the magnetic equator. When the fore and aft forces B and β are compensated the vessel's head is brought N. or S. magnetic. Any P. M. force athwartships represented by C is indicated by the N. end of the needle being attracted towards the vessel's side. The correction is made by placing a magnet athwartships, observing the same rules as for B. The vessel's head is then brought to rest on any quadrantal point to find the value of any N. P. M. force represented by D.

Merchant vessels have their compasses placed in the middle of the vessel, mostly over athwartship beams that cross from side to side producing + D. Soft iron correctors are placed on each side of the compass so as to produce an equal amount of — D, which is done by moving the correctors to or from the compass. The plane of the compass card extended should cut the correctors in halves horizontally when the vessel is upright, and a vertical plane passing athwartships through the centre of the compass card should cut them in halves vertically; and they should be equidistant from the centre of the compass. Because the arrangement of the beams rarely varies, this error is almost always + D, ranging in quantity from two to five degrees. Co-efficient D causes the least trouble, because the disturbing and correcting forces vary together, therefore, neither time nor place affects them unequally. The disturbances grouped under — D, + E, and — E are generally so small that they may be neglected. The compass adjusters should endeavour to use as few correctors of any sort as possible.

The Heeling Error.—So far, all that has been said and done relates to a vessel in the upright position. But a vessel "heels" from side to side while the compass bowl and card swing free, altering its position with reference to the horizontal and vertical iron in its vicinity; new forces are brought into action and the compass is again disturbed. The action of the two

chief forces may be explained as follows: Let B B′ (Fig. 5) represent a beam when the vessel's head is at N. or S. Being at right angles to the mag. meridian it has no disturbing effect, but when the beam is inclined one way

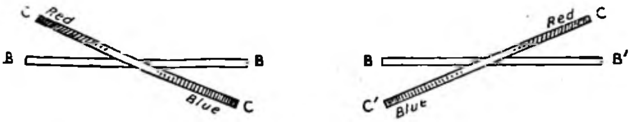


Northern Hemisphere

Fig. 5

Showing upper and lower ends of beams in Northern Hemisphere

or the other the upper ends at C become blue and attract the N. end of the needle towards the high side in the Northern Mag. Hemisphere and repel the N. end of the needle towards the low side in the Southern Mag. Hemisphere where the upper ends at C are red (*see* Fig. 6)



Southern Hemisphere

Fig. 6.

Showing upper and lower ends of beams in Southern Hemisphere

Vertical Iron.—Vertical iron that was under the binnacle when the vessel was upright has no disturbing power, but when the vessel heels the relative positions of compass needle and the vertical iron are altered and a new horizontal force is developed whose action depends upon the colour of the pole nearest the compass.

The induced magnetism in vertical iron varies as the tangent of the dip, and is, therefore, greatest at the magnetic poles and nil on the magnetic equator. In the Northern Magnetic Hemisphere the upper ends of vertical iron are blue and the lower ends are red, therefore when the ship heels in the Northern Magnetic Hemisphere the blue ends of vertical iron below the compass coming out on the high side will draw the needle to the *high side.*

On the magnetic equator the vertical iron is blue on the South side and red on the North side, and the effect on the compass is nil.

In the Southern Magnetic Hemisphere the upper ends of vertical iron are red and the lower ends blue. Therefore when the ship heels in the Southern Magnetic Hemisphere the red ends of vertical iron below the compass coming out on the high side will repel the north end of the needle to the low side. Induction in vertical iron causes semi-circular deviation of a different name in each hemisphere.

The disturbance caused by the beams B B′, (Figs. 5 and 6) is largely compensated automatically for all positions by the spheres used in the D correction. The beams B B′ cause a + D, the spheres a — D; these two forces vary together both in the upright and inclined positions. The heeling error from vertical iron is compensated by a permanent magnet. As induced magnetism in vertical iron is a N. P. M. force this is a contradiction in compensation; it is the only case where a permanent magnet is used to correct

a N. P. M. disturbance. To be consistent a soft iron corrector should be placed vertically above the compass, but this would be so inconvenient that it is never done; instead, a permanent magnet is used, which must be moved up or down as the dip increases or decreases. On the magnetic equator where the N. P. M. force reaches zero the heeling error is caused by the vertical component of sub-permanent magnetism only, and when the ship proceeds into the Southern Hemisphere, if the magnetic induction in vertical iron exceed that from sub-permanent magnetism, the correcting vertical magnet would have to be placed with the other end up.

Heeling.—The adjustment for the heeling error is made either by heeling the vessel, or by the dipping needle.

The vessel is placed heading N. or S. by compass, then heeled about 10 degrees. If the north end of the compass needle deviates towards the high side of the vessel there is a blue vertical force beneath the compass. If the north end of the compass needle deviates towards the low side there is a red vertical force beneath the compass.

The vertical magnet beneath the compass is moved up or down so as to counteract the vertical disturbance, red end up if the north end goes to windward and blue end up if the north end goes to leeward.

With the Dipping Needle.—The dipping needle (*see* p.507) is adjusted on shore. It is placed in the magnetic meridian with its north end towards the N. pole; a small sliding weight in a slot at the other (blue) end is moved along until the needle is horizontal. It is then taken on board, the vessel is placed with her head E. or W. magnetic, the compass card is unshipped, and the dipping needle fixed vertically on the pivot, the north end, as before, towards the N. magnetic pole. The unweighted end dipping downwards indicates a blue force beneath the compass; if it is repelled upward a red force is indicated.

This applies to the Northern Mag. Hemisphere. For the Southern Mag. Hemisphere the dipping needle would have the red end of the needle weighted.

The adjustment is made by means of a vertical magnet the same as when heeling.

If the north end of the needle dips, make it horizontal by placing a magnet vertically under the centre of the compass, *red end uppermost*; if the north end is repelled upwards the above-mentioned magnet must be placed with its *blue end uppermost.*

This adjustment holds good only for places of the same magnetic dip; for greater dip the magnet should be moved up; for lesser, down.

The heeling error can also be corrected by the method of vertical vibration carried out in a similar manner to that explained in finding coefficient λ. (*Lambda*).

Retained Magnetism.—So long as there is any vibration in the hull of the vessel there is an ebb or flow of magnetism more or less temporary in nature, because the vessel never stays in any one fixed direction long enough to secure permanency. If a vessel is laid up in dock for a long time with her head in one direction, or for a lesser time with winches going, or most commonly when she has been kept on one course for a long time, a red polarity is developed in that part of the vessel directed towards the north magnetic pole. This polarity decreases the attractive power of the earth's magnetic pole and causes the compass to become less and less sensitive while continuing on the same course. When the course is changed the red

polarity will have at first a strong repelling effect, which rapidly diminishes in force and eventually disappears.

Retained magnetism always has the effect of carrying the vessel towards the last course steered. The deviation should be obtained directly the course is changed. There is no possible compensation for this error.

The great object in compensating a compass is to entirely eliminate the magnetism of the ship by introducing co-efficients of the same amount and of opposite names by means of magnets and soft iron correctors.

For detailed information read the "Elementary Manual," edition revised by Captain Craik, R.N., etc., and Towson's Deviation.

DEVIATION OF THE COMPASS

SYLLABUS OF EXAMINATION FOR MASTER ORDINARY

1. *State briefly the essentials of an efficient compass.*

It is essential that the card should have the greatest possible magnetic power in its needles, combined with the smallest possible weight in the whole card. The jewelled cap should be sound—that is, not worn nor cracked, and the pivot sharp and free from rust. If the card is placed on the pivot and deflected through a small angle from its position of rest, it should always come back exactly to the same point. The card should be accurately divided and centred, and the point of the pivot should be in the same plane as the gimbals of the bowl.

Standard compasses must be furnished with the means of taking the bearing of an object at any elevation, and of reading it off within a degree. When a compass is placed on board ship, the lubber line should be vertical and exactly in the fore-and-aft line from the centre of the card, and the bowl should swing freely in its gimbals. It is desirable that all compass bowls should be made of pure copper.

2. *State briefly the chief points to be considered when selecting a position for your compass on board ship, and what should be particularly guarded against.*

The standard compass should be in the midship line, in a convenient position for constant watching by the officer of the watch, and for comparison with the steering compass, and should have a clear view all round for taking bearings. It should not be less than five feet from iron of any kind, and the proximity of vertical iron, and of iron which is liable to be changed in position, such as davits, derricks, ventilating cowls, &c., should be particularly guarded against. Where electric lighting is used, the position of the dynamo has also to be considered, as it may disturb a compass at the distance of fifty or sixty feet.

Steering compasses must be so placed that the card can be clearly seen by the helmsman, and should be as far from any iron as circumstances will allow.

3. *What do you mean by deviation of the compass, and how is it caused?*

The deviation of the compass is the angle between the magnetic meridian and the direction of the compass needle, and is caused by the iron of the ship, whether used in her construction, in her equipment, or in her as cargo.

4. *Describe how you would determine the deviation of your compass: (1) by reciprocal bearings; (2) by figures on the dock walls; (3) by bearings of a distant object; (4) by the bearings of the sun or other celestial body.*

To determine the deviation of the compass by reciprocal bearings, a

compass should be placed on shore where there is no iron in the vicinity, and where it can be conveniently seen from the standard compass. As the ship is swung round, observe the bearing of the shore compass from the standard compass when the ship's head is steady on each point; and, by signal, have the bearings of the standard compass taken by the shore compass at the same instant. The difference between the bearing taken on board and the opposite of the shore bearing is the deviation on the respective points.

By figures on the dock wall. Where they are available, as at Liverpool, the difference between the bearing of the object in the background and the bearing marked on the wall exactly in line with the object is the error for any point the ship's head may be on. Apply the variation and get the deviation.

By bearing of distant object. When the magnetic bearing is known, the difference between it and the bearing observed by the standard compass with head on any point is the deviation. When the magnetic bearing is not known it may be taken from the chart, or it may be found by getting the difference of bearing between it and the sun, and applying that difference to the sun's true bearing, computed or found from Azimuth tables, and allowing the variation; the magnetic bearing can be found and thence the deviation.

By bearings of the sun or other celestial object. The exact time must be noted and the bearing of the object taken when the ship's head is on each point. The true bearing of the object at each observation can be computed or found by Azimuth tables. By applying the variation with its proper sign to the true bearing, the magnetic bearing is found. The difference between the magnetic bearing and the compass bearing is the deviation on each point.

5. *Having determined the deviation with the ship's head on the various points of the compass, how do you know when it is easterly and when westerly?*

If the correct magnetic bearing be to the right of the bearing by the compass on board, the deviation is Easterly; if to the left, Westerly.

6. *Why is it necessary, in order to ascertain the deviations, to bring the ship's head in more than one direction?*

Because the deviation changes in amount, as well as in the direction in which it is to be applied, being easterly on some points and westerly on others, when the direction of the ship's head is changed.

7. *For accuracy, what is the least number of points to which the ship's head should be brought for constructing a curve or table of deviations?*

For an accurate table the deviation must be obtained with the ship's head on the four cardinal points, and on the four quadrantal points.

8. *How would you find the deviation when sailing along a well-known coast?*

By taking the compass bearing of any two objects which are marked on the chart, when they are in line. The magnetic bearing can be obtained from the chart, and the difference between it and the compass bearing is the deviation.

9. *Name some suitable objects by which you could readily obtain the deviation of the compass when sailing along the coasts of the Channel you have been accustomed to use.*

The following are a few of the very great number of instances where two objects can be got in line, or in transit as it is commonly called, and thus are available for obtaining the deviation :—The lighthouse and old lighthouse of South Foreland. Tyne Harbour lights. Bolthead and Start lighthouse. Prawle Point and Start Point. Eddystone and Rame Head. The Breakwater lighthouse, with various buoys, beacons, and distant objects laid down on the Channel Chart. Shipwash lightship, with the Orfordness light. Newarp, with Cockle lightship. Skerries and South Stack. Smalls lighthouse and Bishop lighthouse. Wolf and Longship lighthouses. Fastnet lighthouse and Mizen Peak.

10. *Supposing you have no means of ascertaining the magnetic bearing of a distant object when swinging your ship for deviations, how could you find it, approximately from equi-distant compass bearings; and at what distance as a rule should the object be from the ship?*

The magnetic bearing may be found, approximately, by taking the mean of four or more bearings taken with the ship's head on equi-distant points; the mean of the bearings with head on the cardinal points gives a good result if the ship is upright, otherwise the mean of the bearings with head east and west is better, as there should be no heeling error on those points to vitiate the result.

When using a distant mark it should be so far away, that the radius of the circle, along the circumference of which the standard compass moves as the ship goes round, subtends a smaller angle than is of practical consequence in navigating. Using the Traverse table, taking 1° as a course, and the distance of the object as a distance, the length of the radius which will cause 1° of error in the bearing will be the departure.

11. *Having taken the following bearings by your standard compass, of a distant object, find the object's magnetic bearing and also the deviation.*

MAGNETIC BEARING REQUIRED—N. 80° E.

Ship's head by Standard Compass	Bearing by Standard Compass	Deviation required	Ship's head by Standard Compass	Bearing by Standard Compass	Deviation required
North	N. 69° E.	11° E.	South	N. 94° E.	14° W.
N.E.	N. 54° E.	26° E.	S.W.	N. 98° E.	18° W.
East	N. 60° E.	20° E.	West	N. 97° E.	17° W.
S.E.	N. 78° E.	2° E.	N.W.	N. 90° E.	10° W.

To find the magnetic bearing take the mean of the eight bearings

Comp. direction	N.	N.E.	E.	S.E.
Mag. bearing	N. 80° E.	N. 80° E.	N. 80° E.	N. 80° E.
Comp. bearing	N. 69° E.	N. 54° E.	N. 60° E.	N. 78° E.
Deviations	11° E.	26° E	20° E.	2° E.

Comp. direction	S.	S.W.	W.	N.W.
Mag. bearing	N. 80° E.	N. 80° E.	N. 80° E.	N. 80° E.
Comp. bearing	N. 94° E.	N. 98° E.	N. 97° E.	N. 90° E.
Deviations	14° W.	18° W.	17° W.	10° W:

N. 69° E.
N. 54° E.
N. 60° E.
N. 78° E.
N. 94° E.
N. 98° E.
N. 97° E.
N. 90° E.

8)640°

N. 80° E. Mag. bearing required

To find the magnetic bearing.—When some bearings are easterly and others westerly, the difference between the easterly and westerly bearings must be taken before dividing by the number of bearings.

12. *With the above deviations construct a curve of deviations on a Napier's diagram and give the courses you would steer by standard compass to make good the following magnetic courses.*

Magnetic courses	N.N.E.	E.S.E.	S.S.E.	W.N.W.
Compass courses	N. 7° E.	S. 86° E.	S. 14° E.	N. 54½° W.

Explanation of Napier's Diagram.

IT IS THUS HEADED:

DIAGRAM.

NORTH POINT OF COMPASS DRAWN TO THE WEST.	NORTH POINT OF COMPASS DRAWN TO THE EAST.

There is a line drawn through the centre of the diagram (page 522) with a scale of degrees and points of the compass marked on it; this call the mesial line. At each point of the compass the mesial line is intersected at the angle of sixty degrees by dotted lines drawn upwards from right to left, and at the same angle downward from right to left by plain lines. These dotted and plain lines also cross each other where they intersect the mesial line at the angle of sixty degrees, and thus, with the parts of the mesial line intersected, form equilateral triangles.

To construct the Curve of Deviations

Easterly deviation is laid off on the right of the mesial line. Westerly deviation is laid off on the left of the mesial line. The dotted lines are used when the deviations are for ship's head by compass and the plain lines are used when the deviations are for magnetic points.

Commencing at North take in the dividers 11° from the mesial line and lay it off from North along the dotted line to the right, and mark the spot reached A; proceed in the same manner at N.E., East, and S.E., marking them B, C, and D respectively

Lay off the deviation at South, S.W., West, and N.W. in exactly the same manner, but to the left of the mesial line, and mark them E, F, G, and H respectively ; lay off the deviation at North at the bottom of the curve to the right and mark it A′. Now draw a flowing curve through A, B, C, D, E, F, G, H and A′ and it will show the deviations of the compass and the magnetic directions of ship's head for any point.

To find the Compass Course, the Magnetic Course being given

Place one leg of the dividers on the given magnetic course on the mesial line and measure along the plain line until the other leg reaches the curve ; now move the leg which is on the curve in the direction of the dotted line until it reaches the mesial line, and the point arrived at will be the compass course required.

Example.—The magnetic course being N.E. by E., find the compass course.

Ans. Compass course N. 33° E.

Example.—The magnetic course being S.W. by W., find the compass course.

Ans. Compass course S. 75° W.

To find the Magnetic Course, the Compass Course being given

Place one foot of the dividers on the given compass course on the mesial line and measure along the dotted line until the other leg reaches the curve. Now move the leg which is on the curve in the direction of the plain line until it reaches the mesial line, and the point reached is the magnetic course required.

Example.—You have steered S.S.W. by compass ; find the magnetic course.

Ans. Magnetic course S. $5\frac{1}{2}$° W.

Example.—You have steered E. by S. by compass ; find the magnetic course.

Ans. Magnetic course S. $62\frac{3}{4}$° E.

To Correct Bearings

Measure the deviation along the dotted line passing through the compass direction of ship's head on the mesial line and apply it to the bearings.

In the question the ship's head is E. by N. and the deviation from the curve is 22° E., and this applied to the bearings by compass in the usual way gives the magnetic bearings.

Compass bearing	N. 56° E.	Compass bearing	S. 11° E.
Deviation	22° E.	Deviation	22° E.
Magnetic bearing	N. 78° E.	Magnetic bearing	S. 11° W.

An examination of the curve on page 341 will make the subject clear.

13. *You have steered the following courses by the standard compass; find the magnetic courses from the curve of deviations.*

Compass courses	W.S.W.	N.N.W.	E.N.E.	S.S.W.
Magnetic courses	S. 49° W.	N. 23½° W.	S. 89° E.	S. 5½° W.

14. *You have taken the following bearings of two distant objects by your standard compass with the ship's head at E. by N.; find the magnetic bearings.*

Compass bearing	N.E. by E.	and	S. by E.
Magnetic bearing	N. 78° E.		S. 11° W.

15. *Do you expect the deviation to change; if so, state under what circumstances?*

I should expect the deviation to change. First, from lapse of time, especially in a new ship, also when she is kept or steered in a direction opposite to that in which she was built. The deviation may also be expected to change with change of geographical position, from sustaining strains from heavy seas, or shocks, such as from a collision. If a ship is steered for some time on one course, especially if near the east or west point, when the course is altered the north point of the compass may be drawn towards that part of the ship which was previously towards the south.

16. *How often is it desirable to test the accuracy of your table of deviations?*

For the reasons before given, the compasses of a new ship require to be more frequently tested than those of an old one, and those of a ship proceeding to the opposite hemisphere than those of a ship sailing on the same parallel. But under all circumstances the deviation should be tested for the course on which the ship is sailing as frequently as is practicable; and if a change in the deviation on such a course is observed, the accuracy of your table should be tested, at the earliest opportunity and, if necessary, corrected.

17. *What is meant by variation of the compass; what is it caused by; and where can you find the variation for any given position?*

Variation of the compass is the angle, at any place, between the true and magnetic meridians and is caused by the magnetic poles not coinciding with the geographical poles. The variation for any given position can be found on the Admiralty variation chart.

18. *The earth being regarded as a magnet, which is usually termed the blue, and which the red magnetic pole?*

The north magnetic pole is usually termed the blue and the south the red.

19. *Which end of a magnet (or compass needle) is usually termed the red or "marked" end, and which the blue?*

That end of a magnet or compass needle which points towards the north when it is suspended so as to move freely in the horizontal plane is usually termed the red, or marked end; the end which points towards the south is termed the blue.

20. *What effect has the pole of one magnet of either name on the pole of another magnet?*

The pole of one magnet of either name will repel the pole of the same name, and attract the pole of the contrary name of another magnet.

21. *What is meant by transient induced magnetism?*

Transient induced magnetism is magnetism which is instantly produced in soft iron when it is exposed to any magnetic force, such as that of the earth, and is parted with or changed immediately the inducing force is removed or changed. The near poles of the induced magnetism and of the inducing cause are always of contrary names.

22. *Which is the red and which is the blue pole of a mass of soft vertical iron, by induction, and what effect would the upper and lower ends of it have on the compass needle (a) in the northern hemisphere, (b) in the southern hemisphere, (c) on the magnetic equator?*

The red pole is at the lower end in the northern hemisphere, and the blue pole is at the upper end; in the southern hemisphere the red pole is at the upper end, and the blue pole is at the lower end. Therefore the upper end attracts the north end of the compass needle, and the lower end repels it in the northern hemisphere; and in the southern hemisphere the upper end repels the north end of the compass needle, and the lower end attracts it. On the magnetic equator one side is red and the other blue, so that the compass needle should not be affected by either end.

23. *Describe what is usually termed the sub-permanent* magnetism of an iron ship, and state when and how it is acquired, and which is the red and which the blue pole, and why it is called sub-permanent magnetism.*

The sub-permanent magnetism of an iron ship is first acquired from the earth's inductive influence while the ship is being built, but remains after she is launched, although it undergoes a considerable reduction, especially when she proceeds to sea or is subjected to concussion with her head in any other direction than that in which she was built. The red sub-permanent pole of the ship is that which was directed towards the blue or north magnetic pole of the earth, and the opposite extremity of the ship is the blue sub-permanent pole.

It is termed sub-permanent magnetism to distinguish the magnetism which is thus acquired by a ship while building, from the magnetism of a magnetised steel bar, which is of a much more permanent character.

24. *Describe the meaning of the expression co-efficient A.*

Co-efficient A represents a deviation of the same sign and amount, on all points of the compass. It has the sign + when that deviation is easterly, and the sign — when westerly. It should have no value when the iron is symmetrically situated beside the compass.

* The term "sub-permanent" magnetism in these questions is used in the original sense, as proposed by the late Sir G. B. Airy, to denote the character of the permanent magnetism of an iron ship as distinguished from the permanent magnetism of a magnetised steel bar. The terms "sub-permanent" and "permanent" throughout these questions may, therefore, be considered as synonymous.

25. *Describe the meaning of the expression co-efficient B, its signs and effects.*
The expression co-efficient B represents a magnetic force in the fore-and-aft line of the ship. It has the sign + when the north point of the compass is attracted towards the ship's head, and the sign — when the north point of the compass is drawn towards the ship's stern. The effects of a + B are easterly deviations with ship's head in the eastern semicircle of the compass, and westerly deviations in the western semicircle, attaining a maximum value on the east and west points, decreasing to zero on the north and south points, by compass.

The effects of a — B are easterly deviations with ship's head in the western semicircle, and westerly deviations with the ship's head in the eastern semicircle, with a maximum value on the east and west points, decreasing to zero on the north and south points, by compass.

26. *Describe the meaning of the expression co-efficient C, its signs and effects.*
The expression co-efficient C represents the athwartship component of sub-permanent magnetism. It has the sign + when the north point of the compass is attracted towards the starboard side, and the sign — when the north point of the compass is attracted towards the port side. + C causes easterly deviations with ship's head in the northern semicircle, and westerly deviations in the southern semicircle, attaining a maximum value on the north and south points and decreasing to zero on the east and west points, by compass.

— C causes westerly deviations with ship's head in the northern semicircle, and easterly in the southern semicircle, attaining a maximum value on the north and south points and decreasing to zero on the east and west points, by compass.

B and C are termed co-efficients of semicircular deviation, because the deviation they represent is always easterly in one semicircle and westerly in the other.

27. *Describe the meaning of the expression co-efficient D, its signs and effects.*
The expression co-efficient + or — D represents a magnetic force, caused by induction in horizontal soft iron, either fore-and-aft or athwartships, which so changes in direction, as the ship goes round, as to cause a deviation alternately easterly and westerly in successive quadrants, hence the name quadrantal deviation. + D causes easterly deviations with ship's head between N. and E., and S. and W.; and westerly deviations between S. and E., and N. and W.

— D gives results exactly the reverse to + D.

Both + D and — D have a maximum value on the four quadrantal points, and become zero on the cardinal points, by compass.

Co-efficient D very rarely has a — sign.

28. *Describe the meaning of the expression co-efficient E, its signs and effects.*
The expression co-efficient + or — E represents a magnetic force, caused by induction in horizontal iron, diagonal to the fore-and-aft line, or unsymmetrically distributed about the compass, which so changes in direction as the ship goes round, as to cause a deviation alternately easterly and westerly in successive quadrants. + E causes easterly deviations with ship's head

between N.E. and N.W., and S.E. and S.W.; and westerly deviations between N.E. and S.E., and N.W. and S.W.

— E gives results exactly the reverse to + E.

Both + E and — E have a maximum value on the cardinal points, and become zero on the four quadrantal points, but are usually very small in amount in compasses placed in the middle line of the ship.

D and E are termed co-efficients of quadrantal deviation. But as — D is very rare, and E has no value when the iron is symmetrical about the compass, the quadrantal deviation is nearly always due to + D.

29. *Would you expect any change to be caused in the error of your compass by the ship heeling over either from the effect of the wind or the cargo, &c.?*

Yes, because forces which were vertical when the ship was upright and caused no deviation, act obliquely when she heels, and cause deviation.

30. *The compasses of iron ships being more or less affected by what is termed the heeling error, on what courses is this error usually at its minimum, and on what courses at its maximum?*

The heeling error usually has a minimum value on courses east or west by compass, because the disturbing force is then acting in the direction of the compass needle and causes no deflection. It usually has a maximum value on courses near north or south, because the disturbing force is then acting at right angles to the compass needle and causes the greatest amount of deflection.

31. *Describe clearly the three principal causes of the heeling error on board ship.*

The three principal causes of the heeling error are vertical induction in transverse iron, induction in iron vertical to the ship's deck, and the vertical component of the sub-permanent magnetism. The part arising from vertical induction in transverse iron is due to the fact that such iron as beams, by departing from the horizontal position, and inclining to the vertical as the ship heels, acquires polarity in its ends by induction from the earth, of the same sign as that of vertical iron in that hemisphere. This polarity tends to draw the north point of the compass to one side or the other.

The part arising from induction in iron vertical to the ship's deck is due to the fact that such iron is not vertical to the earth when the ship heels. The amount of magnetism induced therein is less in quantity, but by the poles becoming on one side of the compass, instead of vertically under it, the north point is drawn to one side or the other.

The part arising from the vertical component of the ship's sub-permanent magnetism, arises from the fact that although the force remains the same in amount, in all latitudes and both hemispheres it becomes on one side or the other as the ship heels, and so deflects the compass needle.

32. *State to which side of the ship in the majority of cases is the north point of the compass drawn when the ship heels over in the northern hemisphere.*

To the weather or high side, because the higher ends of the beams, and the upper ends of vertical iron which terminate below the compass, and so go towards the high side when the ship heels, have blue magnetism by induction in the northern hemisphere, and draw the north point of the compass

to the high side. If the compass is near vertical iron, such as the funnel, having its upper end above the compass, the force to the high side is diminished.

33. *Under what conditions (that is, as regards position of ship whilst building, and the arrangement of iron in the ship) is the north point of the compass needle usually drawn to windward or the high side of the ship in the northern hemisphere, and if not allowed for, what effect has it on the assumed position of the ship when she is steering on northerly also on southerly courses in the northern hemisphere?*

In the northern hemisphere, if a ship is built head to the northward, and the compass is aft, the north point will be drawn downward by sub-permanent magnetism. If the compass is forward this downward force is much diminished. As the force downward in the ship becomes in part a horizontal force towards the high side when the ship heels, continuous athwartship iron would also draw the needle to the high side.

In the northern hemisphere the error thus caused will put the vessel to windward of her assumed position on northerly courses, and to leeward on southerly courses.

34. *Under what conditions (as in question 33) is the north point of the compass needle usually drawn to leeward or the low side of the ship in the northern hemisphere, and, if not allowed for, what effect would it have on the assumed position of the ship, when she is steering on northerly also on southerly courses, in the northern hemisphere?*

In the northern hemisphere, if a ship is built with head to the southward, and the compass is aft, the north point of the needle will be pushed upward by sub-permanent magnetism. If the compass is forward, this force is diminished. As an upward force in the ship becomes in part a horizontal force towards the low side when the ship heels, if the compass were placed between divided beams they would also repel it to the low side.

In the northern hemisphere the error thus caused, if not allowed for, will put the vessel to windward of her assumed position on southerly courses and to leeward on northerly courses.

35. *The effects being as you state, on what courses would you keep away, and on what courses would you keep closer to the wind in the northern hemisphere in order to make good a given compass course (a) when north point of compass is drawn to windward or the high side of ship; and (b) when drawn to leeward or the low side?*

When the north point of the compass is drawn to windward, keep away on northern courses and closer to the wind on southern courses. When the north point of the compass is drawn to leeward, keep closer to the wind on northern courses and keep away on southern courses.

36. *Does the same rule hold good in both hemispheres with regard to the heeling error?*

Yes.

37. *State clearly how that part of the heeling error due to the permanent part of the magnetism of the ship varies as the ship changes her position on the globe, and what is the reason of this?*

That part of the heeling error due to the permanent part of the ship's magnetism decreases as the ship approaches the magnetic equator where it is least but does not vanish; it increases again as the ship goes towards the south magnetic pole, and has the same name in both hemispheres. The reason for this is that although the sub-permanent magnetism is of the same amount in all latitudes and both hemispheres, the earth's horizontal force which directs the compass needle is greatest on the magnetic equator and least at the magnetic poles, therefore, the sub-permanent magnetism would have least effect where the horizontal directive force is greatest, and greatest effect where the horizontal directive force is least.

38. *State clearly how that part of the heeling error due to the induction in transverse iron (which was horizontal when ship was upright) and iron vertical to the ship's deck, varies as the ship changes her position on the globe.*

That part of the heeling error due to induction in vertical iron and transverse iron which when the ship was upright caused no heeling error varies as the tangent of the dip. It is therefore greatest in high latitudes, decreasing to zero on the magnetic equator where it is nil and increases again as the ship recedes from the magnetic equator into the southern hemisphere where it is of an opposite name.

39. *Your steering compass having a large error, show by "Beall's Compass Deviascope" how you would correct it by compensating magnets and soft iron (as usually practised by compass adjusters in the Mercantile Marine) in order to reduce the error within manageable limits. Show also how the heeling error can be compensated.*

This question is given for examination in practical compass adjustment on the deviascope. In order to prepare for such examination, and also to obtain much valuable information on the magnetism of iron ships, the "Handbook to the Deviascope" should be obtained. It is sold (price 1s., and Supplement 3d.) by J. D. Potter, 145, Minories, E. 1.

40. *As the co-efficient B (capable of being corrected) usually consists of two parts, one due to the permanent magnetism of the ship, and the other to vertical induction in soft iron, how should each of the two parts, strictly speaking, be corrected when compensating the compass?*

The part due to permanent magnetism should be corrected by magnets, the part due to vertical induction by an upright bar of iron on the opposite side to the disturbing force.

41. *If the whole of co-efficient B be corrected by a permanent magnet, as is usually done, what is likely to ensue as the ship changes her magnetic latitude?*

If the whole B is corrected by magnets, + B or — B will probably appear as the ship goes to the northward or southward, because the part of B depending on induction will increase as the ship goes from, and diminish as the ship goes towards the equator, and is of opposite name in the southern hemisphere.

42. *Provided the needles of your compass are not so long and powerful, and so near, as to cause the soft iron correctors to become magnetised by induction, would the co-efficient D if properly compensated be likely to remain so in all magnetic latitudes and both hemispheres? If so, state the reason why.*

If the correctors are not affected by the compass needles, the correction of the D will remain perfect in all magnetic latitudes. Because the D is caused by induction in the horizontal, soft iron of the ship from the earth's horizontal force, and the value of soft iron as a corrector depends upon the same force. Therefore, the disturbing force and the correcting force vary together, and, if once made equal, they remain equal. If D is not corrected it should always be of the same amount for the same reason.

43. *State at what distance, as a general rule, the magnets and soft iron correctors should be placed from the compass needles, and what will be the consequence if they are placed too near the needles.*

Magnets of the length generally used should not be placed nearer than twice the length of the magnets from the centre of the compass. Soft iron correctors should not be nearer than 1½ times the length of the needles from the centre of the compass

If the magnets are placed too near the compass needles, and the semi-circular deviation is exactly corrected on the cardinal points, that correction would not be perfect on other points. The error would be greatest on the quadrantal points.

If the quadrantal correctors are too near, the D would be found to be over-corrected when the ship goes to a place of less horizontal force, and under-corrected if she goes to a place of greater horizontal force.

If the iron bar for correcting the induced part of B is too near, either end of the needle will be drawn towards the bar when in its vicinity, and so vitiate the correction.

It is especially necessary to consider the distance of the correctors from the centre of the compass when the compass is suspended in such a manner as to allow the needles to alter their position relatively to the correctors.

44. *Is it necessary that the magnets used for compensating co-efficients B and C should be placed on the deck? If not, state where they may also be placed, and the rules to be observed in placing them into position.*

It is not necessary that the magnets for correcting the semi-circular deviation should be placed on the deck. The magnets for correcting the B must be horizontal, fore-and-aft, and the middle of the magnets in the same vertical athwartship plane as the centre of the compass. The magnets for correcting the C must be horizontal, athwartships, and the middle of the magnets in the same vertical fore-and-aft plane as the centre of the compass. If these conditions are fulfilled the magnets may be placed inside or on the outside of the binnacle, or in any convenient place. In wheel houses it is sometimes convenient to place them on a near bulkhead.

45. *Can the compensation of the heeling error be depended upon when the ship changes her latitude? If not, state the reason.*

Compensation of the heeling error cannot be depended on when the ship

changes her latitude, because part of it is due to induction in transverse and vertical soft iron. This part decreases as the ship approaches the magnetic equator, where it is zero, and is of contrary name in the two hemispheres. It is impossible that this can be compensated by a fixed magnet, which is the usual practice, except for the place where the ship is, and for places where the induced magnetism will be the same.

SYLLABUS OF EXAMINATION FOR EXTRA MASTER IN THE LAWS OF THE DEVIATION OF THE COMPASSES OF AN IRON SHIP, AND IN THE MEANS OF COMPENSATING OR CORRECTING IT.

From January 1, 1895, the following Syllabus will be substituted for the one at present in use in the examination of all candidates for the voluntary examination in Compass Deviation and for Extra Masters' Certificates. Candidates will be required to give correct written answers to at least 15 of the questions; these will be marked by a cross by the Examiner. They will also be required to prove by "Beall's Compass Deviascope" (1) their knowledge of the Tentative method of Compass Adjustment, and (2) that they really possess a good knowledge of what they have written, by showing on the Deviascope that they are acquainted with the practical application of the same, or of any other questions in the Syllabus that the Examiner in the course of the examination may think proper to touch upon. Questions 31, 61, 62, 69, 70, 72, and 92, will be marked by the Examiner in all cases. The other questions will be *constantly* varied.

1. *Describe an artificial magnet, and how a steel bar or needle is usually magnetised.*

An artificial magnet generally consists of a tempered steel bar, magnetised by the inductive action of another magnet, either natural or artificial. The loadstone and the earth itself are natural magnets. A very general method of magnetising a steel bar or a compass needle is by the means of a horse-shoe magnet; the needle to be magnetised is laid on a flat surface, and one of the poles of the horse-shoe magnet when pressed on the needle should be drawn from end to end. The other pole of the horse-shoe magnet should then be brought to the centre of the needle, and drawn several times in the opposite direction. This process should be repeated on the other side of the needle, care being taken that in all cases the same pole of the horse-shoe should be drawn to the same end of the needle. Steel bars for correcting compasses, and smaller pieces of steel for compass needles, are also magnetised by drawing each half over the poles of a strong compound magnet or of an electro-magnet.

2. *Which end of the compass needle, or a magnet, is commonly termed the red, and which the blue pole?*

That end of the compass needle or magnet which points towards the north when it is suspended so as to move freely in the horizontal plane is usually termed the red pole, and the end which points towards the south is termed the blue pole.

3. *Which is the red magnetic pole of the earth, and which the blue? and give their geographical positions.*

The northern magnetic pole is termed blue, and the southern red. The blue pole is in lat. 70° N. and long. 97° W. The red pole is about 73° S. and 155° E.

4. *What effect has the pole of one magnet of either name on the pole of the same name of another magnet, and what would be the consequence of the pole of one magnet of either name being brought near enough to affect the pole of contrary name, if in these cases both magnets were freely suspended?*

The pole of one magnet of either name will repel the pole of the same name of another magnet. If the pole of one magnet of either name is brought near to the pole of a contrary name of another magnet, the two poles will mutually attract each other.

5. *By applying this law to all magnets, natural as well as artificial, describe what would be the result on a magnetic bar or needle, freely suspended, but by weight or by the nature of its mounting constrained to preserve a horizontal position; and what would be the result, if so mounted, but free to move in every direction, the earth being regarded as a natural magnet?*

If the magnetic bar or needle can only move in a horizontal plane it will take up a position which will coincide with the direction of the magnetic meridian.

If the magnetic bar or needle is free to move in any direction it will take up a position which will coincide with the magnetic meridian, and the north end would dip below the horizon in the north magnetic hemisphere and the south end in the south magnetic hemisphere. At the magnetic equator it would be horizontal and at the magnetic poles vertical.

6. *What is the cause of the variation of the compass?*

Variation of the compass is caused by the magnetic poles of the earth not coinciding with the geographical poles.

7. *What is meant by the deviation of the compass?*

The deviation of the compass is the angle between the magnetic meridian and the direction of the compass needle and is caused by the iron of the ship, whether used in her construction and equipment, or in her as cargo.

8. *What is meant by the term "local attraction"; under what circumstances have ships' compasses, from recent careful investigation, been found to be affected by it, and name some of the localities in different parts of the world where this disturbance is to be found, and consequently where increased vigilance is necessary?*

The term "local attraction" may be understood to mean any disturbance of the compass by the magnetism of objects external to the ship. The term has especial reference to the disturbance caused by the magnetism of the ground in certain localities. As the effects of magnetism are not cut off by the intervening water, a ship's compasses may be affected in shallow water, though she may be at some distance from the land. The following places are given in the "Admiralty Manual" where, from well-authenticated observations, ships' compasses have been found to be disturbed :—

The approaches to Cossack, in North Australia; Cape St. Francis, Labrador; New Ireland and Bougainville, Solomon Islands; and Tumbora Volcano, Sumbawa Island, near Java; the coasts of Madagascar, especially near St. Mary's Island; Iceland and its adjacent waters; Odessa Bay and the shoal south of it; Isle de Los, West Coast of Africa.

9. *What do you understand by the term "soft" iron; and what are its properties as regards acquiring and retaining magnetism?*

The term "soft" iron is applied to iron which becomes instantly magnetised by induction from any magnetic force to which it is exposed, but which loses that magnetism, or changes its magnetic condition, when the inducing cause is removed or changed.

10. *What do you understand by the term "hard" iron; and what are its properties as regards acquiring and retaining magnetism?*

The term "hard" iron is applied to iron which is not easily magnetised by induction, but has the property of retaining the magnetism so acquired more or less permanently.

11. *Describe the meaning of the term "horizontal force" of the earth; where is it the greatest, and where the least, and what effect has it in respect to the increase or decrease of the directive force of the compass needle?*

The term "horizontal force" means the horizontal component of the earth's magnetic force. The earth's force increases and is more inclined from the horizontal as the latitude is increased. The angle at which it is inclined is called the dip. The dip and the earth's total force increase together, but in such a manner that the horizontal force diminishes as the magnetic poles are approached in both hemispheres. At the magnetic poles, the dip being 90°, there is no horizontal force. If the magnetism in the compass needles remains the same, the directive force on the compass needle increases or decreases in proportion to the horizontal force.

12. *Does the magnetic equator coincide with the geographical equator? If not, state clearly how it is situated.*

The magnetic equator—a term applied to a line on the earth's surface where its magnetic force is horizontal—does not coincide with the geographical equator. It intersects the latter in about 12° west longitude; continuing the line to the eastward, it keeps in north latitude, going as high as 10°, and crosses into south latitude in about 170° west longitude. It attains its greatest south latitude (14°) in about 45° west longitude, and rejoins the equator in 12° west longitude.

13. *Where can the values of the magnetic dip, the earth's horizontal force, and the variation, be found?*

Charts showing the magnetic dip and horizontal force are to be found in the "Admiralty Manual of the Deviation of the Compass." The variation is accurately shown on the Admiralty variation chart, by curves of equal variation drawn to each degree.

14. *State in what parts of the globe lying in the usual tracks of navigation the variation changes very rapidly, and what special precautions should be observed when navigating these localities; also why a " variation " chart is then very useful.*

By the variation chart it will be seen that the variation changes very rapidly on the coasts of Newfoundland, the Gulf of and River St. Lawrence, the East Coast of North America, the coasts of Brazil, to the southward and eastward of Madagascar, off the S.W. part of Australia, and the English Channel and its approaches.

In such localities, and in all cases where the ship's course is at right angles to the curves of equal variation, the course must be carefully adjusted to the changing variation. For that purpose the chart of variation curves is especially useful.

15. *Why is a knowledge of the magnetic dip and the earth's horizontal force important in dealing with compass deviations?*

Because the magnetism of the ship and its effects on the compass change when the dip and horizontal force change, and by knowing their values the changes can be calculated.

16. *Describe the meaning of the term " vertical force " of the earth; where is it the greatest and where the least?*

By the term " vertical force " is meant the vertical component of the earth's magnetic force. It is greatest at the magnetic poles where the whole force is vertical, and is zero at the magnetic equator where the whole force is horizontal.

17. *Would you expect a compass to be more seriously affected by any given disturbing force when near the magnetic equator, or near the poles? and state the reason.*

Any disturbing force must affect the compass least where the directive force on the needle is greatest. As the directive force on the needle depends on the horizontal force, which is greatest at the magnetic equator and zero at the poles, the effects of any disturbance on the needle must be least at the equator and greatest at the poles.

18. *State briefly (a) the essentials of an efficient compass; and (b) what you would consider a good arrangement of the needles (that is—whether long or short, single or double, &c.) with the view to good compensation.*

It is essential that the card should have the greatest possible magnetic power in its needles combined with the smallest possible weight in the whole card. The jewelled cap should be sound—that is, not worn nor cracked, and the pivot sharp and free from rust. If the card is placed on the pivot and deflected through a small angle from its position of rest, it should always

come back to exactly the same point. The card should be accurately divided and centred, and the point of the pivot should be in the same plane as the gimbals of the bowl. Standard compasses must be furnished with the means of taking the bearings of an object at any elevation, and of reading it off within a degree. When a compass is placed on board ship, the lubber line should be vertical and exactly in the fore-and-aft line from the centre of the card, and the bowl should swing freely in its gimbals. It is desirable that all compass bowls should be made of pure copper.

In order that the ends of the needles should be the required distance from the soft iron correctors, and that the effects of the correcting magnets on them should be uniform, it is desirable that the needles should be short. This necessitates the use of two or more needles to get the requisite magnetic power. By placing the needles parallel to each other, and an equal number on each side of the centre of the card, the space is free for the cap. Needles made of two laminæ of steel are said to be stronger for the same weight than when in one piece.

19. *In stowing away spare compass cards or magnets how would you place them with regard to each other, or what might be the probable consequence?*

Spare compass cards or magnets may be stowed over, or beside each other, or end on; but always with their unlike poles together. If poles of the same name are together they will probably weaken the magnetism of each other.

20. *State briefly the chief points to be considered when selecting a position for your compass on board ship, and what should be particularly guarded against?*

The standard compass should be in the midship line, in a convenient position for constant watching by the officer of the watch, and comparing with the steering compass, and should have a clear view all round for taking bearings. It should not be less than five feet from iron of any kind, and the proximity of vertical iron, and of iron which is liable to be changed in position, such as davits, derricks, ventilating cowls, &c., should be particularly guarded against. When electric lighting is used, the position of the dynamo has also to be considered, as it may disturb a compass at a distance of fifty or sixty feet.

Steering compasses must be so placed that the card can be clearly seen by the helmsman, and should be as far from any iron as circumstances will allow.

21. *What is meant by transient induced magnetism?*

Transient induced magnetism is magnetism which is instantly produced in soft iron when it is exposed to any magnetic force, such as that of the earth, and is parted with or changed immediately the inducing force is removed or changed. The near poles of the induced magnetism and of the inducing cause are always of contrary names.

22. *Which is the red and which the blue pole of a mass of soft vertical iron (or indeed of any soft iron not in a horizontal position) by induction, and what effect would the upper and lower ends of it have on a compass needle in the northern hemisphere?*

The red pole is at the lower end, and the blue pole is at the upper end,

therefore the upper end attracts the north end of the compass needle, and the lower end repels it, in the northern hemisphere.

23. *Which is the red and which the blue pole of a mass of soft vertical iron by induction, and what effect would the upper and lower ends of it have on the compass needle in the southern hemisphere?*

The red pole is at the upper end, and the blue pole is at the lower end, therefore the lower end attracts the north end of the compass needle, and the upper end repels it, in the southern hemisphere.

24. *What effect would a bar of soft vertical iron have on the compass needle on the magnetic equator?*

It would have no effect. As the earth's force is horizontal at magnetic equator, there could be no vertical induction, and therefore no disturbing force in the bar.

25. *Describe what is usually termed the sub-permanent* magnetism of an iron ship, and state when and how it is acquired, and which is the sub-permanent red and which is the blue pole, and why it is called sub-permanent magnetism.*

The sub-permanent magnetism of an iron ship is first acquired from the earth's inductive influence while the ship is being built, but remains after she is launched, although it undergoes a considerable reduction, especially when she proceeds to sea or is subjected to concussion with her head in any other direction than it was when she was built. The red sub-permanent pole of the ship is that which was directed towards the blue or north magnetic pole of the earth, when building, and the opposite extremity of the ship is the blue sub-permanent pole.

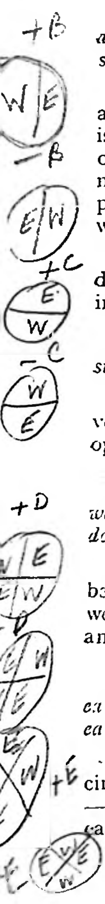
It is termed sub-permanent magnetism to distinguish the magnetism which is thus acquired by a ship while building, from the magnetism of a magnetised steel bar, which is of a much more permanent character.

26. *What is meant by "the composition of forces" and "the parallelogram of forces," and show how the knowledge of these is valuable in ascertaining and compensating the sub-permanent magnetism of an iron ship?*

A force which is the effect of two or more forces is called their *resultant*, and each of these forces separately is called a *component* of such resultant. The composition of forces is the method by which we find the resultant of two or more components. The resolution of forces is the means by which we separate each component from the resultant. Having observed the deviation of the compass with the ship's head in several directions, we employ a method for the resolution of forces to determine the value of co-efficients A, B, C, D, and E; and, having determined the value of these co-efficients, we make use of the composition of forces to ascertain the deviation of the compass while the ship's head is in any direction. The resultant of two forces acting in the same line is either the sum or difference of the two. But if the

* The term "sub-permanent" magnetism in the syllabus is used in the original sense, as proposed by the late Sir G. B. Airy, to denote the character of the permanent magnetism of an iron ship as distinguished from the permanent magnetism of a magnetised steel bar. The terms "sub-permanent" and "permanent" throughout the syllabus may, therefore, be considered as synonymous.

two forces act obliquely to each other, the parallelogram of forces must be employed in determining both the magnitude and direction of the resultant. For this purpose both forces must be represented in magnitude and in direction by the adjacent sides of a parallelogram, and the diagonal will represent both the magnitude and the direction of the resultant. Unless a ship is built with her head on one of the magnetic cardinal points, the sub-permanent magnetism will be oblique to the fore-and-aft midship line. The compass adjuster therefore, following the instructions of the Astronomer-Royal, resolves this force into two components, the force towards or from the bows by the co-efficient + B or — B, and that towards or from the starboard side by + C or — C. The former he compensates while the ship's head is east or west magnetic by a fore-and-aft magnet, the latter by an athwartship magnet while her head is north or south.



27. *Describe the nature of the co-efficients B and C plus (+), and minus (—) and the different magnetic forces they represent; also why they are said to produce semicircular deviations.*

The expression co-efficient B represents a magnetic force in the fore-and-aft line of the ship. It has the sign + when the north point of the compass is attracted towards the ship's head, and the sign — when the north point of the compass is drawn towards the ship's stern. Co-efficient C represents a magnetic force in the athwartship line. It has the sign + when the north point of the compass is attracted towards the starboard side, and the sign — when the north point of the compass is attracted towards the port side.

B and C are termed co-efficients of semicircular deviation, because the deviation they represent is always easterly in one semicircle, and westerly in the other.

28. *Can semicircular deviations be produced by any other force than the sub-permanent magnetism of the ship? If so, by what?*

Yes, —, a semicircular deviation is produced by vertical induction when vertical iron is nearer to the compass in one direction in the ship than in the opposite direction.

29. *On what points, by compass bearings of the ship's head, does + B give westerly deviation, and on what points does it give easterly; also on what points does — B give westerly, and on what points easterly?*

+ B gives westerly deviation with ship's head in the westerly semicircle between north and south, and easterly in the easterly semicircle; — B gives westerly deviation on the points of the compass where + B gives easterly, and easterly where + B gives westerly.

30. *On what points does + C give westerly deviation, and on what points easterly; also on what points does + C give westerly, and on what points easterly deviation?*

+ C gives westerly deviation with ship's head in the southern semicircle from east to west, and easterly deviation in the northern semicircle; — C gives westerly deviation with ship's head in the northern semicircle, and easterly deviation in the southern semicircle.

31. *The value of either co-efficient B or C being given, also the magnetic direction of the ship's head while she was being built, determine by the traverse tables the approximate value of the other co-efficient C or B; and the value of both these co-efficients being given, determine approximately the direction by compass of the ship's head whilst being built, assuming, of course, that these co-efficients resulted altogether from sub-permanent magnetism.*

If B be given to find C, enter the traverse table, the course of which agrees with the angle between the magnetic meridian and the direction of the ship's head while she was being built; adjacent to the value of B in the "latitude" column will be found that of C in the "departure" column; or if C be given, C in the "departure" column will adjoin B in the "latitude" column; but if B and C are given to find the direction of the ship's head when she was being built, enter the traverse table in which the value of B in the "latitude" column agrees with that of C as "departure"; the course will approximately give the angle between the magnetic meridian and the slip on which the ship was built. by Trig. or drawing Parallelogram to Scale.

See problems on compass deviation on pp. 555-6.

32. *Would you expect the greatest disturbance of the needle from the effects of sub-permanent magnetism alone to take place when ship's head is in same direction as when building, or when her head is at right angles to that direction; and in what direction of the ship's head would you expect to find the least disturbance?*

I should expect the greatest disturbance when the ship's head is at right angles to the direction in which she was built, and the least disturbance when in the same direction in which she was built. In the former case the ship's force is at right angles to the compass needle, and, in the latter case, in line with it.

33. *Describe quadrantal deviation, and state what co-efficients represent it; also what points of the ship's head, by compass, each of these co-efficients gives the greatest amount of deviation, and why it is called quadrantal deviation?*

Quadrantal deviation is produced by the induced magnetism of horizontal iron; it is represented by the co-efficients + D and — D and + E and — E. D gives the maximum deviation with the ship's head N.E., S.W., N.W., and S.E. by compass. Co-efficient E gives the greatest deviation when the ship's head bears N., S., E., or W. They are called quadrantal because the deviation produced by them is of a different name in successive quadrants.

34. *On what points of the compass will each of the co-efficients, D and E + and —, give easterly, and on what points westerly deviation?*

+ D gives easterly deviation on the N.E. and S.W. quadrants, and westerly deviation on the N.W. and S.E. quadrants. — D gives westerly deviation in the quadrants where + D gives easterly deviation, and easterly deviation where + D gives westerly. + E gives easterly deviation in the northerly and southerly quadrants, and westerly in easterly and westerly quadrants. — E gives westerly deviation in the quadrants where + E gives easterly deviation, and easterly deviation where + E gives westerly.

35. *What conditions of the iron of a ship will produce + D and what — D?*

Induction in continuous transverse iron, such as beams, produces + D; but when a beam is divided for any purpose,—D is produced in a compass placed between its divided parts. Induction in continuous fore-and-aft iron produces — D, but a compass placed between divided parts would have + D.

36. *State clearly, then, which end of horizontal iron running athwartship (such as beams, &c.), and of horizontal iron running fore-and-aft of a ship, acquires* red *and which* blue *polarity, by induction, when ship's head is at N.E., S.E., S.W., and N.W. respectively.*

When the ship's head is N.E. the fore end of fore-and-aft iron acquires red polarity, and the after end blue; and the port end of athwartship iron acquires red polarity, and the starboard end blue.

With head S.E. the fore end of fore-and-aft iron acquires blue polarity, and the after end red; and the port end of athwartship iron acquires red polarity, and the starboard end blue.

With head S.W. the fore end of fore-and-aft iron acquires blue polarity, and the after end red; and the port end of athwartship iron acquires blue polarity, and the starboard end red.

With head N.W., the fore end of fore-and-aft iron acquires red polarity. and the after end blue; and the port end of athwartship iron acquires blue polarity, and the starboard end red.

37. *Describe the nature of the deviation represented by co-efficients + A and — A, and describe the errors in the construction of the compass, and other causes, that frequently produce it.*

Co-efficient A represents a constant deviation—that is, a deviation of the same sign and amount on all points of the compass. It has the sign + when the deviation is easterly, and the sign — when westerly. Co-efficient A is sometimes produced by a certain unsymmetrical arrangement of iron, but more frequently from the poles of the needles not being parallel to a line drawn through the north and south points of the card, or from the lubber line being misplaced, or from an error in observing the magnetic bearing of the object on shore, or from an error in the estimated value of variation; also, in combination with E of the same sign and value, it is produced by magnetism induced and retained while the ship is being swung. If the cap or pivot is defective, there will be an A from the card being drawn round by friction in the direction in which the ship's head is moving.

38. *What is the object of compensating the compass by magnets, &c., and what are the general advantages of a compensated compass over an uncompensated one?*

The primary object is to equalize the directive force of the compass on all points and to reduce the deviation to the smallest possible amount. The advantages are that a small deviation is obviously more convenient for navigation than a large one; also with a large deviation the directive force on the compass needle is proportionally diminished with the ship's head on some points.

39. *Before adjusting the compass of an iron ship, what is it desirable to do with the view to eliminating, as far as possible, what may be termed the unstable part of the magnetism of the ship?*

It is desirable to lay the ship with her head in the opposite direction to that in which she was built.

40. *Describe clearly the tentative method of compass adjustment (that is, the compensation of co-efficients B, C, and D, with ship upright) as generally practised by compass adjusters in ships of the mercantile marine.*

The D, presumed to be known or previously found, is first corrected by soft iron placed on each side of the binnacle. The ship's head is then placed in succession on any two adjacent cardinal points magnetic, and the deviation reduced to zero on those points by magnets placed fore-and-aft, and athwartships, thus correcting the B and C.

41. *State at what distance, as a general rule, the magnets and soft iron correctors should be placed from the compass needles, and what will be the consequence if they are placed too near the needles.*

Magnets should not be placed nearer the centre of the compass than twice the length of the magnets. Soft iron corrector should not be nearer the centre of the compass than one and a quarter times the length of the needles.

If the magnets are placed too near the compass needles, and the deviation is exactly corrected on the cardinal points, that correction will not be perfect on other points. The error would be greatest on the quadrantal points.

If the quadrantal correctors are too near, the spheres would probably become magnetised by induction from the compass needles, and rendered useless as correctors.

If the iron bar for correcting the induced part of B is too near, either end of the needle will be drawn towards the bar when in its vicinity, because the bar will be magnetised by induction from the compass needles.

42. *Is it necessary that the magnets used for compensating co-efficients B and C should be placed on the deck? If not, state where they may also be placed, and the rules to be observed in placing them into position.*

It is not necessary that the magnets for correcting the semicircular deviation should be placed on the deck. The magnets for correcting the B must be horizontal, fore-and-aft, and the middle of the magnets in the same vertical athwartship plane as the centre of the compass. The magnets for correcting the C must be horizontal, athwartships, and the middle of the magnets in the same vertical fore-and-aft plane as the centre of the compass. If these conditions are fulfilled, the magnets may be placed inside or on the outside of the binnacle, or in any convenient place. In wheel houses it is sometimes convenient to place them on a near bulkhead.

43. *Does the B found on board ship usually arise altogether from sub-permanent magnetism, or does part of it usually arise from some other cause or causes?*

In the positions compasses are necessarily placed, part of the B is usually caused by magnetism induced by the earth's vertical force in vertical iron.

44. *If the part of B due to induced magnetism in vertical soft iron, as well as the part due to sub-permanent magnetism, is corrected by a magnet alone, as is generally the case, what is frequently the consequence on the ship changing her magnetic latitude and hemisphere?*

If the whole B is corrected by magnets, + B or — B will probably appear as the ship goes to the northward or southward. Because the part of B depending on induction will increase as the ship goes from, and diminish as the ship goes towards the magnetic equator, and of reverse sign when the ship gets in the southern hemisphere.

45. *How should each of these two parts of B then, strictly speaking, be compensated?*

The part due to sub-permanent magnetism should be corrected by magnets, the part due to vertical induction by an upright bar of iron on the opposite side to the disturbing force.

46. *Assuming, for the sake of clearness, that your steering compass is unavoidably placed very near to the head of the stern-post (and other vertical iron at the stern), thereby causing a very large — B from induced magnetism; describe briefly any method by which the approximate position for the compensating vertical iron bar (Flinder's or Rundell's) could be estimated in order to reduce the error; describe also how you would proceed, in order to improve, if not to perfect, its position after observations have been made on the magnetic equator.*

In the case mentioned a correcting bar would be required before the compass. In some cases the value of the part of B which is caused by vertical induction may be found approximately from the direction of ship's head while building, and the value of B and C found by observation. As the C in a compass placed amidships is generally caused by sub-permanent magnetism only, if the traverse table is entered with the direction of head when building as a course, and the C as a departure, the diff. lat. will represent the value of the sub-permanent part of B. It will have the same sign as the C when the ship was built with her head in the S.E. or N.W. quadrants, and the contrary sign to C when built with head in N.E. and S.W. quadrants. The difference between the B thus found, and that found by observation, will be the part due to induction, which should be corrected by a vertical bar. At the magnetic equator there can be no B from vertical induction, therefore the deviation of the uncorrected compass, with head east or west, is the B due to sub-permanent magnetism. If that is corrected by magnets, any B which appears as the ship leaves the equator will be due solely to vertical induction, and if that is corrected by a vertical bar, the B will be perfectly corrected for both induced and sub-permanent magnetism.

47. *State if standard compasses, as well as steering compasses, are generally subject to this disturbance from induced magnetism in vertical iron; also whether the attraction in all cases is found to be towards the stern; and if not, state the conditions under which it might be toward the bow, and how the compensating soft iron bar should then be placed.*

Standard compasses, from the position in which they are placed, are not generally so subject to disturbance from vertical induction as steering compasses, and the attraction is not always towards the stern. If the standard compass is near the funnel, and abaft it, the north point of the compass will be drawn towards the bow in the northern hemisphere, and the compensating bar should then be placed abaft the compass.

48. *Generally speaking, does the magnetism induced in vertical iron usually have any effect in producing the co-efficient C, ship upright, or is it generally produced by sub-permanent magnetism alone? State also your reasons for saying so.*

Generally, when the compass is in the midship line, no part of the C is caused by vertical induction, but is caused by sub-permanent magnetism alone. Because the compass is at equal distance from the upper part of vertical iron of the ship's frame on each side, and the attraction to one side which would cause a C is counteracted by the attraction to the other side.

49. *Provided the needles of your compass are not so long and powerful, and so near, as to cause the soft iron correctors to become magnetised by induction, would the co-efficient D, if properly compensated as you have described (Ans. 40), be likely to remain so in all latitudes and both hemispheres? If so, state the reason why.*

If the correctors are not affected by the compass needles, the correction of the D will remain perfect in all magnetic latitudes. Because the D is caused by induction in the iron of the ship from the earth's horizontal force, and the value of soft iron as a corrector depends upon the same force. Therefore the disturbing force and the correcting force vary together, and if once made equal they remain equal.

50. *Under what circumstances does the character of A and E so change as to render it desirable that these co-efficients should be disregarded or modified?*

When a ship's head is moving round, the north point of the compass stands a little from its proper position in the opposite direction to that in which the head is going. The error thus caused is greatest when the head is north or south, and least when east or west. The effect on the co-efficients is to give + A and + E when the ship is swung to the left, and — A and — E when swung to the right, unless the ship's head is steadied on each point. A and E may, therefore, be disregarded when small, as they should be in a standard compass.

51. *Supposing your compasses were allowed to remain uncompensated, explain clearly what would be the probable changes (ship upright) in the deviations produced separately, by (1) the sub-permanent magnetism of the ship alone, (2) by the induced magnetism in vertical soft iron; (a) on reaching the equator, (b) in the southern hemisphere.*

The deviation due to the sub-permanent magnetism would decrease as the ship approached the magnetic equator, would be least when on it, but

does not vanish, and would increase again as the ship receded from the equator into the southern magnetic hemisphere, still retaining the same name. The part due to induction in vertical soft iron decreases as the ship approaches the magnetic equator, where it is zero, and increases again as the ship goes into the southern magnetic hemisphere and is of an opposite name.

52. *Assuming you were able to arrive at the proper proportions to be corrected and were then to exactly compensate the sub-permanent magnetism of the ship by means of a permanent magnet, and the induced magnetism in vertical iron by a soft iron bar, would you expect any deviation to take place in your compass as the ship changed her latitude and hemisphere? And state your reasons for saying so.*

I should expect but little change in the deviation, especially in an old ship. Because the sub-permanent magnetism of the ship, being corrected by permanent magnets, the only change that could take place would be from the less permanent character of the ship's magnetism. As the vertical induction in the ship's iron is compensated by vertical induction of the same amount in the bar, both being magnetised by the same force, there can be no change from that cause.

53. *Supposing the co-efficient D from horizontal soft iron were allowed to remain uncompensated, would you, or would you not, expect the D to differ in name or amount on the ship changing her magnetic latitude and hemisphere? And state the reason.*

I should not expect the D to change its name or amount. The directive force of the needle depends on the earth's horizontal force, and the disturbing force causing the D depends on induction from the same force. Therefore, the pointing force of the needle and the disturbing force maintain the same relative value, and the amount of deflection produced by the latter remains the same.

54. *Describe how you would determine the deviation of your compass* (1) *by reciprocal bearings,* (2) *by figures on the dock walls,* (3) *by bearings of a distant object.*

To determine the deviation of a compass by reciprocal bearings, a compass should be placed on shore where there is no iron in the vicinity, and where it could be conveniently seen from the standard compass. As the ship is swung round, observe the bearing of the shore compass from the standard when the ship's head is steady on each required point; and, by signal, have the bearing of the standard taken by the shore compass at the same instant. The difference between the bearing taken on board and the opposite of the shore bearing is the deviation.

By figures on the dock wall. When they are available, as at Liverpool, the difference between the bearing of the object in the background by the standard compass, and the bearing marked on the wall exactly in line with the object at the same instant, is the error for any point the ship's head may be on. Apply the variation and get the deviation.

By bearing of distant object. When the magnetic bearing is known, the difference between it and the bearing observed by the standard, with head

on any point, is the deviation for that point. When the magnetic bearing is not known it may be taken from the chart, or it may be found by getting the difference of bearing between it and the sun, and applying that difference to the sun's true bearing, computed or found from azimuth tables, and allowing the variation to find the magnetic bearing.

55. *Describe, in detail, how you would determine the deviation of your compass by the bearings of the sun. Also by a star or planet.*

To determine the deviation by bearings of the sun or other celestial object, the exact time must be noted and the bearing of the object taken, when the ship's head is on each point. The true bearing of the object at each observation can be computed or found by azimuth tables. By applying the variation with its proper sign to the true bearing, the magnetic bearing is found. The difference between the magnetic bearing and the compass bearing is the deviation on each point.

56. *Describe the uses to which the Napier's diagram can be applied, and its special advantages.*

By Napier's diagram, a complete table of deviation can be formed from observations made on a small number of points, and from observations made at irregular intervals round the compass. Its special advantage is the facilities it gives for converting compass courses or bearings into magnetic courses or bearings, and the reverse.

57. *Describe clearly how the Napier's diagram is constructed.*

(Refer to Curve in Section, Napier's diagram, p. 341.)

58. *For accuracy, what is the least number of points to which the ship's head should be brought for constructing a complete curve of deviations, or a complete table of deviations?*

For an accurate table the deviation must be obtained with the ship's head on the four cardinal points and on the four intermediate or quadrantal points.

59. *Nearing land, and being anxious to check your deviations on a few courses you may probably require to steer, what is the least number of points it would be necessary to steady the ship's head upon, if making use of a Napier's diagram, in order to ascertain the deviation on each of the points, say in a quadrant of the compass? and describe clearly how you would do this at sea.*

If the deviation is small, probably an observation on the course, and on the fourth point on each side of it, will be sufficient; but it is preferable to take every alternate point.

60. *Supposing you have no means of ascertaining the magnetic bearing of the distant object when swinging your ship for deviations, how could you find it, approximately, from equi-distant compass bearings; and how far, as a rule, should the object be from the ship when swinging, or steaming round?*

The magnetic bearing may be found approximately by taking the mean of four or more bearings taken with head on equi-distant points; the mean of bearings with head on the cardinal points gives a good result if the ship is upright, otherwise the mean of the bearings with head East and West is better, as there would be no heeling error on those points.

When using a distant mark, it should be so far away that the radius of the circle, along the circumference of which the standard compass moves as the ship goes round, subtends a smaller angle than is of practical consequence in navigating. Using the traverse table, taking 1° as a course, and the distance of the object as a distance, the length of the radius which will cause 1° of error in the bearing will be the departure.

61. *Having taken the following equi-distant compass bearings of a distant object, find the object's magnetic bearings, and thence the deviations:—*

(See Answer to Question 11, page 522.)

62. *Assuming the deviations observed with ship's head by compass to be as follows [or as in question 61, whichever may be given], determine the value of the co-efficients A, B, C, D, and E, and from them construct a complete table of deviations (or for as many points as the Examiner may direct).*

See p. 558.

63. *When swinging your ship, if it be required to construct deviation tables for two or more compasses situated in different parts of the vessel, describe the process, and how you would employ the Napier's diagram for this purpose.*

Note the direction of head by each compass, at the same instant that the deviation is observed at the standard. Find the magnetic direction of head at each observation, by allowing the observed deviation to the direction of head by standard. By comparing the magnetic direction of head thus found, with the noted direction of head by each compass, its deviation may be found at irregular intervals round the compass. If the deviation for each compass is plotted on the diagram and the curve drawn, the deviation both for the compass points and the magnetic points can be obtained.

64. *State your rule for determining whether deviation is easterly or westerly.*

If the magnetic bearing be to the right of the compass bearing the deviation it is easterly, but if it be to the left it is westerly.

65. *Is a knowledge of the value of the various co-efficients of any advantage? If so, state why.*

A knowledge of the value of the various co-efficients is of advantage, because by their means the whole effect of the ship's magnetism in causing deviation at any particular compass can be expressed, and the parts due to the several forces defined in five terms of which two are generally zero. Also a table of deviation can be computed from any given co-efficients.

66. *Describe (a) what is commonly known by the term "retentive" or "retained" magnetism, and how the ship acquires it when in port and at sea; (b) its effect on the compass needle whilst ship's head continues in the same direction; (c) the immediate consequence when the direction of the ship's head is altered; and (d) the special precautions to be invariably observed at sea on the alteration of the ship's course.*

Retentive is a term applied to magnetism of a more transient character than that termed sub-permanent. It presents itself when a vessel has been kept for some time in one direction, either in port or at sea, especially if during that time she has been subjected to strains or shocks. She thus acquires magnetism by induction from the earth, which, without being permanent, may remain for some time. As the earth's force induces blue magnetism towards the south, and red magnetism towards the north, it causes no deviation while the ship remains on the same course, but a small diminution of the directive force on the needle. When the course is altered the needle will be drawn towards that part of the ship which had been previously south. This is especially the case when a ship has been steering near east or west, and the course is altered to near north or south. The deviation should always be observed and watched when the course is changed, and when the deviation cannot be observed the possible error in the course from retentive magnetism should be guarded against.

67. *Describe a "dumb-card" or "Pelorus," and its use (a) in compensating a compass, (b) in determining the deviation.*

The "dumb-card" or "Pelorus" consists of a circular plate of metal, graduated like a compass card, and so gimballed that it may be revolved round a central pivot, in a horizontal plane. Adjacent to the circumference is a mark, similar to the lubber line of a compass. It is fitted with sight-vanes, shades, and reflector, for taking bearings.

To Compensate the Compass

Clamp the sight-vanes at the known magnetic bearing of the sun or other celestial object, then clamp the required point at the lubber line of the Pelorus and swing the ship until the object is seen through the sight-vanes, when her head will be in the magnetic direction shown at the lubber line of the Pelorus; the magnets can then be moved until the compass shows the ship's head to be in the same direction as that shown by the Pelorus.

The lubber line of the instrument having been fixed parallel to the fore-and-aft midship line of the ship, screw the sight-vanes to the card at the known magnetic bearing of the distant object; then, on turning the card with sight-vanes to the object the bearing of the ship's head correct magnetic will be found opposite the lubber line; the difference between the magnetic direction as shown by Pelorus and the direction as shown by compass will be the deviation for that point.

68. *If you determine the deviation by an azimuth or an amplitude of a heavenly body, it is then combined with variation, which together is sometimes called the* correction *for the compass. State when the deviation is the difference between the variation and the* correction, *and when the sum; and when it is of the same name as that of the* correction, *and when of the contrary name.*

When the correction and the variation are of the same name, their difference is the deviation. If the correction is greater than the variation, the deviation takes the name of both. But when the correction is less, the deviation takes the opposite name. When the correction and the variation are of different names, the deviation is their sum, and is east when the correction is east, and west when the correction is west.

69. *In observing azimuths of heavenly bodies, the best method is by "time azimuths," since these can be observed without any altitude when the ship is in port, or when the horizon cannot be defined from any cause. Given the sun's declination, the hour of the day, and the latitude, to find the true bearing of the sun.** See problems, p. 561.

70. *By night, if it be desirable to observe the* correction *of the compass. Given the day of the year, and time at ship, also the latitude of the place, to determine what stars will be in good position for this purpose.*

See problems, p. 562.

71. *If your correcting magnets are so mounted that their positions can be altered, describe the process by which, on open sea, you can place the ship's head correct magnetic N. (or S.), and correct magnetic E. (or W.), and can make the correction perfect.*

The correct magnetic bearing of a heavenly body having been previously computed for the time the correction is to be made, fix the cardinal point to which the ship's head is to be brought to the lubber line of a dumb-card, placed as before directed in answer to question 67; then screw the sight-vanes to the card at the computed bearing of the heavenly body, and swing the ship till the sights are directed to the body. An azimuth compass with its card stopped at the cardinal point may be used instead of a dumb-card.

72. *Given the name of a star, the time, the place of ship, the variation of the compass, and the bearing of the star by compass. Determine the deviation, and name it east or west.*

See problems, p. 563.

73. *Would you expect any change to be caused in the error of your compass by the ship heeling over either from the effect of the wind or the cargo?*

Yes, because when a ship heels a magnetic force previously vertical may be no longer so, and the position of the iron about a compass may be so changed with reference to the earth's magnetic force, that new forces may arise from induction.

74. *Describe clearly the three principal causes of the heeling error on board an iron ship.*

The three principal causes of the heeling error are vertical induction in transverse iron, induction in iron vertical to the ship's deck, and the vertical component of the sub-permanent magnetism. The part arising

* The process of finding time azimuths by the ordinary formulæ of spherical trigonometry is tedious, and since on board an iron ship these observations should be often repeated, the candidate will be allowed to use any table or graphic or linear method that will solve the problem within a half of a degree, the altitude of the heavenly body not being given.

from vertical induction in transverse iron is due to the fact that such iron as beams, by departing from the horizontal position and inclining to the vertical as the ship heels, acquires polarity in its ends by induction from the earth, of the same sign as that of vertical iron in that hemisphere. This polarity tends to draw the north point of the compass to one side or the other.

The part arising from induction in iron vertical to the ship's deck is due to the fact that such iron is not vertical to the earth when the ship heels. The amount of magnetism induced therein is less in quantity, but by the poles becoming on one side of the compass instead of vertically under it, the north point is drawn to one side or the other.

The part arising from the vertical component of the ship's sub-permanent magnetism arises from the fact that, although the force remains the same in amount, it becomes on one side or the other as the ship heels, and so deflects the compass needle.

75. *Towards which side of the ship would that part of magnetism induced in continuous transverse iron (which was horizontal while ship was upright) help to draw the north point of the needle when ship heels over (a) in the northern hemisphere, (b) in the southern hemisphere?*

The north point of the needle would be drawn towards the windward or high side in the northern hemisphere, and towards the low side in the southern hemisphere.

76. *Supposing the compass were placed between the two parts of a divided beam or other athwartship iron, towards which side of the ship would iron so situated help to draw the north point of the needle when ship heels over (a) in the northern hemisphere, (b) in the southern hemisphere?*

In the case of divided athwartship iron, the poles beside the compass would have contrary polarity to those of the undivided iron of the previous question. Therefore the north point of the needle would be drawn to the low side in the northern hemisphere, and to the high side in the southern hemisphere.

77. *Would you expect that part of the magnetism induced in iron exactly perpendicular to the ship's deck, such as stanchions, bulkheads, &c., if below the compass, to cause any part of the heeling error when ship heels over, and if so, towards which side of the ship (a) in the northern hemisphere, (b) in the southern hemisphere?*

Yes, for the reason stated in answer to question 74. The north point of the compass will be drawn to the weather or high side in the northern hemisphere, and to the low side in the southern hemisphere, when the ship heels.

78. *If an ordinary standard compass placed higher than the iron top sides be compensated whilst the ship is upright, what co-efficient will be affected by heeling?*

Co-efficient C.

79. *Under what conditions (that is, as regards position whilst building and the arrangement of iron in the ship) is the north point of the compass needle usually drawn to windward, or the high side of the ship, in the northern hemisphere?*

In the northern hemisphere, if a ship is built head to the northward, and the compass is placed aft, the north point of the compass needle will be drawn downward by sub-permanent magnetism; this downward force, when the ship heels over, becomes in part a horizontal force to the high side; also, if the beams extend across the ship they will tend to draw the needle to the high side. If the compass is placed forward the downward force is much diminished.

80. *Under what conditions, as a rule, is the north point of the compass needle usually drawn to leeward, or the low side of the ship, in the northern hemisphere?*

In the northern hemisphere, if a ship is built with head to the southward, and the compass is aft, the north point of the needle will be pushed upward by sub-permanent magnetism; if the compass is forward this force is diminished. The upward force in the ship becomes in part a horizontal force when the ship heels, drawing the north point of the compass to the low side. Also if the transverse beams were divided and the compass placed between the divided ends the north point of the compass would be repelled to the low side when the ship heeled over.

81. *State to which side of the ship, in the majority of cases, is the north point of the compass drawn when ship heels over in the northern hemisphere, and when this is the case, and it is not allowed for, what effect has it on the assumed position of the ship when she is steering on northerly, and also on southerly courses?*

In the majority of cases the north point of the needle is drawn to the weather or high side in the northern hemisphere. When this is the case and it is not allowed for, a ship will be found to windward of her assumed position when steering to the northward, and to leeward when steering to the southward.

82. *On what courses would you keep away, and on what courses would you keep closer to the wind in both the northern and southern hemispheres in order to make good a given compass course (a) when north point of compass is drawn to windward, or the high side of ship, and (b) when drawn to leeward or the low side?*

When the north point is drawn to windward, keep away on northerly courses, and closer to the wind on southerly courses. When the north point is drawn to leeward, keep away on southerly courses, and closer to the wind on northerly courses.

83. *If a ship is beating to windward; when she tacks, under what circumstances will the heeling error retain the same name, and under what circumstances will it take the contrary name?*

When a ship is beating to windward the error will retain the same name—if on one tack her head is northerly and on the other tack southerly; but if on both tacks her head is either northerly or southerly, the deviation will be easterly on one tack and westerly on the other.

84. *If a ship is placed on the opposite tack by the change of wind, the ship's course being the same by compass, will the heeling error change its name ?*

In such a case the name of the heeling error will change.

85. *In which direction of the ship's head does the heeling error attain its maximum value, and in which direction does it generally vanish ?*

The heeling error attains its maximum value when the ship's head is north or south, and it generally vanishes when the ship's head is east or west ; because in the first case the disturbing force is at right angles to the compass needle, and in the latter case in line with it.

86. *Explain clearly how that part of the heeling error due to the permanent part of the magnetism of the ship varies as the ship changes her geographical position, and what is the reason of this ?*

That part of the heeling due to the permanent part of the ship's magnetism decreases as the ship approaches the magnetic equator, but does not vanish, and increases as she recedes from the magnetic equator into the southern hemisphere, still retaining the same name. This is due to the permanent magnetism being always of the same amount, but the horizontal directive force of the compass needle is greatest on the magnetic equator and least at the poles, therefore the heeling error must be greatest in high latitudes and least on the magnetic equator. It varies inversely as the horizontal force.

87. *Explain clearly how that part of the heeling error due to the induction in transverse iron (which was horizontal when ship was upright), and iron vertical to the ship's deck, varies as the ship changes her geographical position.*

When the ship heels, the upper ends of transverse iron, such as beams, acquire by induction magnetism of the same name as the upper ends of iron vertical to the deck, both being of the same name as the magnetism of the earth in each hemisphere. The amount of magnetism induced varies as the earth's vertical force, which is zero about the magnetic equator, and increases in both hemispheres going from the equator. Therefore, induced magnetism must draw the north point of the compass to the high side in the northern hemisphere, and to the low side in the southern hemisphere, in an increasing amount as the ship goes away from the equator. The heeling error from vertical induction varies as the tangent of the dip.

88. *What, then, would be the probable nature of the heeling error, that is, whether to high or low side of the ship, and whether the error would be equal to the sum or difference, &c., of the forces given* (1) *in high north latitudes*, (2) *on magnetic equator*, (3) *in high south latitudes? Assuming the polarity of the sub-permanent magnetism of the ship under, and affecting, the compass, to be as given below; the vertical induction in soft iron, of course, obeying the ordinary laws in the above geographical positions* (1), (2), (3).

(*a*) *In cases where the effect of* red *vertical sub-permanent magnetism is equal to that of the vertical induction in the soft iron of the ship.*

In high north latitude, no heeling error, it depending on the difference of the forces given. On the equator, heeling error from sub-permanent magnetism only, drawing north point towards low side. In high south latitude, north point drawn towards low side by the sum of the forces given.

(*b*) *Where the effect of* red *vertical sub-permanent magnetism is greater than that of the vertical induction in the soft iron.*

In high north latitude, north point drawn towards low side by the difference in the forces given. On the equator, north point drawn towards low side by sub-permanent magnetism only. In high south latitude, north point drawn towards low side by sum of forces given.

(*c*) *Where the effect of* red *vertical sub-permanent magnetism is less than that of the vertical induction in the soft iron.*

In high north latitude, north point drawn towards high side by difference of forces given. On the equator, north point drawn towards low side by sub-permanent magnetism only. In high south latitude, north point drawn towards low side by sum of forces given.

(*d*) *Where the effect of* blue *vertical sub-permanent magnetism is equal to that of the vertical induction in the soft iron.*

In high north latitude, north point drawn to high side by the sum of the forces given On the equator, north point to high side by sub-permanent magnetism only. In high south latitude, no heeling error, it depending on the difference of the forces given.

(*e*) *Where the effect of* blue *vertical sub-permanent magnetism is greater than that of the vertical induction in the soft iron.*

In high north latitude, north point drawn towards high side by the sum of the forces given. On the equator, north point to high side by sub-permanent magnetism only. In high south latitude, north point is drawn to high side by the difference of the forces given.

(*f*) *Where the effect of* blue *vertical sub-permanent magnetism is less than that of the vertical induction in the soft iron.*

In high north latitude, north point is drawn towards the high side by the sum of the forces given. On the equator, the north point is drawn towards the high side by sub-permanent magnetism only. In high south latitude, the north point is drawn towards the low side by the difference of the forces given.

89. *Can the heeling error be compensated? If so, state the means to be employed, and how the compensation may be effected.*

The heeling error can be compensated. The means employed is to place a magnet vertically under the centre of the compass, with the upper end so near to the needles as to counteract the ship's vertical force, whether arising from sub-permanent magnetism or from induction in vertical iron. The dipping needle is taken on shore in a place free from *local attraction*, and made to assume a horizontal position by means of a small sliding weight; it is then brought on board and put in the exact position which the compass is to occupy, with the ship's head east magnetic (although the direction makes very little difference); then, if the north end dips down, the north end of the vertical magnet is placed uppermost, but if the north end is repelled upwards the south end of the vertical magnet is placed uppermost to draw it back to the horizontal.

Method II.—By placing the ship's head north or south, and heeling the ship and observing to which side the north end of the compass is drawn; if it is drawn to the high side, place the vertical magnet " red end up " and move it to and fro until the ship's head is north or south by compass; if it is drawn to the lee side place the magnet blue end up.

90. *Can the compensation of the heeling error be depended on in every latitude? If not, state the reason.*

Compensation of the heeling error cannot be depended on when the ship changes her latitude, because part of it is due to induction in transverse and vertical iron. This part decreases as the ship approaches the magnetic equator, where it is zero, and is of contrary names in the two hemispheres. It is impossible that this can be compensated by a fixed magnet, which is the usual practice, except for the place where the ship is, and for places where the induced magnetism will be the same.

91. *Do the soft iron correctors used for compensating the co-efficient + D have any effect on the compass needle when the ship heels over, and if so, do they draw the needle towards the low or the high side of the ship, and do they counteract, or otherwise, the effect produced by the vertical induction in the soft iron; (a) in the northern hemisphere, (b) in the southern hemisphere: and what is the reason of this?*

The soft iron correctors for + D have the same effect as the divided beam mentioned in question 76. They draw the north point of the compass towards the low side in the northern hemisphere, and towards the high side in the southern hemisphere. Thus they tend to correct the effects of vertical induction in both hemispheres.

92. *Given the heel, the direction of the ship's head by compass, and the heeling error observed, to find the approximate heeling error, with a greater or less given heel, and with the ship's head on some other named point of the compass, the ship's magnetic latitude being in both cases the same.*

When the heeling error has been observed, it is desirable to reduce it to the error for 10° heel, when the ship's head is north or south by compass. This can be effected, first, by placing a cypher to the right of the heeling error observed, and dividing by the number of degrees the ship was heeling: this will give the error for 10° heel with the ship's head in the same direction.

Next, with this as "difference of latitude," enter the traverse table in which the course agrees with the number of degrees the ship's head was from the north or south when the error was observed, and the "distance" adjacent will be the error for 10° heel when the ship's head is north or south, and a record of this should be retained. Then, by the inverse process, we can find the approximate error for any number of degrees heel with the ship's head in any direction while she is in the same magnetic latitude. If it is recorded that 18° 5′ is the heeling error for 10°, when the ship's head is north, then, with the ship's head N. 20° E., if she heeled 6°, to ascertain the heeling error we multiply 18° 5′ by 6, and remove the decimal point one figure to your left: this gives 11° 1′, the error if her head had been north. With this enter a traverse table for a course of 20°, and adjacent to "distance" 11° 1′ will be found 10° 4′ as "difference of latitude," which is the number of degrees error for 6° heel with the ship's head N. 20° E. See problems, p. 565.

93. *Describe any instrument to show the ship's heel (generally called a clinometer), and state how and where it should be fixed.*

A clinometer consists of an arc correctly divided, and the degrees numbered both to the right and left of the vertical line which passes through its centre when the ship is upright, and at which is suspended an index weighted so as to be at all times perpendicular, with a point showing on the arc the number of degrees the ship is heeling. This instrument should be fixed in a place convenient for inspection, in the centre of an athwartship bulkhead.

94. *Should the clinometer be observed when the ship is swung to determine the deviation when the ship is upright? If so, state the reason why.*

Yes; otherwise the heeling error would be combined with the deviation. The clinometer should also be carefully observed when compensation by the athwartship magnet is made. If the ship is then heeling this part of the compensation should be regarded as unsatisfactory, and a readjustment should be effected as soon as the ship's head could be brought north or south magnetic with the clinometer at zero (0). When, however, the fore-and-aft magnets are placed, the clinometer may be disregarded.

95. *Would you expect the table of deviations supplied by the compass adjuster from observations made in swinging the ship to remain good during the voyage, or would you expect the deviations to change? If so, state under what circumstances.*

I should expect the deviation to change. First, from lapse of time, especially in a new ship, especially when she is kept or steered in a direction opposite to that in which she was built. The deviation may also be expected to change with change of geographical position, from sustaining strains from heavy seas, or shocks such as from a collision. If a ship is steered for some time on one course, especially if near the east or west points, when the course is altered the north point of the compass may be drawn to that part of the ship which was previously towards the south.

96. *Is it desirable that a record of your observations for deviations should be kept as a guide for any subsequent voyage in case the ship should be in the same locality, or for further correction of the compass? If so, describe some suitable form for keeping such record.*

It is very desirable to keep a record of the deviation of the compass at all times. A good form is to have a page or opening of a book for each point of the compass, so that all the observations made on any point may be

seen at once. At the same time, the date, apparent time, latitude and longitude, true bearing of object, variation allowed, heel of ship, and any circumstance that may affect the value of the observation should be recorded. By these means an accurate course can generally be shaped when observations cannot be obtained; also, the position and amount of vertical soft-iron bar required to perfect the correction of the compass can be subsequently found.

97. *Would you, under any circumstances, consider it a safe and proper procedure to place implicit confidence in your compasses, however skilfully they may have been adjusted? If not, what precautions is it your duty to take at all times?*

Under no circumstances is it safe to place implicit confidence in the compasses, however closely they may have been adjusted. It is an essential duty to watch and record the deviation at all times, and when observations cannot be obtained, to take precautions to guard against the effects of possible compass error.

Problems for Compass Deviation Certificates and Master Extra

31 (a). Ship's head while building N. 26° E. magnetic, and co-efficient — B 11°·7. Required co-efficient C with its proper sign assuming both co-efficients to be due to sub-permanent magnetism.

In the right-angled triangle A F C, Fig. 1, given side — B and ∠ A, to find side + C.

By Calculation.

Formula—

$$\frac{C}{B} = \text{Tan. } \angle A$$

$$\therefore C = B \times \text{Tan. } \angle A$$

— B 11°·7	Log.	1·068186
26°	Tan.	9·688182
+ C 5°·7	Log.	0·756368

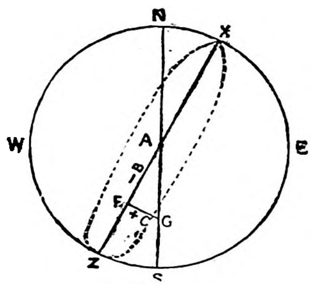


Fig. 1.

By Inspection

With 26° as course and co-efficient — B 11°·7 in lat. col. in Traverse Table, Norie's Tables, will give co-efficient + C 5°·7 in dep. col.

The circle represents the magnetic compass, N A S the magnetic meridian, Z A X direction of ship's head when building, A F, co-efficient B, F G, co-efficient C.

31 (b). Given co-efficient + B 16°·2 and co-efficient + C 11°·8, to find the direction of ship's head while building, assuming both B and C to arise from sub-permanent magnetism.

In the right-angled triangle A F G, Fig. 2, given the sides + B and + C, to find ∠ A.

BY CALCULATION

Formula—

$$\frac{B}{C} = \text{Co-tangent } A$$

+ B 16°·2	Log.	11·209515
+ C 11°·8	Log.	1·071882
S. 36°·4 E.	Cot.	10·137633

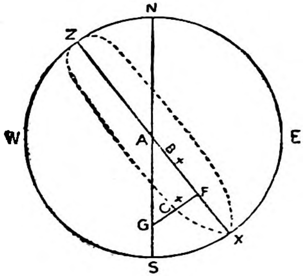


Fig 2.

Direction of ship's head while building, S. 36°·4 E. South because B is +, and East because C is +.

BY INSPECTION

B 16°·2 in d. lat. col. and C 11°·8 in dep. col. give S. 36° E. in Traverse Table.

The circle represents the magnetic compass, N A S the magnetic meridian, Z A X direction of ship's head when building, A F, co-efficient B, and G F, co-efficient C.

When the ship is built with her head north or south magnetic, there will be no co-efficient C, and when her head while building was east or west magnetic there will be no co-efficient B.

With a + B her head was in the Southern semicircle while building.
With a — B ,, ,, Northern ,, ,,
With a + C ,, ,, Eastern ,, ,,
With a — C ,, ,, Western ,, ,,
+ B and no C, head South while building.
— B ,, ,, C ,, North ,, ,,
+ C ,, ,, B ,, East ,, ,,
— C ,, ,, B ,, West ,, ,,

Examples for Practice

1. Ship's head while building S. 30° W. magnetic, and co-efficient + C 10°. Required co-efficient B with its proper sign, assuming both co-efficients to be due to sub-permanent magnetism.

Ans. Co-efficient B 17°·3 +.

2. Given co-efficient — B 10°·6 and co-efficient — C 14°·6: to find the magnetic direction of ship's head while building, assuming both co-efficients to be due to sub-permanent magnetism.

Ans. N. 54° W. magnetic.

3. Ship's head while building, East magnetic, and co-efficient + C 21°. Required co-efficient B.

Ans. Co-efficient B nil.

4 Given co-efficient — C 20° and co-efficient B nil. Required direction of ship's head while building.

Ans. Ship's head West magnetic.

62. *Assuming the deviations observed with ship's head by compass to be as follows (or as in question 61, whichever may be given), determine the value of the co-efficients A, B, C, D, and E, and from them construct a complete table of deviations (or for as many points as the Examiner may direct).*

North	— 7°	10′	South	+ 2°	10′
N.E.	— 11	15	S.W.	+ 20	5
East	— 18	20	West	+ 17	20
S.E.	— 16	29	N.W.	+ 2	9
— sign = westerly dev.			+ sign = easterly dev.		

Determination of the Co-efficients A, B, C, D, E.

2. To find the value of A, add together algebraically the four deviations determined, with the ship's head on the cardinal points by compass, and divide the sum by four, and the result will be the value of A.

North	— 7°	10′	South	+ 2°	10′
East	— 18	20	West	+ 17	20
	— 25	30		+ 19	30
	+ 19	30			
	4) 6	0			
A =	— 1	30			

To find B add together the deviation observed with the ship's head east and west by compass, reversing the sign of the latter, and half the sum will be the value of B.

East	— 18°	20′	
West	— 17	20	sign reversed
sum	— 35	40	
½ sum	— 17	50	= B

To find C add together the deviation observed when ship's head was north and south by compass, while the ship is upright, reversing the sign of the latter, and half the sum will be the value of C.

North	— 7°	10′	
South	— 2	10	sign reversed
sum	— 9	20	
½ sum	— 4	40	= C

To find D add together the deviation observed with ship's head N.E., S.W., S.E., and N.W., by compass, reversing the signs of the last-named two, and divide the sum by four.

N.E.	— 11° 45′		S.W.	+ 20° 5′
N.W.	— 2 9 sign reversed		S.E.	+ 16 29 sign reversed
	13 54			+ 36 34
				— 13 54
				4) 22 40
				+ 5 40 = D

To find E add together the deviations when ship's head was north and south, and those observed when east and west, reversing the signs of the last-named two, and divide the sum by four.

North	— 7° 10′		South	+ 2° 10′
West	— 17 20 sign reversed		East	+ 18 20 sign reversed
	— 24 30			+ 20 30
	+ 20 30			
	4) 4 0			
	— 1 0 = E			

Construction of a Deviation Table for all Bearings, by use of the Co-efficients A, B, C, D, E.

The co-efficients being given, a table of deviations may be constructed for every point of the compass, by finding the deviation due from each co-efficient, with the ship's head by compass towards either or each point that is thought necessary. For this purpose co-efficient A with its sign, being a constant quantity, is the same on each point of the compass.

To determine the deviation due from B with the ship's head on any given point, reduce the minutes to tenths of a degree by dividing them by six, and placing this to the right of the number of degrees we have the number of tenths. Co-efficient B in the table of deviations given is — 17° 50′, and is therefore approximately — 178 tenths of a degree. Enter the Traverse Table with the tenths of a degree in the distance column, and the given number of points from the north or south by compass at the top or bottom of the table, and the corresponding departure will be the number of tenths of a degree due to B when the ship's head is on the given point. To determine whether it should be easterly or westerly, the sign will be the same as that of the co-efficient, if the given point is east of north or south, but if west it must have the contrary sign. If we desire,

to know the deviation represented by B with ship's head by compass N.N.E., we enter traverse table with 178 in the distance column, and two points at the top; the departure, 68 tenths of a degree = 6° 48′, is the deviation required on this point, and is — 6° 48′, having the same sign as the co-efficient, according to the above rule.

In the same manner the value from C on any given point may be determined with this difference, that the deviation due from C must be taken out from the latitude column, instead of that of the departure, as in the case of B, and it has the same sign as the co-efficient if north of the east and west points, but a contrary sign if south of these points. The co-efficient C is — 4° 40′ = 47 tenths of a degree nearly; this at N.N.E. gives in latitude column 43 tenths or — 4° 18′, having the same sign as the co-efficient, because the ship's head is in the northern semi-circle.

To determine the deviation due to D for any given point, with this co-efficient reduced to tenths of a degree, enter the table as before described, but at top or bottom with twice the number of points that the ship's head is from either of the cardinal points, and the departure will give the number of tenths of a degree due to D. This will have the same sign as the co-efficient in the quadrants from north to east, and from south to west, but a contrary sign from east to south and from west to north. Thus the co-efficient D being + 5° 40′ = 57 tenths at N.N.E., being two points from north, we enter traverse table with four points at the top, and distance 57 tenths; the departure will give 40 tenths = + 4°, having the same sign as the co-efficient according to the above rule.

In like manner we find the value of E, only the result must be taken from the latitude column, instead of that of the departure. In the quadrants between N.E. and N.W., or S.E. and S.W., the sign is the same as that of the co-efficient, but between N.E. and S.E., or S.W. and N.W., it has the contrary sign. Co-efficient E being — 1° 0′ we enter the table with 10 distance, and for N.N.E. being two points from north we enter the table with four points and take out latitude 7 = — 0° 42′, the sign of the same name as the co-efficient according to the above rule.

Having the deviation due to each co-efficient, we sum them up algebraically thus—

Direction of ship's head N.N.E.	A gives	—	1°	30′
	B ,,	—	6	48
	C ,,	—	4	20
	E ,,	—	0	42
		—	13	20
	D ,,	+	4	0
Deviation ship's head N.N.E. by compass.		—	9	20

$$\text{Dev.} = A + B \sin.\ \theta + C \cos.\ \theta + D \sin.\ 2\ \theta + E \cos.\ 2\ \theta$$

where A, B, C, D, and E are the co-efficients of deviation and θ the direction of ship's head by compass.

Ship's Head by Compass.	Co-Efficients. A — 1° 30′	B — 17° 50′	C — 4° 40′	D + 5° 40′	E — 1° 0′	Total Deviation.	Ship's Head Correct Magnetic.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′
North	— 1 30	0 0	— 4 40	0 0	— 1 0	— 7 10	N. 7 10 W.
N. by E.	— 1 30	— 3 29	— 4 35	+ 2 10	— 0 55	— 8 19	N. 2 56 E.
N.N.E.	— 1 30	— 6 50	— 4 19	+ 4 0	— 0 42	— 9 21	N. 13 9 E.
N.E. by N.	— 1 30	— 9 54	— 3 53	+ 5 14	— 0 23	— 10 26	N. 23 19 E.
N.E.	— 1 30	— 12 37	— 3 18	+ 5 40	0 0	— 11 45	N. 33 15 E.
N.E. by E.	— 1 30	— 14 50	— 2 36	+ 5 14	+ 0 23	— 13 19	N. 42 56 E.
E.N.E.	— 1 30	— 16 29	— 1 47	+ 4 0	+ 0 42	— 15 4	N. 52 26 E.
E. by N.	— 1 30	— 17 29	— 0 55	+ 2 10	+ 0 55	— 16 49	N. 61 56 E.
East	— 1 30	— 17 50	0 0	0 0	+ 1 0	— 18 20	N. 71 40 E.
E. by S.	— 1 30	— 17 29	+ 0 55	— 2 10	+ 0 55	— 19 19	N. 81 56 E.
E.S.E.	— 1 30	— 16 29	+ 1 47	— 4 0	+ 0 42	— 19 30	S. 87 0 E.
S.E. by E.	— 1 30	— 14 50	+ 2 36	— 5 14	+ 0 23	— 18 35	S. 74 50 E.
S.E.	— 1 30	— 12 37	+ 3 18	— 5 40	0 0	— 16 29	S. 61 29 E.
S.E. by S.	— 1 30	— 9 54	+ 3 53	— 5 14	— 0 23	— 13 8	S. 46 53 E.
S.S.E.	— 1 30	— 6 50	+ 4 19	— 4 0	— 0 42	— 8 43	S. 31 13 E.
S. by E.	— 1 30	— 3 29	+ 4 35	— 2 10	— 0 55	— 3 29	S. 14 44 E.
South	— 1 30	— 0 0	+ 4 40	0 0	— 1 0	+ 2 10	S. 2 10 W.
S. by W.	— 1 30	+ 3 29	+ 4 35	+ 2 10	— 0 55	+ 7 49	S. 19 4 W.
S.S.W.	— 1 30	+ 6 50	+ 4 19	+ 4 0	— 0 42	+ 12 57	S. 35 27 W.
S.W. by S.	— 1 30	+ 9 54	+ 3 53	+ 5 14	— 0 23	+ 17 8	S. 50 53 W.
S.W.	— 1 30	+ 12 37	+ 3 18	+ 5 40	0 0	+ 20 5	S. 65 5 W.
S.W. by W.	— 1 30	+ 14 50	+ 2 36	+ 5 14	+ 0 23	+ 21 33	S. 77 48 W.
W.S.W.	— 1 30	+ 16 29	+ 1 47	+ 4 0	+ 0 42	+ 21 28	S. 88 58 W.
W. by S.	— 1 30	+ 17 29	+ 0 55	+ 2 10	+ 0 55	+ 19 59	N. 81 16 W.
West	— 1 30	+ 17 50	0 0	0 0	+ 1 0	+ 17 20	N. 72 40 W.
W. by N.	— 1 30	+ 17 29	— 0 55	— 2 10	+ 0 55	+ 13 49	N. 64 56 W.
W.N.W.	— 1 30	+ 16 29	— 1 47	— 4 0	+ 0 42	+ 9 54	N. 57 36 W.
N.W. by W.	— 1 30	+ 14 50	— 2 36	— 5 14	+ 0 23	+ 5 53	N. 50 22 W.
N.W.	— 1 30	+ 12 37	— 3 18	— 5 40	0 0	+ 2 9	N. 42 51 W.
N.W. by N.	— 1 30	+ 9 54	— 3 53	— 5 14	— 0 23	— 1 6	N. 34 51 W.
N.N.W.	— 1 30	+ 6 50	— 4 19	— 4 0	— 0 42	— 3 41	N. 26 11 W.
N. by W.	— 1 30	+ 3 29	— 4 35	— 2 10	— 0 55	— 5 41	N. 16 56 W.

69. June 20th, 8h. 41m. a.m. App. time at ship in lat. 41° 30′ N., and long. 75° W. Required the sun's true bearing.

	D. H. M. S.		° ′ ″		″		° ′
June	19 20 41 00	Dec.	23 27 1·3 N.	Var.	1·03	p	66 33
Long. in time	+ 5 00 00		+ 1·8		1·7	l'	48 30
A.T. Gr.	20 1 41 00		23 27 3·1	Corr.	1·751	Sum	115 03
	H. M. S.		90 00 0			Diff.	18 03
Hour Angle or P	3 19 0		66 32 56·9			$\frac{p+l'}{2}$	57 32
$\frac{P}{2}$	1 39 30					$\frac{p \sim l'}{2}$	9 2

Formula—

$$\text{Tan. } \frac{Z+X}{2} = \frac{\text{Cos. } \frac{p \sim l'}{2}}{\text{Cos. } \frac{p+l'}{2}} \text{ Cot. } \frac{P}{2} \qquad \text{Tan. } \frac{Z \sim X}{2} = \frac{\text{Sin. } \frac{p \sim l'}{2}}{\text{Sin. } \frac{p+l'}{2}} \text{ Cot. } \frac{P}{2}$$

$$\frac{Z+X}{2} + \frac{Z-X}{2} = \angle Z, \text{ the Azimuth.}$$

When the co-lat. exceeds the polar distance, the Azimuth equals $\frac{Z+X}{2} - \frac{Z \sim X}{2}$

	H. M. S.						
$\frac{P}{2}$	1 39 30	cot.	10·333806			cot.	10·333806
$\frac{p \sim l'}{2}$	° ′ ″ 9 2 00	cos.	9·994580			sin.	9·195925
$\frac{p+l'}{2}$	57 32 00	sec.	10·270180			cosec.	10·073810
$\frac{Z+X}{2}$	75 51 18	tan.	10·598566	$\frac{Z \sim X}{2}$	° ′ ″ 21 52 8	tan.	9·603541
				$\frac{Z+X}{2}$	75 51 18		
				$\angle$ Z or Azimuth N.	97 43 26 E.		

Explanation of the Figure.

N W S E	Rational Horizon.
P	Pole.
$d\ d$	Parallel of declination.
W Z E	Prime vertical.
W Q E	Equinoctial.
l'	Co-lat.
p	Polar distance.
Z X	Zenith distance.
X	Position of sun.
$\angle$ Z	Azimuth.

For further Examples *see* Time Azimuths.

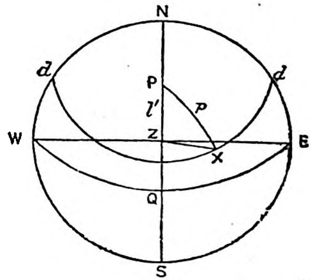


Fig. 3.

70. April 12th, at 9h. 30m. p.m. App. time at ship in lat. 30° N. and long. 30° West. Required: what stars will be in good position for Azimuth.

	D. H. M. S.		H. M. S.	Hourly var sec.
App. time at ship	1 9 30 00	App. ⊙'s R.A.	1 23 18·49	9·20
Long. in time	+ 2 00 00	Correction	+ 1 45·80	11·5
A.T. Greenwich	1 11 30 00	Corr. R.A.	1 25 04·29	4600
				10120
				60)105·800
				1m. 45·80s.

	H. M. S.
App. time at ship	9 30 00
App. ⊙'s R.A.	1 25 24
R.A.M.	10 55 24

To find the Hour-Angles

	H. M.
R.A.M.	10 55
Libræ R.A.	14 46
H.A.	3 51 E.

Table 1 gives 22. E. / 2.16.3 W.

	H. M.
R.A.M.	10 55
Sirius R.A.	6 41
H.A.	4 14 W.

			H. M		° ′	
22a Libræ	E., R.A.		14 46	Dec.	15 39	S.
16 Sirius	W., R.A.		6 41	Dec.	16 35	S.
3 Betelguese	W., R.A.		5 50	Dec.	7 23	N.
2 Capella	W., R.A.		5 10	Dec.	45 54	N.

	H. M.
R.A.M.	10 55
Betelguese R.A.	5 50
H.A.	5 5 W

	H. M
R.A.M.	10 55
Capella R.A.	5 10
H.A.	5 45 W

Explanation of the Figure

N W S E	Rational horizon.
N Z S	Observer's meridian.
W Z E	Prime vertical.
W Q E	Equinoctial.
Z Q	Latitude.
P Z	Co-latitude.
P	Pole.
P2, 3, 16, 22	Hour circles.

dd The respective Parallels of Dec., 16 and 22 being nearly on the same parallel of Dec., only one has been drawn.

When mean time at ship is given, use the mean ⊙'s R.A. or Sid. Time to find R.A.M.

For names and positions see Tables I. and II., pages 567-569.

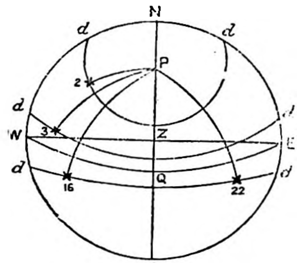


Fig. 4.

Examples for Practice.

(1) April 1st, at 10h. 20m. p.m., mean time at ship in lat. 50° and long. 45° W. Find what stars will be in good position for Azimuth.

Ans. R.A.M. 11h. 1m. 23s. $\frac{10.20\ \text{E.}}{2.3.5.18\ \text{W.}}$

$\frac{\text{Vega, Spica} \quad \text{E}}{\text{Capella, Betelguese, Procyon, Alphard, W.}}$

(2) May 30th, at 4h. 30m. a.m., mean time at ship in lat. 40° N. and long. 60° W. Find what stars will be in good position for Azimuth.

Ans. R.A.M. 21h. 1m. 13s. $\frac{2\ \text{E.}}{7.8\ \text{W.}}$ $\frac{\text{Capella} \quad \text{E.}}{\text{Dubhe, Benetnach, W.}}$

(3) June 27th, at 1h. 30m. a.m., mean time at ship in lat. 60° S. and long. 80° E. Find what stars will be in good position for Azimuth.

Ans. R.A.M. 19h. 49m. 35s. $\frac{15\ \text{E.}}{11.22.\ \text{W.}}$ $\frac{\text{Canopus} \quad \text{E.}}{\text{Altair, } \alpha^2 \text{ Libiæ W.}}$

72. Jan. 25th, at 2h. 10m. a.m., mean time at ship, lat. 49° 2′ N., long. 25° 30′ W. Find the true bearing of α Aurigæ (Capella); also if Capella bore N. 64° 30′ W. by compass and the variation by chart is 36° 40′ W. Required the error of the compass and the deviation for the direction of the ship's head.

	D. H. M.		H. M. S.		° ′ ″
M.T. ship Jan.	24 14 10	M. ⊙ R.A.	20 15 2·05	Capella's Dec.	45 53 12N.
Long. in time	+ 2 22	Accel. for	+ 2 37·70		90 00 00
M.T.G.	24 16 32	16h. 23m.	+ 5·26	p	44 6 48
		Corr. M. ⊙ R.A.	20 17 45·01	l'	40 58 00
		M.T. ship	14 10 00	Sum	85 4 48
		R.A.M.	10 27 45	Diff.	3 8 48
		Capella's R.A.	5 8 34	$\frac{p+l'}{2}$	42 32 24
		P. or H.A.	5 19 11 W.		
		$\frac{P}{2}$	2 39 35·5	$\frac{p \sim l'}{2}$	1 34 24

In the spherical triangle P Z X, fig. 5. given ∠ P the hour angle, l the co-lat. and p the Polar dist. to find ∠ Z.

Formula—

$$\text{Tan.}\,\frac{Z+X}{2} = \frac{\text{Cos.}\,\frac{p \sim l'}{2}}{\text{Cos.}\,\frac{p+l'}{2}}\ \text{Cot.}\,\frac{P}{2} \qquad \text{Tan.}\,\frac{Z \sim X}{2} = \frac{\text{Sin.}\,\frac{p \sim l'}{2}}{\text{Sin.}\,\frac{p+l'}{2}}\ \text{Cot.}\,\frac{P}{2}$$

$$Z = \frac{Z+X}{2} + \frac{Z \sim X}{2}$$

$\frac{P}{2}$	30° 53′ 52″	cot.	10·077761	cot.	10·077761
$\frac{p - l'}{2}$	1 34 24	cos.	9·999839	sin.	8·438544
$\frac{p + l'}{2}$	42 32 24	sec.	10·132663	Cosec.	10·169970
$\frac{A + B}{2}$	58 21 28	tan.	10·210263 $\frac{A \sim B}{2}$ 2° 46′ 55″	tan.	8·686375
$\frac{A \sim B}{2}$	2 46 55				
Az. N.	61 08 23 W.				
Comp. bearing N.	64 30 00 W.				
Error	3 21 37 E.				
Var.	36 40 00 W.				
Dev.	40 1 37 E.				

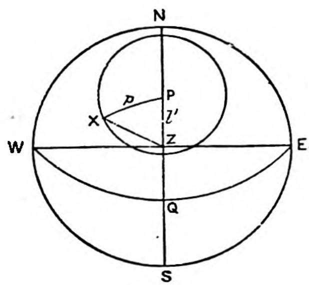


Fig. 5.

The star is circumpolar and therefore does not rise or set. The explanation of the figure is the same as in similar problems, and need not be repeated here.

For further examples, *see* Time Azimuths.

Heeling Error

92. Ship's head N.E. × E. and heeling 10° to port; the heeling error observed was 5° 30′ east.

Required the probable heeling error, ship's head S.S.E., and heeling 8° to starboard.

Formula—

$$\text{Heeling error} = \frac{\text{observed error} \times \text{cos. new course} \times \text{new heel}}{\text{cos. old course} \times \text{old heel}}$$

$$\text{Log. heeling error} = \begin{cases} \text{Log. observed error} + \text{log. cos. new course} + \text{log. new heel} \\ + \text{log. sec. old course} + \text{arith.-co, log. old heel} - \text{tens} \end{cases}$$

Error 5°·5		log.	0·740363
New course 22° 30′		cos.	9·945430
New heel 8°		log.	0·903090
Old course 56° 15′		sec.	10·255261
Old heel 10°	Arith.-co.	log.	9·000000
Heeling error 7° E.		log.	0·844144

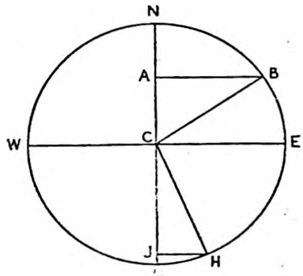


Fig. 6.

Explanation of the Figure

The circle represents the compass, N C S the compass needle, ∠ N C B first compass course, ∠ S C H second compass course, C A heeling error for 1° of heel at N E × E, C B heeling error for 1° of heel at N or S, C J the heeling error for 1° of heel at S.S.E. It is obvious that the heeling error varies as the cosine of the compass course, as shown in the formula.

Ship's head N.E. × E. and heeling 10° to starboard, the heeling error observed was 6° west.

Required the probable heeling error, ship's head north and heeling 10° to port.

Fig. 6. Let C A = heeling error at N.E. × E., then C B will be the heeling error for ship's head north, equal to C N, both being radii of the same circle.

Then in the right angled plane triangle, C A B, Fig. 6, given ∠ C and side C A: to find side C B.

Formula—

$$\frac{CB}{CA} = \text{Sec.} \angle C \therefore CB = CA \times \text{Sec.} \angle C.$$

C A 6°	log.	0·778151
∠ C 56° 15′	sec.	10·255261
Heeling error at North 10°·8 E.	log.	1·033412

Let us suppose that the ship, in the last example, had been heeling, say, 13° on north; you would have to multiply the error just found by 13° and divide the product by 10°, thus—

$$\begin{array}{r} 10^\circ{\cdot}8 \\ 13^\circ \\ \hline 324 \\ 108 \\ \hline 10^\circ)\,140{\cdot}4 \\ \hline 14^\circ{\cdot}0 \text{ E.} \end{array}$$

The heeling error for 13° of port heel head North = 14° E.
The error would be the same at South but would be West.
At East or West there should be no heeling error.

Examples for Practice

1. Ship's head S.E. and heeling 6° to starboard, the heeling error observed was 8° East.

Required the probable heeling error, ship's head N.N.E. and heeling 10° to port.

Ans. Heeling error 17°·42 East.

2. Ship's head N.N.E. and heeling 10° to port, the heeling error observed was 17°·42 East.

Required the probable heeling error, ship's head S.E. and heeling 6° to port.

Ans. Heeling error 8° West.

3. Ship's head S.E. and heeling 6° to starboard, the heeling error observed was 8° East.

Required the probable heeling error, ship's head N.N.E. and heeling 10° to starboard.

Ans. Heeling error 17°·42 West.

4. Ship's head north heeling 7° to port, the heeling error observed was 11° West.

Required the probable heeling error, ship's head south and heeling 10° to port.

Ans. Heeling error 15°·7 East.

The heeling error changes its name if on both tacks her head is either northerly or southerly; it retains the same name if on one tack her head is southerly and on the other tack northerly. A ship with a heavy list to port or starboard would change the name of the heeling error on changing from a northerly to a southerly course, or from a southerly to a northerly course.

Table I.—The Number of the Stars in Table II. that are in best Position for having their Azimuths observed at different Latitudes for each Hour of Sidereal Time.

Latitude North Degrees.		Sidereal Time. 0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Latitude North. Degrees.
	E.	2.14.1.	2.3.14.1.	2.4.5.15.16.	16.4.5.15.	18.15.	17.18.6.	17.6.7.	17.7.	19.20.7.	9.19.20.	21.9.22.8.7.	21.19.8.	
0	W.	11.24.	11.24.		12.	12.13.	13.	13.		2.	2.1.14.15.	14.15.3.2.	15.16.17.4.	0
	E.	2.14.1.13.	2.3.14.13.	16.3.4.2.5.	16.15.5.4.	15.	18.15.6.7.	7.6.15.18.	7.17.	17.20.8.7.	17.9.20.19.	19.22.9.8.	19.21.22.	
10	W.	10.11.	11.	13.	12.13.	12.13.	13.		15.	15.2.1.	1.14.15.	2.15.14.	3.16.	10
	E.	13.2.1.	13.14.3.2.	16.4.	16.5.4.	7.15.	15.18.16.7.	15.7.6.	7.8.	8.	9.20.	22.9.	22.	
20	W.	10.11.	11.	13.	13.	12.			15.	15.	1.15.14.	2.3.14.	2.3.16.	20
	E.	1.2.	3.4.14.	3.4.14.16.	16.5.7.	7.15.	6.18.7.	15.6.18.	8.	8.9.	9.20.	9.20.	22.	
30	W.	10.11.	10.11.		12.	12.			15.	14.1.	1.14.16.	2.16.3.	2.16.3.	30
	E.	1.3.	4.3.14.	4.3.14.5.	7.5.4.	6.7.16.	6.8.16.	18.6.8.	8.18.	8.	9.20.	9.20.	22.20.	
40	W.	10.11.	10.11.	10.	12.	12.	12.	14.	16.14.	16.14.1.	16.14.3.1.	16.3.1.	2.3.5.	40
	E.	1.1.7.	4.3.7.	4.3.7.14.	14.5.7.	16.5.14.	16.14.6.8.	16.8.6.18.	18.8.	9.18.	18.9.	20.10.9.	10.20.	
50	W.	10.8.11.	10.11.	10.	10.12.	12.	12.	14.	16.14	16.14.1.	16.14.2.1.	1.3.16.5.	2.3.5.18.	50
	E.	1.4.7.	1.3.4.7.	3.4.8.	14.5 3.8.	14.5.6.8.	14.5.6.8.	16.5.18.6.	9.10.18.6.	9.10.18.	10.9.18.	10.9.20.	10.20.	
60	W.	10.11.	10.11.	10.12.	10.12.	10.12.	10.12.	10.14.	16.14	14.16.3.1.	16.3.5.1.	5.3.1.18.	3.5.18.1.	60

Latitude North. Degrees.		Sidereal Time. 12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Latitude North. Degrees.
	E.	21.19.23.	21.23.	10.21	11.10.	24.11.	24.	24.12.	12.24.	24.	13.	13.	1.13.	
0	W.	17.16.7.4.	7.18.	7.18.19.6.	7.6 19.21.	19.21.8.	20.21.8.9.	21.9.20.8	9.21.22.	22:23.	23.24.10.	10.24.	10.24.	0
	E.	22.23.	21.10.23	21.10.	10.11.	11.	24.	12.24.	12.24.	24.		13.	13.1.	
10	W.	4.5.	19.18.7.	19.6.7.	21.19.7.	21.7.	21.20.8.	20.8.21.	9.22.8.	23.	24.23.	24.10.	24.10.	10
	E.	23.	10.23.21.	10.21.23.	11.	11.		12.	12.24.				1.2.	
20	W.	4.5.	18.4.5.	6.18.	21.6.7.	7.21.	7.20.	20.7.8.9.	9.8.22.	8.9.23.	24.	24.	10.	20
	E.	10.22.	10.23.	10.23.	11.23.	11.	12.	12.	12.			2.	2.1.	
30	W	2.5.4.	4.5.18.	4.6.18.	6.	20.7.	20.7.	9.7.20.23.	9.8.22.3.	9.8.23.	8.		·10.11	30
	E.	22.10.	10.23.22.	23.10.22.	11.23.	11.23.	12.	12.	12.		2.	2.	1.2	
40	W.	2.4.5.18.	2.4.5.18.	4.6.18.	6.	20.22.	20.22.23.	22.23.20.9.	9.7.23.8.	9.7.8.	7.8.	7.8.	10.11.	40
	E.	10.20.22.	20.22.10.	22.	23.11.	23.11.	12.11.	12.2	2.12.	2.	2.	2.1.	2.1.7.	
50	W.	2.5.4.18.	2.4.18.5.	2.4.6.20.	2.4.6.20.	20.22.6.	20.22.23	9.7.22.23.	9.7	9.7.8.	7.8.9.	7.8.	8.11.	50
	E.	20.10.	20.22.10.	22.11.	11.	11.	2.12.11.	2.12.11.	2.12.	2.12.	2.	1.2.	1.2.4.7.	
60	W.	5.18.2.6.	2.4.5.6.18.	20.2.4.6.	20.2.4.6.22.	20.2.4.6.	20.4.	9.	9.	9.7.	7.9.11.	7.8.11.	8.11.	60

TABLE I.—*continued*.

Latitude South. Degrees.		SIDEREAL TIME. 0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	Latitude South. Degrees.
	E.	1.14.	2.15.3.14.	2.15.16.	2.15.5.4.	17.4.	17.6.18.	17.6.	19.17.	19.17.20.	19.21.20.7.	9.21.8.22.	21.23.8.	
10	W.	11.24.	24.	24.	12.	12.	13.	13.2.	13.2.	2.1.	2.1.14.	15.17.3.14.	17.16.15.4	10
	E.	15.14.1.	15.16.1.3.	16.15.3.2.	17.5.2.	17.18.4.2.	17.18.4.2.	19.6.	19.	19.21.20.	21.20.	21.9.22.	9.23.8.	
20	W.	11.	24.	24.12.	12.	12.		2.13.	2.13	2.1.13.	1.	4.3.14.	15.16.4.	20
	E.	15.17.1.14.	17.15.16.1.	17.16.3.	17.5.	2.19.18.	19.18.4.2.	19.6.21.	21.	21.20.	21.22.20.	22.	9.23	
30	W.	11.	24.	12.24.	12.24.	24.		2.	2.1.13.	4.1.13.	4.13.3.14.	4.3.14.	16.5.	30
	E.	17.15.14.	19.17.16.1.	19.16.3.1.	19.21.5.	19.21.5.	19.21.18.	21.6.4.	21.6.	20.6.	20.22.	23.22.24.	9.23.24.	
40	W.	21.11.	21.12.	21.12.	12.24.	24.	24.	1.24.	1.	4.1.13.	4.13.3.14.	13.3.14.5.	5.16.13	40
	E.	14	19.16.14.	19.16.3.	21.19.3.1.	21.19.5.	21.18.5.	21.18.5.	6.	20.6	22.20.6.24.	24.23.22.	23.24.	
50	W.	19.21.12.	21.12.	21.12.	24.	24.	1.24.	1.24.	24.1.3.	3.	13.14.3.5.	13.5.14.	13.6.5.16.	50
	E.	14.	19.14.	16.14.	1.3.16.21.	1.3.5.18.21.	3.5.18.	5.18.	5.6.	6.20.	6.20.22.24.	6.20.22.23.24.	20.22.23.24.	
60	W.	19.21.12.	21.12.	21.			1.24.	3.1.24.	3.24.	5.3.14.	5.14.	5.14.	16.6.5.13.	60

Latitude South. Degrees.		SIDEREAL TIME. 12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	Latitude South. Degrees.
	E.	8.23.	8.	10.	10.24.11.	10.24.	24.	12.	12.	13.	13.	13.		
10	W.	17.5.15.16.	17.7.	17.6.8.18.	19.17.8.	19.8.	19.21.8.	19.21.20.9.	21.9.22.	21.22.	10.23.	10.	10.24.	10
	E.	23.8.	8.	24.	24.10.	10.11	10.	10.	13.12.	13.	13.			
20	W.	16.15.6.	15.17.	17.6.18.8.	17.8.	17.8.	19.9.	19.9.20.	10.19.21.22	21.10.22.	10.21.23.	10.23.	11.	20
	E.	9.	24.	24.	11.24.	8.11	10.11	10.13.	13.12	13.12.		17.	15.17.	
30	W.	15.16.5.	15.6.	15.6.18.	17.	17.9.	17.9.	17.19.9.20.	19.17.10.22.	19.17.10.21.	21.10.19.23.	21.23.	11.	30
	E.	9.24.	24.9.	9.24	13.	13.11.	13.11.	13.10.	13.12.	12.	17.12.15.	17.15.	17.15.	
40	W.	16.5.6.	6.15.	15.6.18.	9.15.	9.17.	17.9.	17.20.	17.10.19.22.	17.19.22.	19.21.23.	19.21.23.11.	19.21.11.	40
	E.	9.24.	9.	9.13.	13.	13.11.	11.13.	11.13.	11.15.	15.12.	15.12.17.	17.15.12.	17.15.	
50	W.	13.5.16.6.	6.15.18.	15.18.	9.15.	9.15.	15.20.	15.20	17.22.	17.19.22.11.	19.23.11.	19.21.23.11.	19.21.11.23.	50
	E.		9.	9.13.	13.	13.	11.	11.	15.11.	15.	15.12.	12.15.	15.	
60	W.	16.13.6.8.	16.13.18.	18.	15.9.	20.15.	15.20.	15.20.22	22.	11.22.	23.11.	23.11.	12.19.23.	60

Table I. inserted by kind permission of J. D. Potter, 145, Minories, E.

TABLE II.

		R. A.		Declination				R. A.		Declination	
		H.	M.	°	′			H.	M.	°	′
1	Aldebaran	4	31	16	19 N.	13	Achernar	1	34	57	43 S.
2	Capella	5	10	45	54 N.	14	Rigel	5	10	8	19 S.
3	Betelguese	5	50	7	23 N.	15	Canopis	6	22	52	39 S.
4	Castor	7	29	32	6 N.	16	Sirius	6	41	16	35 S.
5	Procyon	7	34	5	27 N.	17	β Argus	9	12	69	20 S.
6	Regulus	10	3	12	26 N.	18	Alphard	9	23	8	16 S.
7	Dubhe	10	58	62	16 N.	19	α^1 Crucis	12	21	62	35 S.
8	Benetnach	13	44	49	47 N.	20	Spica	13	20	10	40 S.
9	Arcturus	14	11	19	40 N.	21	α^2 Centauri	14	33	60	27 S.
10	Vega	18	34	38	42 N.	22	α^2 Librae	14	46	15	39 S.
11	Altair	19	46	8	37 N.	23	Antares	16	24	26	13 S.
12	Markab	23	0	14	42 N.	24	α Pavonis	20	18	57	2 S.

PROOFS OF FORMULÆ

PLANE TRIGONOMETRY

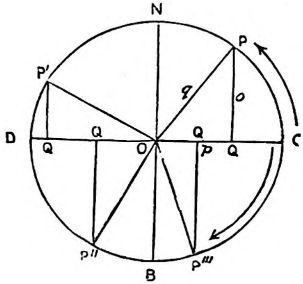


Fig. 1.

It is agreed that when the straight line O C, Fig. 1, revolves in the direction of the outer arrow all the angles generated will be positive, and when it revolves in the direction of the inner arrow all the angles generated will be negative. Let the straight line O C, commencing at C, take up the positions P, P′, P″ and P‴, then the angles P O Q, P′ O Q, P″ O Q and P‴ O Q will all be positive. All vertical lines above D O C are positive or plus, and all vertical lines below D O C are minus; all horizontal lines to the right of N O B are plus, and all horizontal lines to the left are minus; the revolving line O C is always plus.

In forming the ratios like signs give plus and unlike signs give minus.

It will be observed that all the lines in the first quadrant, N C, are plus, consequently all the ratios are plus. In the second quadrant, N D, the vertical lines are plus and the horizontal lines are minus; consequently all the ratios in the second quadrant are minus, except the sine and cosecant. The student should prove this for himself by forming the ratios and observing the rule that like signs give plus and unlike signs minus. It is of the utmost importance to know whether a ratio is plus or minus, because it enables you to recognise at once the magnitude of the angle with which you are dealing.

In the plane triangle P O Q, Fig. 1, let the sides be denoted by the letters p, o, q. Let $\angle O = \theta$.

Then— $$q^2 = o^2 + p^2$$

Divide these equals by q^2, o^2 and p^2, thus—

$$\frac{q^2}{q^2} = \frac{o^2}{q^2} + \frac{p^2}{q^2} \qquad \frac{q^2}{o^2} = \frac{o^2}{o^2} + \frac{p^2}{o^2} \qquad \frac{q^2}{p^2} = \frac{o^2}{p^2} + \frac{p^2}{p^2}$$

i.e., $1 = \sin.^2\theta + \cos.^2\theta$; *i.e.*, $\text{cosec.}^2\theta = 1 + \cot.^2\theta$; *i.e.*, $\sec.^2\theta = \tan.^2\theta + 1$

Also $$1 - \sin.^2\theta = \cos.^2\theta, \text{ and } 1 - \cos.^2\theta = \sin.^2\theta$$

$$\text{Cosec.}^2\theta - 1 = \cot.^2\theta, \text{ and cosec.}^2\theta - \cot.^2\theta = 1$$

$$\text{Sec.}^2\theta - \tan.^2\theta = 1, \text{ and sec.}^2\theta - 1 = \tan.^2\theta$$

When one ratio multiplied by another produces unity one is said to be the reciprocal of the other. (*See* Fig. 1 for triangle.)

$$\begin{cases} \text{Sin. } \theta = \dfrac{o}{q} \\ \text{Cosec. } \theta = \dfrac{q}{o} \text{ and } \dfrac{o}{q} \times \dfrac{q}{o} = 1 \therefore \text{ the sine and cosec. are reciprocals.} \end{cases}$$

$$\begin{cases} \text{Cos. } \theta = \dfrac{p}{q} \\ \text{Sec. } \theta = \dfrac{q}{p} \text{ and } \dfrac{p}{q} \times \dfrac{q}{p} = 1 \therefore \text{ the cosine and sec. are reciprocals.} \end{cases}$$

$$\begin{cases} \text{Tan. } \theta = \dfrac{o}{p} \\ \text{Cot. } \theta = \dfrac{p}{o} \text{ and } \dfrac{o}{p} \times \dfrac{p}{o} = 1 \therefore \text{ the tan. and cot. are reciprocals.} \end{cases}$$

It is very frequently a source of great saving of figures to substitute the reciprocal of a ratio in an equation.

The above results may be stated thus, and should be remembered—

(1) Sine squared of an angle plus cosine squared of the *same* angle equals 1.

(2) Cosecant squared of an angle equals 1 plus cotangent squared of the *same* angle.

(3) Secant squared of an angle equals tangent squared of the *same* angle, plus 1.

As the sine of an angle increases from 0 at 0° to unity at 90°; the cosine decreases from unity at 0° to 0 at 90°.

The tangent of an angle increases from 0 at 0° to ∞ (infinity) at 90°, and the cotangent decreases from infinity at 0° to 0 at 90°.

The secant increases from unity at 0° to infinity at 90°, and the cosecant decreases from infinity at 0° to unity at 90°.

Values for Ratios of 30° and 60°

Let A B C, Fig. 2, be an equilateral triangle, then by Euclid, Book I., Prop. 32, each of its angles equals 60°. Bisect the angle B by a line cutting the base A C in D, then the angle A B D is 30°. Because A B = B C and B D is common to both, and angle A B D = angle C B D, therefore B D bisects A C at right angles.

Let	$AD = c$
then	$AB = 2c$
and	$BD^2 + DA^2 = AB^2$
i.e.,	$BD^2 + c^2 = 4c^2$
and	$BD^2 = 3c^2$
therefore	$BD = \sqrt{3c^2} = c\sqrt{3}$

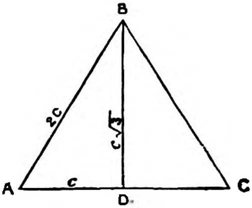


Fig. 2.

Now— $\text{Sin. } 60° = \text{sin. } BAD = \frac{BD}{BA} = \frac{c\sqrt{3}}{2c} = \frac{\sqrt{3}}{2}$

and $\text{Sin. } 30° = \text{sin. } ABD = \frac{AD}{AB} = \frac{c}{2c} = \frac{1}{2}$

All the other ratios of 30° and 60° are found in like manner.

Values for Ratios of 45°

Let A B C, Fig. 3, be a right-angled triangle, then by Euclid, Book I., Prop. 32, A C B = 45°.

Let	$CB = c$, then $AB = c$
and	$CA^2 = CB^2 + AB^2 = c^2 + c^2 = 2c^2$
therefore	$CA = c\sqrt{2}$

Now $\text{Sin. } 45° = \text{sin. } ACB = \frac{AB}{AC} = \frac{c}{c\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\text{Tan. } 45° = \text{tan. } ACB = \frac{AB}{CB} = \frac{c}{c} = 1$

$\text{Cosec. } 45° = \text{cosec. } ACB = \frac{AC}{AB} = \frac{c\sqrt{2}}{c} = \sqrt{2}$

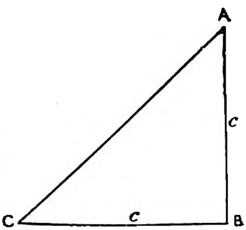


Fig. 3.

Any ratio of 45° can be found in a similar manner.

Values for Ratios of A + B

Let M C N, Fig. 4, be angle A and N C O be angle B, then M C O = (A + B).

In C O take any point D, from D draw D E perpendicular to C M, also draw D G perpendicular to C N; through G draw G H parallel to D E, also draw G F parallel to C M.

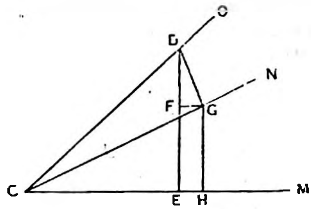


Fig. 4.

Now $FDG + FGD = 90°$
and $FGD + FGC = 90°$
therefore $FDG = FGC$ because each is the complement of F G D
and $FGC = MCN = A$
therefore $FDG = A$

Now— $$\text{Sin. }(A + B) = \text{sin. } MCO = \frac{DE}{DC} = \frac{EF + FD}{DC} = \frac{HG + FD}{DC}$$

$$= \frac{HG}{DC} + \frac{FD}{DC} = \frac{HG \times GC}{GC \times DC} + \frac{FD \times DG}{DG \times DC}$$

$$= \text{sin. } A \times \text{cos. } B + \text{cos. } A \times \text{sin. } B$$

$$\text{Cos. }(A + B) = \text{cos. } MCO = \frac{CE}{CD} = \frac{CH - HE}{CD} = \frac{CH - FG}{CD} = \frac{CH}{CD} - \frac{FG}{CD}$$

$$= \frac{CH \times CG}{CG \times CD} - \frac{FG \times GD}{GD \times CD}$$

$$= \text{cos. } A \times \text{cos. } B - \text{sin. } A \times \text{sin. } B$$

$$\text{Tan. }(A + B) = \text{tan. } MCO = \frac{\text{sin. } A + B}{\text{cos. } A + B}$$

$$= \frac{\text{Sin. } A \times \text{cos. } B + \text{cos. } A \times \text{sin. } B}{\text{Cos. } A \times \text{cos. } B - \text{sin. } A \times \text{sin. } B}$$

Dividing numerator and denominator by cos. A × cos. B we get—

$$\text{Tan }(A + B) = \frac{\dfrac{\text{sin. } A \times \text{cos. } B}{\text{cos. } A \times \text{cos. } B} + \dfrac{\text{cos. } A \times \text{sin. } B}{\text{cos. } A \times \text{cos } B}}{\dfrac{\text{cos. } A \times \text{cos. } B}{\text{cos. } A \times \text{cos. } B} - \dfrac{\text{sin. } A \times \text{sin. } B}{\text{cos. } A \times \text{cos. } B}}$$

$$= \frac{\text{tan. } A + \text{tan. } B}{1 - \text{tan. } A \times \text{tan. } B}$$

The above results are very important, and should be committed to memory, thus—

The sine of the sum of two angles equals the sine of the first into the cosine of the second plus the cosine of the first into the sine of the second.

The cosine of the sum of any two angles equals the cosine of the first angle into the cosine of the second angle minus the sine of the first into the sine of the second.

The tangent of the sum of any two angles equals the tangent of the first angle plus the tangent of the second, divided by 1 minus the tangent of the first into the tangent of the second.

VALUES FOR RATIOS OF (A — B)

Let M C O, Fig. 5, be an angle A, and O C N an angle B; then M C N = (A—B).

In C N take a point G, and from G draw G H perpendicular to C M; from G draw G D perpendicular to C O, through D draw D E parallel to G H, and through D draw D F parallel to C M, meeting H G produced, in F.

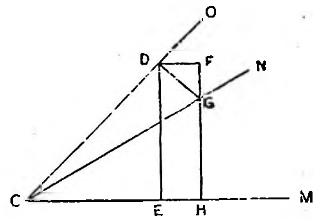


Fig. 5.

Now	F G D + F D G = 90°
and	F D G + F D O = 90°
therefore	F G D + F D G = F D G + F D O
and	F G D = F D O because each is the complement of F D G.
Now	F D O = D C M = A
therefore	F G D = A

$$\text{Sin. }(A - B) = \text{sin. } MCN = \frac{GH}{GC} = \frac{HF - FG}{GC} = \frac{DE - FG}{GC}$$

$$= \frac{DE}{GC} - \frac{FG}{GC} = \frac{DE \times DC}{DC \times GC} - \frac{FG \times DG}{DG \times GC}$$

$$= \text{sin. } A \times \text{cos. } B - \text{cos. } A \times \text{sin. } B$$

$$\text{Cos. }(A - B) = \text{cos. } MCN = \frac{CH}{CG} = \frac{CE + EH}{CG} = \frac{CE + DF}{CG}$$

$$= \frac{CE}{CG} + \frac{DF}{CG} = \frac{CE \times CD}{CD \times CG} + \frac{DF \times DG}{DG \times CG}$$

$$= \text{cos. } A \times \text{cos. } B + \text{sin. } A \times \text{sin. } B$$

$$\text{Tan. }(A - B) = \text{tan. } MCN = \frac{\text{sin. }(A - B)}{\text{cos. }(A - B)}$$

$$= \frac{\text{sin. } A \times \text{cos. } B - \text{cos. } A \times \text{sin. } B}{\text{cos. } A \times \text{cos. } B + \text{sin. } A \times \text{sin. } B}$$

Dividing numerator and denominator by cos. A × cos. B then—

$$\text{Tan. }(A - B) = \frac{\dfrac{\text{sin. } A \times \text{cos. } B}{\text{cos. } A \times \text{cos. } B} - \dfrac{\text{cos. } A \times \text{sin. } B}{\text{cos. } A \times \text{cos. } B}}{\dfrac{\text{cos. } A \times \text{cos. } B}{\text{cos. } A \times \text{cos. } B} + \dfrac{\text{sin. } A \times \text{sin. } B}{\text{cos. } A \times \text{cos. } B}}$$

$$= \frac{\text{tan. } A - \text{tan. } B}{1 + \text{tan. } A \times \text{tan. } B}$$

These results should be committed to memory. The sine of the *difference* of any two angles equals the sine of the first into the cosine of the second minus the cosine of the first into the sine of the second.

The cosine of the *difference* of any two angles equals the cosine of the first into the cosine of the second plus the sine of the first into the sine of the second.

The tangent of the *difference* of any two angles equals the tangent of the first minus the tangent of the second divided by 1 plus the tangent of the first into the tangent of the second.

Values for Sin. A + Sin. B, etc.

$$\frac{A+B}{2}+\frac{A-B}{2}=A, \text{ and } \frac{A+B}{2}-\frac{A-B}{2}=B$$

and by substituting these values for A and B we get—

$$\text{Sin. } A=\sin.\frac{A+B}{2}\times\cos.\frac{A-B}{2}+\cos.\frac{A+B}{2}\times\sin.\frac{A-B}{2}$$

and $$\text{Sin. } B=\sin.\frac{A+B}{2}\times\cos.\frac{A-B}{2}-\cos.\frac{A+B}{2}\times\sin.\frac{A-B}{2}$$

by addition $$\text{Sin. } A+\sin. B=2\sin.\frac{A+B}{2}\times\cos.\frac{A-B}{2}$$

and by subtraction—

$$\text{Sin. } A-\sin. B=2\cos.\frac{A+B}{2}\times\sin.\frac{A-B}{2}$$

$$\text{Cos. } A=\cos.\frac{A+B}{2}\times\cos.\frac{A-B}{2}-\sin.\frac{A+B}{2}\times\sin.\frac{A-B}{2}$$

and $$\text{Cos. } B=\cos.\frac{A+B}{2}\times\cos.\frac{A-B}{2}+\sin.\frac{A+B}{2}\times\sin.\frac{A-B}{2}$$

by addition $$\text{Cos. } A+\cos. B=2\cos.\frac{A+B}{2}\times\cos.\frac{A-B}{2}$$

and by subtraction—

$$\text{Cos. } A-\cos. B=-2\sin.\frac{A+B}{2}\times\sin.\frac{A-B}{2}$$

$$=2\sin.\frac{A+B}{2}\times\sin.\frac{B-A}{2}$$

These results are most important, and should be thoroughly understood and committed to memory, as follows—

The sum of the sines of two angles equals twice the sine of half their sum into the cosine of half their difference.

The difference of the sines of two angles equals twice the cosine of half their sum into the sine of half their difference.

The sum of the cosines of two angles equals twice the cosine of half their sum into the cosine of half their difference.

The difference of the cosines of two angles equals twice the sine of half the sum into the sine of half their difference, reversing the order of the terms.

In addition to the above the following results, which are easily proved, should be remembered.

The cosine of any angle equals the cos.^{2} of its half, minus sin.^{2} of its half, *i.e.*

$$\text{Cos. } A = \cos.^2 \frac{A}{2} - \sin.^2 \frac{A}{2}$$

The cosine of any angle equals $1 - 2$ sin.^{2} of its half, *i.e.*

$$\text{Cos. } A = 1 - 2 \sin.^2 \frac{A}{2}$$

$$\therefore 1 - \cos. A = 2 \sin.^2 \frac{A}{2}$$

The cosine of any angle equals twice the cos.^{2} of its half, minus 1, *i.e.*

$$\text{Cos. } A = 2 \cos.^2 \frac{A}{2} - 1.$$

$$\therefore 1 + \cos. A = 2 \cos.^2 \frac{A}{2}$$

$$\text{Sin.}^2 A - \sin.^2 B = \sin. (A + B) \sin. (A - B).$$

Now

$$\frac{\text{Sin. } A + \sin. B}{\text{Cos. } A + \cos. B} = \frac{2 \sin. \frac{A+B}{2} \times \cos. \frac{A-B}{2}}{2 \cos. \frac{A+B}{2} \times \cos. \frac{A-B}{2}}$$

$$= \tan. \frac{A+B}{2}$$

and

$$\frac{\text{Sin. } A - \sin. B}{\text{Cos. } A + \cos. B} = \frac{2 \cos. \frac{A+B}{2} \times \sin. \frac{A-B}{2}}{2 \cos. \frac{A+B}{2} \times \cos. \frac{A-B}{2}}$$

$$= \tan. \frac{A-B}{2}$$

also

$$\frac{\text{Sin. A} + \text{sin. B}}{\text{Sin. A} - \text{sin. B}} = \frac{2 \text{ sin. } \frac{A+B}{2} \times \text{cos. } \frac{A-B}{2}}{2 \text{ cos. } \frac{A+B}{2} \times \text{sin. } \frac{A-B}{2}}$$

$$= \text{tan. } \frac{A+B}{2} \times \text{cot. } \frac{A-B}{2}$$

which by substituting the reciprocal of cot. $\frac{A-B}{2} = \frac{\text{tan. } \frac{A+B}{2}}{\text{tan. } \frac{A-B}{2}}$

Proof of Rule of Sines

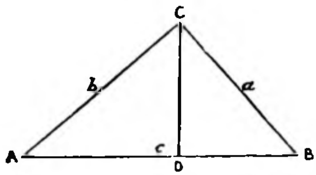


Fig. 6.

Case I.—When the angles are acute.

Let A B C, Fig. 6, be a triangle having the angles acute; from C draw C D perpendicular to A B. Let C A = *b* and C B = *a*.

Now

$$\frac{\text{Sin. A}}{\text{Sin. B}} = \frac{\frac{CD}{CA}}{\frac{CD}{CB}} = \frac{CD}{CA} \times \frac{CB}{CD} = \frac{CB}{CA} = \frac{a}{b}$$

Case II.—When one of the angles is obtuse, as B.

Let A B C, Fig. 7, be a triangle having angle B obtuse; produce A B to D and draw D C perpendicular to A D.

Now

$$\frac{\text{Sin. A}}{\text{Sin. B}} = \frac{\text{sin. A}}{\text{sin. } (180 - B)} = \frac{\text{sin. A}}{\text{sin. C B D}} = \frac{\frac{CD}{CA}}{\frac{CD}{CB}} = \frac{CD}{CA} \times \frac{CB}{CD} = \frac{CB}{CA} = \frac{a}{b}$$

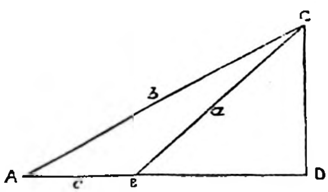


Fig. 7.

Proof of Rule of Cosines

Case I.—When the angle is acute.

Let A B C, Fig. 8, be a triangle having A acute; from B draw B D perpendicular to A C.

$$B\,C^2 = C\,A^2 + A\,B^2 - 2\,C\,A\,.\,A\,D$$
$$= C\,A^2 + A\,B^2 - 2\,C\,A\,.\,A\,B\cos.\,A;\ \text{because } A\,D = A\,B\,.\cos.\,A.$$

Let $a = B\,C;\ b = C\,A$ and $c = A\,B$

then $a^2 = b^2 + c^2 - 2\,b\,c\,.\cos.\,A$

$$\therefore 2\,b\,c\,.\cos.\,A = b^2 + c^2 - a^2$$

and $$\cos.\,A = \frac{b^2 + c^2 - a^2}{2\,b\,c}$$

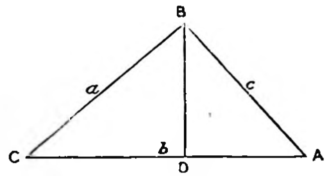


Fig 8

Case II.—When the angle is obtuse.

Let A B C, Fig. 9, be an obtuse-angled triangle having the angle A obtuse. From B drop a perpendicular on C A produced to D.

Now $B\,C^2 = C\,A^2 + A\,B^2 + 2\,C\,A\,.\,A\,D$
$$= C\,A^2 + A\,B^2 + 2\,C\,A\,.\,B\,A\,.\,(-\cos.\,A);\ \text{because } A\,D = B\,A \times \cos.\,A$$
$$= C\,A^2 + A\,B^2 - 2\,C\,A\,.\,B\,A\,.\cos.\,A$$

and using the same notation as before—

$$a^2 = b^2 + c^2 - 2\,b\,c\,.\cos.\,A$$

therefore $2\,b\,c\,.\cos.\,A = b^2 + c^2 - a^2$

and $$\cos.\,A = \frac{b^2 + c^2 - a^2}{2\,b\,c}$$

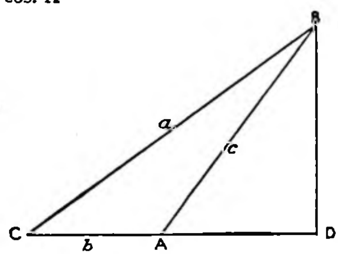


Fig. 9.

To adapt the above formulæ to logarithmic calculation proceed thus—

$$\text{Cos. } A = \frac{b^2 + c^2 - a^2}{2\,b\,c}$$

$$1 + \cos.\,A = 1 + \frac{b^2 + c^2 - a^2}{2\,b\,c}$$

i.e., $$2\cos.^2\frac{A}{2} = \frac{2\,b\,c + b^2 + c^2 - a^2}{2\,b\,c}$$

$$= \frac{(b + c)^2 - a^2}{2\,b\,c}$$

$$= \frac{(b + c + a)\,(b + c - a)}{2\,b\,c}$$

Let $b + c + a = 2s$

and $b + c - a = 2s - 2a = 2(s - a)$

then $2\cos.^2\frac{A}{2} = \frac{2s \,.\, 2(s-a)}{2bc}$

and $\cos.^2\frac{A}{2} = \frac{s\,.\,(s-a)}{bc}$

and $\cos.\frac{A}{2} = \sqrt{\frac{s\,.\,(s-a)}{bc}}$

and $\log.\cos.\frac{A}{2} = \frac{1}{2}\left\{\log. s + \log.(s-a) + 20 - (\log. b + \log. c)\right\}$

Logarithmic formula for sine of half an angle of a triangle in terms of the sides—

$$\text{Cos. A} = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\text{I} - \cos. A = \text{I} - \frac{b^2 + c^2 - a^2}{2bc}$$

therefore

$$2\sin.^2\frac{A}{2} = \frac{2bc - b^2 - c^2 + a^2}{2bc}$$

$$= \frac{a^2 - (b^2 + c^2 - 2bc)}{2bc}$$

$$= \frac{a^2 - (b-c)^2}{2bc}$$

$$= \frac{(a-b+c)(a+b-c)}{2bc}$$

As before, let $a + b + c = 2s$

then $a - b + c = 2s - 2b = 2(s - b)$

and $a - c + b = 2s - 2c = 2(s - c)$

therefore $2\sin.^2\frac{A}{2} = \frac{2(s-b)\,2(s-c)}{2bc}$

and $\sin.^2\frac{A}{2} = \frac{(s-b)(s-c)}{bc}$

and $\sin.\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$

$$\text{Log. sin.}\frac{A}{2} = \frac{\text{I}}{2}\left\{\log.(s-b) + \log.(s-c) + 20 - (\log. b + \log. c)\right\}$$

Proof of Rule of Tangents

$$\frac{a}{b} = \frac{\sin. A}{\sin. B}$$

and $$\frac{a}{b} + 1 = \frac{\sin. A}{\sin. B} + 1 \text{ and } \frac{a}{b} - 1 = \frac{\sin. A}{\sin. B} - 1$$

i.e., $$\frac{a+b}{b} = \frac{\sin. A + \sin. B}{\sin. B} \text{ and } \frac{a-b}{b} = \frac{\sin. A - \sin. B}{\sin. B}$$

hence by division— $$\frac{\frac{a+b}{b}}{\frac{a-b}{b}} = \frac{\frac{\sin. A + \sin. B}{\sin. B}}{\frac{\sin. A - \sin. B}{\sin. B}}$$

i.e., $$\frac{a+b}{a-b} = \frac{\sin. A + \sin. B}{\sin. A - \sin. B} = \frac{2 \sin. \frac{A+B}{2} \cos. \frac{A-B}{2}}{2 \cos. \frac{A+B}{2} \sin. \frac{A-B}{2}}$$

$$= \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}$$

$$a + b : a - b :: \tan. \frac{1}{2}(A+B) : \tan. \frac{1}{2}(A-B)$$

The above result may be enunciated thus—

The sum of two sides is to their *difference* as the tangent of half the sum of their opposite angles is to the tangent of half their difference.

By combining the last two formulæ the formula for the tangent of half an angle in terms of the sides is easily deduced, as follows—

$$\text{Tan. } \frac{A}{2} = \frac{\sqrt{\frac{(s-b)(s-c)}{b\,c}}}{\sqrt{\frac{s\,.\,(s-a)}{b\,c}}} = \sqrt{\frac{(s-b)(s-c)}{b\,c}} \times \sqrt{\frac{b\,c}{s\,.\,(s-a)}}$$

Cancelling common factors

$$= \sqrt{\frac{(s-b)(s-c)}{s\,.\,(s-a)}}$$

$$\text{Log. tan. } \frac{A}{2} = \frac{1}{2}\left\{\log. (s-b) + \log. (s-c) + 20 - (\log. s + \log. (s-a))\right\}$$

SPHERICAL TRIGONOMETRY

The fundamental formula in spherical trigonometry as deduced from the spherical triangle having its solid angle at the centre of the sphere is that which connects the cosine of an angle with sines and cosines of the three sides of the triangle.

In the spherical triangle A B C, Fig. 10, given the three sides a b c to find angle A.

$$\text{Cos. } a = \cos. b \, . \cos. c + \sin. b \, . \sin. c \, . \cos. A.$$

Now $$\sin. b \, . \sin. c \, . \cos. A = \cos. a - \cos. b \, . \cos. c$$

and $$\cos. A = \frac{\cos. a - \cos. b \, . \cos. c}{\sin. b \, . \sin. c}$$

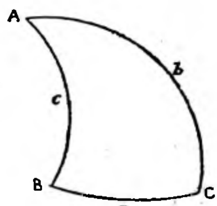


Fig. 10.

$$\cos. A + 1 = \frac{\cos. a - \cos. b \, . \cos. c}{\sin. b \, . \sin. c} + 1$$

i.e., $$2 \cos.^2 \frac{A}{2} = \frac{\cos. a - \cos. b \, . \cos. c + \sin. b \, . \sin. c}{\sin. b \, . \sin. c}$$

$$= \frac{\cos. a - (\cos. b \, . \cos. c - \sin. b \, . \sin. c)}{\sin. b \, . \sin. c}$$

$$= \frac{\cos. a - \cos. (b + c)}{\sin. b \, . \sin. c}$$

$$= \frac{2 \sin. \frac{1}{2} (b + c + a) \, . \sin. \frac{1}{2} (b + c - a)}{\sin. b \, . \sin. c}$$

Let $$\tfrac{1}{2} (b + c + a) = s \text{; then } \tfrac{1}{2} (b + c - a) = s - a$$

then $$\cos.^2 \frac{A}{2} = \frac{\sin. s \, . \sin. (s - a)}{\sin. b \, . \sin. c}$$

and $$\cos. \frac{A}{2} = \sqrt{\frac{\sin. s \, . \sin. (s - a)}{\sin. b \, . \sin. c}}$$

and substituting the reciprocals of sin. b and sin. c—

$$\text{Cos. } \frac{A}{2} = \sqrt{\sin. s \, . \sin. (s - a) \, . \text{cosec. } b \, . \text{cosec. } c}$$

$$\text{Log. cos. } \frac{A}{2} = \frac{1}{2} \left\{ \text{L. sin. } s + \text{L. sin. } (s - a) + \text{L. cosec. } b \, . \text{ L. cosec. } c - 20 \right\}$$

TO FIND LOGARITHMIC FORMULA FOR THE SINE OF AN ANGLE IN TERMS OF THE SIDES

$$\text{Cos. } A = \frac{\cos. a - \cos. b \,.\, \cos. c}{\sin. b \,.\, \sin. c}$$

$$1 - \cos. A = 1 - \left\{\frac{\cos. a - \cos. b \,.\, \cos. c}{\sin. b \,.\, \sin. c}\right\}$$

i.e.,
$$2 \sin.^2 \frac{A}{2} = \frac{\sin. b \,.\, \sin. c - \cos. a + \cos. b \,.\, \cos. c}{\sin. b \,.\, \sin. c}$$

$$= \frac{\cos. b \,.\, \cos. c + \sin. b \sin. c - \cos. a}{\sin. b \,.\, \sin. c}$$

$$= \frac{\cos. (b - c) - \cos. a}{\sin. b \,.\, \sin. c}$$

Now
$$2 \sin.^2 \frac{A}{2} = \frac{\sin. \frac{1}{2}(a + b - c) \,.\, \sin. \frac{1}{2}(a - b + c)}{\sin. b \,.\, \sin. c}$$

Let $\frac{1}{2}(a + b + c) = s$; then $\frac{1}{2}(a + b - c) = s - c$; and $\frac{1}{2}(a - b + c) = s - b$

Then
$$\sin.^2 \frac{A}{2} = \frac{\sin. (s - c) \sin. (s - b)}{\sin. b \,.\, \sin. c}$$

and
$$\sin. \frac{A}{2} = \sqrt{\frac{\sin. (s - c) \sin. (s - b)}{\sin. b \,.\, \sin. c}}$$

and substituting the reciprocals of sin. b and sin. c.

$$\text{Sin. } \frac{A}{2} = \sqrt{\sin. (s - c) \sin. (s - b) \text{ cosec. } b. \text{ cosec. } c}$$

$$\text{L. sin. } \frac{A}{2} = \frac{1}{2}\left\{\text{L. sin. } (s - c) + \text{L. sin. } (s - b) + \text{L. cosec. } b + \text{L. cosec. } c - 20\right\}$$

By combining these two formulæ the tangent formula is easily deduced as follows—

$$\text{Tan. } \frac{A}{2} = \frac{\sin. \frac{A}{2}}{\cos. \frac{A}{2}} = \frac{\sqrt{\dfrac{\sin. (s - c) \sin. (s - b)}{\sin. b \,.\, \sin. c}}}{\sqrt{\dfrac{\sin. s \sin. (s - a)}{\sin. b \,.\, \sin. c}}}$$

$$= \sqrt{\frac{\sin. (s - c) \sin. (s - b)}{\sin. b \,.\, \sin. c}} \times \sqrt{\frac{\sin. b \,.\, \sin. c}{\sin. s \sin. (s - a)}}$$

cancelling common factors

$$\text{Tan. } \frac{A}{2} \sqrt{\frac{\sin. (s - c) \sin. (s - b)}{\sin. s \,.\, \sin. (s - a)}}$$

To prove a formula for finding A direct through the Natural Haversine

$$\text{Cos. } a = \cos. b \,.\, \cos. c + \sin. b \,.\, \sin. c - \sin. b \,.\, \sin. c\,(1 - \cos. A)$$

$$= \cos. (b \sim c) - \sin. b \,.\, \sin. c\,(1 - \cos. A)$$

$$1 - \cos. a = 1 - \cos. (b \sim c) + \sin. b \,.\, \sin. c\,(1 - \cos. A)$$

$$\frac{1 - \cos. a}{2} = \frac{1 - \cos. (b \sim c)}{2} + \sin. b \,.\, \sin. c\,\frac{(1 - \cos. A)}{2}$$

Now $\quad 1 - \cos. A = 2 \sin.^2 \frac{A}{2} = \text{versine } A$

and $\quad \frac{1 - \cos. A}{2} = \sin.^2 \frac{A}{2} = \frac{\text{versine } A}{2} = \text{haversine } A$

therefore $\quad \text{Hav. } a = \text{hav. } (b \sim c) + \sin. b \,.\, \sin. c \,.\, \text{hav. } A$

For the further use of the natural haversine, *see* the Explanation of the Haversine Tables in Norie's Tables.

The usual formula for finding an angle is as follows—

where a = the alt. l = the latitude, p = the polar dist., and $\angle$ P the hour angle.

$$\cos. P = \frac{\cos. (90° - a) - \cos. p \cos. (90° - l)}{\sin. p \sin. (90° - l)}$$

$$= \frac{\sin. a - \cos. p \sin. l}{\sin. p \cos. l.}$$

Subtracting each side of the equation from 1—

$$1 - \cos. P = 1 - \frac{\sin. a - \cos p \sin. l}{\sin. p \cos. l}$$

i.e., $$2 \sin.^2 \frac{P}{2} = \frac{\sin p \cos l + \cos p \sin. l - \sin. a}{\sin p \cos. l}$$

$$= \frac{\sin. (p + l) - \sin. a}{\sin p \cos. l}$$

$$= \frac{2 \cos \frac{1}{2} (p + l + a) \sin. \frac{1}{2} (p + l - a)}{\sin. p \cos. l}$$

Now, if $\quad s = \frac{1}{2} (p + l + a)$, then $(s - a) = \frac{1}{2} (p + l - a)$

therefore $$\sin.^2 \frac{P}{2} = \frac{\cos. s \sin (s - a)}{\sin. p \cos l}$$

$$= \text{co-sec. } p \sec. l \cos. s \sin. (s - a)$$

And $$\sin. \frac{P}{2} = \sqrt{\text{co-sec. } p \sec. l \cos. s \sin. (s - a)}$$

$$\text{Hav. } P = \sin.^2 \frac{P}{2}$$

In finding the azimuth when the altitude and latitude are substituted for zenith distance and co-latitude, the following modification of the formula is necessary.

$$-\text{Cos. } Z = \frac{\cos. p - \cos.(90° - a)\cos.(90° - l)}{\sin.(90° - a)\sin.(90° - l)}$$

$$= \frac{\cos. p - \sin. a \sin l}{\cos. a \cos. l}$$

adding 1 to each side of the equation—

$$1 - \cos. Z = 1 + \frac{\cos. p - \sin. a \sin. l}{\cos. a \cos. l}$$

$$\therefore 2 \sin.^2 \frac{Z}{2} = \frac{(\cos. a \cos. l - \sin. a \sin. l) + \cos. p}{\cos. a \cos. l}$$

$$= \frac{\cos.(a + l) + \cos. p}{\cos. a \cos. l}$$

$$\text{Sin.}^2 \frac{Z}{2} = \frac{\cos. \frac{1}{2}(a + l + p)\cos. \frac{1}{2}(a + l - p)}{\cos. a \cos. l}$$

$$\therefore \sin.^2 \frac{Z}{2} = \frac{\cos. s \cos.(s - p)}{\cos. a \cos. l}, \text{ where } s = \frac{(a + l + p)}{2}$$

i.e., $$\sin. \frac{Z}{2} = \cos. s \cos.(s - p) \sec. a \sec. l$$

or $$\sin. \frac{Z}{2} = \sqrt{\cos. s \cos.(s - p) \sec. a \sec. l}$$

$$\text{Hav. } Z = \sin.^2 \frac{Z}{2}$$

REQUISITE ELEMENTS

FROM THE

NAUTICAL ALMANAC

TO BE USED

FOR THE EXAMPLES IN

NORIE'S EPITOME OF NAVIGATION.

SUN'S DECLINATION; SUN'S SEMIDIAMETER; EQUATION OF TIME: AND SIDEREAL TIME.

N.B.—The elements on the *left* of the double lines are from page I. of the Nautical Almanac, those on the *right* are from page II. The sign + or — to the left of the Equation of Time, indicates its application to *Mean Time; reverse* the sign for application to *Apparent Time*.

Apparent Noon. (N.A. p. I.)			Mean Noon. (N. A. p. II.)				
DATE.	DECLINATION.	VAR. IN 1 HOUR.	DECLINATION.	SUN'S SEMI-DIAMETER.	EQUATION OF TIME.	VAR. IN 1 HOUR.	SIDEREAL TIME.
	° ′ ″	″	° ′ ″	′ ″	m. s.	s.	h. m. s.
Jan. 1	S. 22 59 22·8	12·73	S. 22 59 23·6	16 18·2	−3 53·80	1·177	18 44 21·23
2	22 51 3·6	13·87	22 54 4·6	16 18·2	4 21·86	1·161	18 48 17·79
3	22 48 17·1	15·00	22 48 18·3	16 18·2	4 49·54	1·145	18 52 14·35
4	22 42 3·4	16·13	22 42 4·8	16 18·2	5 16·83	1·128	18 56 10·90
5	22 35 22·7	17·25	22 35 24·3	16 18·2	5 43·68	1·110	19 0 7·46
6	22 28 15·2	18·36	22 28 17·1	16 18·2	6 10·08	1·090	19 4 4·02
7	22 20 41·2	19·46	22 20 43·4	16 18·2	6 36·01	1·070	19 8 0·58
8	22 12 40·9	20·56	22 12 43·3	16 18·1	7 1·43	1·049	19 11 57·14
9	22 4 14·4	21·64	22 4 17·1	16 18·1	7 26·33	1·026	19 15 53·69
10	21 55 22·0	22·72	21 55 25·0	16 18·0	7 50·68	1·003	19 19 50·25
11	21 46 4·0	23·78	21 46 7·3	16 18·0	8 14·46	0·979	19 23 46·81
12	21 36 20·7	24·83	21 36 24·2	16 17·9	8 37·66	0·954	19 27 43·36
13	21 26 12·2	25·87	21 26 16·1	16 17·8	9 0·25	0·929	19 31 39·92
14	21 15 38·9	26·90	21 15 43·1	16 17·8	9 22·22	0·902	19 35 36·48
15	21 4 41·1	27·91	21 4 45·6	16 17·7	9 43·55	0·875	19 39 33·04
16	20 53 19·0	28·92	20 53 23·8	16 17·6	10 4·22	0·847	19 43 29·59
17	20 41 33·0	29·91	20 41 38·2	16 17·5	10 24·22	0·819	19 47 26·15
18	20 29 23·4	30·88	20 29 28·9	16 17·5	10 43·52	0·789	19 51 22·71
19	20 16 50·6	31·84	20 16 56·5	16 17·4	11 2·11	0·759	19 55 19·26
20	20 3 54·9	32·79	20 4 1·1	16 17·3	11 19·96	0·728	19 59 15·82
21	19 50 36·7	33·72	19 50 43·2	16 17·2	11 37·05	0·696	20 3 12·38
22	19 36 56·3	34·64	19 37 3·2	16 17·1	11 53·37	0·663	20 7 8·93
23	19 22 54·1	35·54	19 23 1·3	16 17·0	12 8·89	0·630	20 11 5·49
24	19 8 30·5	36·42	19 8 38·0	16 16·9	12 23·61	0·596	20 15 2·05
25	18 53 45·8	37·29	18 53 53·7	16 16·7	12 37·52	0·562	20 18 58·60
26	18 38 40·5	38·14	18 38 48·6	16 16·6	12 50·60	0·527	20 22 55·16
27	18 23 14·9	38·98	18 23 23·3	16 16·5	13 2·86	0·493	20 26 51·72
28	18 7 29·4	39·80	18 7 38·2	16 16·4	13 14·28	0·459	20 30 48·27
29	17 51 24·5	40·60	17 51 33·6	16 16·3	13 24·87	0·424	20 34 44·83
30	17 35 0·6	41·39	17 35 9·9	16 16·1	13 34·62	0·389	20 38 41·38
Jan. 31	17 18 18·0	42·15	17 18 27·6	16 16·0	−13 43·54	0·354	20 42 37·94

Apparent Noon. (N.A. p.I.)			Mean Noon. (N. A. p. II.)				
Date.	Declination.	Var. in 1 hour.	Declination.	Sun's semi-diameter.	Equation of time.	Var. in 1 hour.	Sidereal time.
	° ′ ″	″	° ′ ″	′ ″	m. s.	s.	h. m. s.
Feb. 1	S. 17 1 17·2	42·91	S. 17 1 27·1	16 15·9	−13 51·63	0·319	20 46 34·50
2	16 43 58·5	43·64	16 44 8·7	16 15·7	13 58·88	0·284	20 50 31·05
3	16 26 22·4	44·36	16 26 32·9	16 15·5	14 5·30	0·250	20 54 27·61
4	16 8 29·3	45·06	16 8 40·0	16 15·4	14 10·90	0·216	20 58 24·16
5	15 50 19·6	45·74	15 50 30·5	16 15·2	14 15·68	0·182	21 2 20·72
6	15 31 53·7	46·41	15 32 4·7	16 15·0	14 19·64	0·148	21 6 17·27
7	15 13 11·9	47·06	15 13 23·2	16 14·9	14 22·81	0·115	21 10 13·83
8	14 54 14·7	47·70	14 54 26·2	16 14·7	14 25·18	0·082	21 14 10·38
9	14 35 2·5	48·31	14 35 14·1	16 14·5	14 26·78	0·050	21 18 6·94
10	14 15 35·6	48·92	14 15 47·4	16 14·3	14 27·60	0·018	21 22 3·49
11	13 55 54·5	49·50	13 56 6·4	16 14·1	14 27·67	0·013	21 26 0·05
12	13 35 59·5	50·07	13 36 11·6	16 13·9	14 26·99	0·014	21 29 56·60
13	13 15 51·2	50·62	13 16 3·4	16 13·7	14 25·58	0·074	21 33 53·16
14	12 55 29·8	51·15	12 55 42·1	16 13·5	14 23·45	0·104	21 37 49·71
15	12 34 55·8	51·67	12 35 8·1	16 13·3	14 20·60	0·134	21 41 46·26
16	12 14 9·6	52·17	12 14 22·0	16 13·1	14 17·04	0·163	21 45 42·82
17	11 53 11·7	52·65	11 53 24·2	16 12·9	14 12·78	0·192	21 49 39·37
18	11 32 2·4	53·11	11 32 14·9	16 12·7	14 7·84	0·221	21 53 35·93
19	11 10 42·3	53·56	11 10 54·8	16 12·5	14 2·20	0·249	21 57 32·48
20	10 49 11·7	53·98	10 49 24·2	16 12·2	13 55·90	0·277	22 1 29·03
21	10 27 31·1	54·39	10 27 43·6	16 12·0	13 48·92	0·304	22 5 25·59
22	10 5 40·8	54·79	10 5 53·3	16 11·8	13 41·30	0·331	22 9 22·14
23	9 43 41·4	55·16	9 43 53·9	16 11·6	13 33·04	0·357	22 13 18·70
24	9 21 33·2	55·51	9 21 45·6	16 11·3	13 24·15	0·383	22 17 15·25
25	8 59 16·7	55·85	8 59 29·0	16 11·1	13 14·66	0·408	22 21 11·80
26	8 36 52·3	56·17	8 37 4·5	16 10·9	13 4·58	0·432	22 25 8·36
27	8 14 20·3	56·48	8 14 32·5	16 10·7	12 53·92	0·456	22 29 4·91
Feb. 28	7 51 41·3	56·76	7 51 53·3	16 10·4	−12 42·70	0·479	22 33 1·46
Mar. 1	S. 7 28 55·6	57·01	S. 7 29 7·5	16 10·2	−12 30·95	0·501	22 36 58·02
2	7 6 3·5	57·29	7 6 15·3	16 10·0	12 18·68	0·522	22 40 54·57
3	6 43 5·6	57·53	6 43 17·2	16 9·7	12 5·90	0·542	22 44 51·12
4	6 20 2·1	57·75	6 20 13·5	16 9·5	11 52·61	0·562	22 48 47·68
5	5 56 53·5	57·96	5 57 4·7	16 9·2	11 38·94	0·580	22 52 44·23
6	5 33 40·1	58·15	5 33 51·1	16 9·0	11 24·80	0·598	22 56 40·78
7	5 10 22·2	58·33	5 10 33·1	16 8·7	11 10·24	0·615	23 0 37·34
8	4 47 0·3	58·49	4 47 11·0	16 8·4	10 55·30	0·630	23 4 33·89
9	4 23 34·7	58·64	4 23 45·2	16 8·2	10 39·99	0·645	23 8 30·44
10	4 0 5·8	58·77	4 0 16·0	16 7·9	10 24·34	0·659	23 12 27·00
11	3 36 33·9	58·89	3 36 43·8	16 7·6	10 8·37	0·671	23 16 23·55
12	3 12 59·3	58·99	3 13 9·0	16 7·4	9 52·12	0·683	23 20 20·10
13	2 49 22·4	59·08	2 49 31·8	16 7·1	9 35·59	0·694	23 24 16·65
14	2 25 43·6	59·15	2 25 52·7	16 6·8	9 18·81	0·704	23 28 13·21
15	2 2 3·2	59·21	2 2 12·1	16 6·5	9 1·81	0·713	23 32 9·76
16	1 38 21·6	59·25	1 38 30·3	16 6·3	8 44·59	0·721	23 36 6·31
17	1 14 39·3	59·27	1 14 47·6	16 6·0	8 27·18	0·729	23 40 2·86
18	0 50 56·5	59·29	0 51 4·6	16 5·8	8 9·59	0·736	23 43 59·42
19	0 27 13·6	59·28	0 27 21·4	16 5·5	7 51·84	0·743	23 47 55·97
20	S. 0 3 31·1	59·26	S. 0 3 38·6	16 5·2	7 33·94	0·748	23 51 52·52
21	N. 0 20 10·7	59·22	N. 0 20 3·5	16 4·9	7 15·91	0·753	23 55 49·08
22	0 43 51·4	59·16	0 43 44·5	16 4·6	6 57·78	0·757	23 59 45·63
23	1 7 30·6	59·09	1 7 24·0	16 4·4	6 39·56	0·761	0 3 42·18
24	1 31 7·9	59·01	1 31 1·7	16 4·1	6 21·26	0·764	0 7 38·73
25	1 54 43·0	58·91	1 54 37·1	16 3·8	6 2·90	0·765	0 11 35·29
26	2 18 15·5	58·79	2 18 9·9	16 3·5	5 44·50	0·767	0 15 31·84
27	2 41 45·0	58·66	2 41 39·7	16 3·3	5 26·08	0·767	0 19 28·39
28	3 5 11·1	58·51	3 5 6·1	16 3·0	5 7·66	0·767	0 23 24·94
29	3 28 33·6	58·35	3 28 28·9	16 2·7	4 49·26	0·766	0 27 21·50
30	3 51 52·0	58·17	3 51 47·6	16 2·4	4 30·90	0·764	0 31 18·05
Mar. 31	4 15 5·9	57·98	4 15 1·8	16 2·2	− 4 12·59	0·761	0 35 14·60

Apparent Noon. (N. A. p. I.)			Mean Noon. (N. A. p. II.)				
Date.	Declination.	Var. in 1 hour.	Declination.	Sun's semi-diameter.	Equation of time.	Var. in 1 hour.	Sidereal time.
	° ′ ″	″	° ′ ″	′ ″	m. s.	s.	h. m. s.
April 1	N. 4 38 15·1	57·78	N. 4 38 11·3	16 1·9	− 3 54·36	0·757	0 39 11·16
2	5 1 19·2	57·56	5 1 15·7	16 1·6	3 36·23	0·753	0 43 7·71
3	5 24 17·9	57·33	5 24 14·7	16 1·4	3 18·21	0·747	0 47 4·26
4	5 47 10·8	57·08	5 47 7·9	16 1·1	3 0·34	0·741	0 51 0·82
5	6 9 57·6	56·82	6 9 55·1	16 0·8	2 42·64	0·734	0 54 57·37
6	6 32 38·1	56·55	6 32 35·8	16 0·5	2 25·13	0·725	0 58 53·92
7	6 55 11·9	56·26	6 55 9·9	16 0·2	2 7·83	0·716	1 2 50·48
8	7 17 38·7	55·96	7 17 36·9	16 0·0	1 50·77	0·706	1 6 47·03
9	7 39 58·1	55·65	7 39 56·7	15 59·7	1 33·96	0·695	1 10 43·58
10	8 2 9·9	55·33	8 2 8·8	15 59·4	1 17·42	0·683	1 14 40·14
11	8 24 13·8	54·99	8 24 12·9	15 59·1	1 1·18	0·670	1 18 36·69
12	8 46 9·4	54·64	8 46 8·8	15 58·9	0 45·25	0·657	1 22 33·24
13	9 7 56·4	54·27	9 7 56·0	15 58·6	0 29·65	0·643	1 26 29·80
14	9 29 34·5	53·89	9 29 34·2	15 58·3	− 0 14·39	0·628	1 30 26·35
15	9 51 3·2	53·49	9 51 3·2	15 58·0	+ 0 0·52	0·613	1 34 22·90
16	10 12 22·2	53·09	10 12 22·5	15 57·8	0 15·06	0·598	1 38 19·46
17	10 33 31·3	52·66	10 33 31·7	15 57·5	0 29·22	0·582	1 42 16·01
18	10 54 30·0	52·22	10 54 30·6	15 57·2	0 42·98	0·565	1 46 12·56
19	11 15 18·0	51·77	11 15 18·8	15 57·0	0 56·34	0·548	1 50 9·12
20	11 35 55·0	51·30	11 35 55·9	15 56·7	1 9·29	0·530	1 54 5·67
21	11 56 20·5	50·82	11 56 21·6	15 56·4	1 21·81	0·512	1 58 2·23
22	12 16 34·2	50·32	12 16 35·6	15 56·2	1 33·89	0·494	2 1 58·78
23	12 36 35·9	49·81	12 36 37·4	15 55·9	1 45·54	0·476	2 5 55·31
24	12 56 25·1	49·28	12 56 26·7	15 55·7	1 56·73	0·457	2 9 51·89
25	13 16 1·4	48·74	13 16 3·2	15 55·4	2 7·46	0·437	2 13 48·45
26	13 35 24·7	48·19	13 35 26·5	15 55·2	2 17·72	0·418	2 17 45·00
27	13 54 34·4	47·62	13 54 36·4	15 55·0	2 27·50	0·398	2 21 41·55
28	14 13 30·4	47·04	14 13 32·4	15 54·7	2 36·80	0·377	2 25 38·11
29	14 32 12·2	46·44	14 32 14·3	15 54·5	2 45·60	0·356	2 29 34·66
April 30	14 50 39·6	45·83	14 50 41·8	15 54·2	+ 2 53·89	0·335	2 33 31·22
May 1	N. 15 8 52·2	45·21	N. 15 8 54·5	15 54·0	+ 3 1·66	0·313	2 37 27·77
2	15 26 49·8	44·58	15 26 52·1	15 53·8	3 8·91	0·291	2 41 24·33
3	15 44 32·0	43·93	15 44 34·4	15 53·6	3 15·63	0·268	2 45 20·88
4	16 1 58·5	43·28	16 2 1·0	15 53·3	3 21·79	0·245	2 49 17·44
5	16 19 9·2	42·61	16 19 11·6	15 53·1	3 27·39	0·222	2 53 14·00
6	16 36 3·6	41·92	16 36 6·1	15 52·9	3 32·43	0·198	2 57 10·55
7	16 52 41·5	41·23	16 52 44·0	15 52·7	3 36·89	0·174	3 1 7·11
8	17 9 2·8	40·53	17 9 5·3	15 52·4	3 40·77	0·149	3 5 3·66
9	17 25 7·0	39·81	17 25 9·5	15 52·2	3 44·06	0·125	3 9 0·22
10	17 40 53·9	39·09	17 40 56·4	15 52·0	3 46·76	0·100	3 12 56·77
11	17 56 23·1	38·35	17 56 25·6	15 51·8	3 48·87	0·076	3 16 53·33
12	18 11 34·5	37·59	18 11 36·9	15 51·6	3 50·39	0·051	3 20 49·89
13	18 26 27·6	36·83	18 26 30·0	15 51·4	3 51·31	0·026	3 24 46·44
14	18 41 2·2	36·05	18 41 4·6	15 51·2	3 51·65	0·002	3 28 43·00
15	18 55 18·1	35·26	18 55 20·3	15 50·9	3 51·39	0·023	3 32 39·55
16	19 9 14·8	34·46	19 9 17·0	15 50·7	3 50·55	0·047	3 36 36·11
17	19 22 52·2	33·65	19 22 54·4	15 50·5	3 49·14	0·071	3 40 32·67
18	19 36 9·9	32·82	19 36 12·0	15 50·4	3 47·15	0·094	3 44 29·22
19	19 49 7·7	31·99	19 49 9·7	15 50·2	3 44·60	0·117	3 48 25·78
20	20 1 45·2	31·14	20 1 47·1	15 50·0	3 41·51	0·140	3 52 22·34
21	20 14 2·3	30·28	20 14 4·1	15 49·8	3 37·88	0·162	3 56 18·89
22	20 25 58·6	29·41	20 26 0·4	15 49·7	3 33·72	0·184	4 0 15·45
23	20 37 33·9	28·53	20 37 35·6	15 49·5	3 29·05	0·205	4 4 12·01
24	20 48 48·0	27·64	20 48 49·6	15 49·3	3 23·88	0·226	4 8 8·56
25	20 59 40·5	26·74	20 59 42·0	15 49·2	3 18·22	0·246	4 12 5·12
26	21 10 11·3	25·83	21 10 12·7	15 49·0	3 12·08	0·265	4 16 1·68
27	21 20 20·2	24·91	21 20 21·5	15 48·9	3 5·48	0·284	4 19 58·23
28	21 30 6·9	23·98	21 30 8·1	15 48·7	2 58·48	0·303	4 23 54·79
29	21 39 31·2	23·04	21 39 32·2	15 48·6	2 50·93	0·321	4 27 51·35
30	21 48 32·9	22·10	21 48 33·8	15 48·4	2 43·01	0·339	4 31 47·91
May 31	21 57 11·8	21·14	21 57 12·7	15 48·3	+ 2 34·67	0·356	4 35 44·46

Apparent Noon. (N. A. p. I.)			Mean Noon. (N. A. p. II.)				
Date.	Declination.	Var. in 1 hour.	Declination.	Sun's semi-diameter.	Equation of time.	Var. in 1 hour.	Sidereal time.
	° ′ ″	″	° ′ ″	′ ″	m. s.	s.	h. m. s.
June 1	N. 22 5 27·8	20·19	N. 22 5 28·6	15 48·2	+2 25·93	0·372	4 39 41·02
2	22 13 20·7	19·22	22 13 21·5	15 48·0	2 16·80	0·388	4 43 37·58
3	22 20 50·4	18·25	22 20 51·1	15 47·9	2 7·29	0·401	4 47 34·14
4	22 27 56·8	17·27	22 27 57·4	15 47·8	1 57·41	0·419	4 51 30·70
5	22 34 39·6	16·29	22 34 40·1	15 47·7	1 47·18	0.433	4 55 27·25
6	22 40 58·8	15·30	22 40 59·2	15 47·5	1 36·61	0·447	4 59 23·81
7	22 46 54·2	14·31	22 46 54·5	15 47·4	1 25·71	0·460	5 3 20·37
8	22 52 25·7	13·31	22 52 26·0	15 47·3	1 14·51	0·472	5 7 16·93
9	22 57 33·2	12·31	22 57 33·4	15 47·2	1 3·03	0·484	5 11 13·48
10	23 2 16·5	11·30	23 2 16·7	15 47·1	0 51·28	0·495	5 15 10·04
11	23 6 35·5	10·29	23 6 35·6	15 47·0	0 39·28	0·505	5 19 6·60
12	23 10 30·2	9·27	23 10 30·3	15 46·9	0 27·05	0·514	5 23 3·16
13	23 14 0·4	8·25	23 14 0·5	15 46·8	0 14·62	0·521	5 26 59·71
14	23 17 6·1	7·22	23 17 6·1	15 46·7	+0 2·02	0·528	5 30 56·27
15	23 19 47·1	6·20	23 19 47·1	15 46·6	−0 10·74	0·534	5 34 52·83
16	23 22 3·5	5·17	23 22 3·5	15 46·5	0 23·62	0·539	5 38 49·39
17	23 23 55·2	4·14	23 23 55·1	15 46·5	0 36·60	0·542	5 42 45·95
18	23 25 22·0	3·10	23 25 22·0	15 46·4	0 49·64	0·544	5 46 42·50
19	23 26 24·1	2·07	23 26 24·1	15 46·4	1 2·73	0·546	5 50 39·06
20	23 27 1·3	1·03	23 27 1·3	15 46·3	1 15·83	0·545	5 54 35·62
21	23 27 13·7	0·00	23 27 13·7	15 46·3	1 28·91	0·544	5 58 32·18
22	23 27 1·3	1·04	23 27 1·3	15 46·2	1 41·94	0·542	6 2 28·74
23	23 26 24·0	2·07	23 26 24·1	15 46·2	1 54·91	0·539	6 6 25·30
24	23 25 22·0	3·10	23 25 22·1	15 46·1	2 7·79	0·534	6 10 21·85
25	23 23 55·1	4·14	23 23 55·3	15 46·1	2 20·55	0·529	6 14 18·41
26	23 22 3·5	5·16	23 22 3·8	15 46·1	2 33·16	0·522	6 18 14·97
27	23 19 47·2	6·19	23 19 47·5	15 46 1	2 45·61	0·515	6 22 11·53
28	23 17 6·4	7·21	23 17 6·7	15 46·0	2 57·88	0·507	6 26 8·09
29	23 14 1·0	8·23	23 14 1·5	15 46·0	3 9·94	0·498	6 30 4·64
June 30	23 10 31·2	9·25	23 10 31·7	15 46·0	−3 21·78	0·488	6 34 1·20
July 1	N. 23 6 37·1	10·26	N. 23 6 37·7	15 46·0	−3 33·38	0·478	6 37 57·76
2	23 2 18·7	11·27	23 2 19·4	15 46·0	3 44·71	0·466	6 41 54·32
3	22 57 36·3	12·27	22 57 37·1	15 46·0	3 55·76	0·454	6 45 50·88
4	22 52 29·9	13·26	22 52 30·8	15 46·0	4 6·52	0·442	6 49 47·43
5	22 46 59·7	14·25	22 47 0·7	15 46·0	4 16·97	0·429	6 53 43·99
6	22 41 5·7	15·24	22 41 6·9	15 46·0	4 27·09	0·415	6 57 40·55
7	22 34 48·1	16·22	22 34 49·4	15 46·0	4 36·87	0·400	7 1 37·11
8	22 28 7·1	17·19	22 28 8·5	15 46·0	4 46·30	0·385	7 5 33·66
9	22 21 2·8	18·16	22 21 4·3	15 46·1	4 55·36	0·369	7 9 30·22
10	22 13 35·2	19·13	22 13 36·8	15 46·1	5 4·03	0·353	7 13 26·78
11	22 5 44·7	20·08	22 5 46·4	15 46·1	5 12·29	0·335	7 17 23·34
12	21 57 31·4	21·03	21 57 33·2	15 46·2	5 20·13	0·317	7 21 19·89
13	21 48 55·4	21·97	21 48 57·4	15 46·2	5 27·53	0·299	7 25 16·45
14	21 39 57·0	22·90	21 39 59·2	15 46·3	5 34·46	0·279	7 29 13 01
15	21 30 36·4	23·82	21 30 38·7	15 46·3	5 40·92	0·259	7 33 9·57
16	21 20 53·8	24·73	21 20 56·2	15 46·4	5 46·88	0·238	7 37 6·12
17	21 10 49·4	25·63	21 10 52·0	15 46·4	5 52·32	0·216	7 41 2·68
18	21 0 23·5	26·52	21 0 26·2	15 46·5	5 57·24	0·193	7 44 59·24
19	20 49 36·3	27·41	20 49 39·0	15 46·5	6 1·61	0·171	7 48 55·80
20	20 38 27·9	28·28	20 38 30·7	15 46·6	6 5·43	0·147	7 52 52·35
21	20 26 58·7	29·15	20 27 1·7	15 46·7	6 8·68	0·123	7 56 48·91
22	20 15 8·9	30·00	20 15 12·0	15 46·8	6 11·34	0·099	8 0 45·47
23	20 2 58·8	30·84	20 3 2·0	15 46·9	6 13·42	0·074	8 4 42·02
24	19 50 28·6	31·67	19 50 31·9	15 47·0	6 14·90	0·049	8 8 38·58
25	19 37 38·6	32·49	19 37 42·0	15 47·1	6 15·77	0·024	8 12 35·14
26	19 24 29·0	33·30	19 24 32·5	15 47·2	6 16·03	0·002	8 16 31·69
27	19 11 0·2	34·09	19 11 3·8	15 47·3	6 15·69	0·027	8 20 28·25
28	18 57 12·5	34·88	18 57 16·1	15 47·4	6 14·74	0·053	8 24 24·81
29	18 43 6·1	35·65	18 43 9·7	15 47·5	6 13·17	0·079	8 28 21·36
30	18 28 41·3	36·41	18 28 45·0	15 47·7	6 10·98	0·104	8 32 17·92
July 31	18 13 58·3	37·16	18 14 2·1	15 47·8	−6 8·18	0·129	8 36 14·48

Apparent Noon. (N.A. p.I.)			Mean Noon. (N. A. p. II.)				
DATE.	DECLINATION.	VAR. IN 1 HOUR.	DECLINATION.	SUN'S SEMI-DIAMETER.	EQUATION OF TIME.	VAR. IN 1 HOUR.	SIDEREAL TIME.
	° ′ ″	″	° ′ ″	′ ″	m. s.	s.	h. m. s.
Aug. 1	N. 17 58 57·5	37·90	N. 17 59 1·4	15 47·9	− 6 4·77	0·155	8 40 11·03
2	17 43 39·2	38·62	17 43 43·0	15 48·0	6 0·76	0·180	8 44 7·59
3	17 28 3·6	39·34	17 28 7·4	15 48·2	5 56·15	0·205	8 48 4·14
4	17 12 10·9	40·05	17 12 14·8	15 48·3	5 50·95	0·229	8 52 0·70
5	16 56 1·4	40·74	16 56 5·3	15 48·4	5 45·16	0·253	8 55 57·26
6	16 39 35·4	41·42	16 39 39·3	15 48·6	5 38·80	0·277	8 59 53·81
7	16 22 53·2	42·09	16 22 57·1	15 48·7	5 31·88	0·300	9 3 50·37
8	16 5 55·1	42·75	16 5 58·9	15 48·9	5 24·39	0·324	9 7 46·92
9	15 48 41·3	43·39	15 48 45·1	15 49·0	5 16·33	0·347	9 11 43·48
10	15 31 12·3	44·02	15 31 16·1	15 49·2	5 7·72	0·370	9 15 40·03
11	15 13 28·3	44·64	15 13 32·0	15 49·3	4 58·56	0·394	9 19 36·59
12	14 55 29·6	45·25	14 55 33·2	15 49·5	4 48·84	0·416	9 23 33·14
13	14 37 16·5	45·84	14 37 20·1	15 49·7	4 38·57	0·439	9 27 29·70
14	14 18 49·4	46·41	14 18 52·9	15 49·8	4 27·76	0·461	9 31 26·25
15	14 0 8·6	46·98	14 0 12·0	15 50·0	4 16·42	0·484	9 35 22·81
16	13 41 14·4	47·53	13 41 17·7	15 50·2	4 4·55	0·506	9 39 19·36
17	13 22 7·2	48·07	13 22 10·3	15 50·4	3 52·15	0·527	9 43 15·92
18	13 2 47·2	48·59	13 2 50·2	15 50·6	3 39·24	0·549	9 47 12·47
19	12 43 14·8	49·10	12 43 17·6	15 50·7	3 25·82	0·569	9 51 9·03
20	12 23 30·3	49·60	12 23 33·0	15 50·9	3 11·90	0·590	9 55 5·58
21	12 3 34·1	50·08	12 3 36·6	15 51·1	2 57·50	0·610	9 59 2·14
22	11 43 26·5	50·55	11 43 28·8	15 51·4	2 42·62	0·630	10 2 58·69
23	11 23 7·8	51·00	11 23 9·9	15 51·6	2 27·28	0·648	10 6 55·24
24	11 2 38·3	51·44	11 2 40·2	15 51·8	2 11·49	0·667	10 10 51·80
25	10 41 58·5	51·87	10 42 0·1	15 52·0	1 55·27	0·685	10 14 48·35
26	10 21 8·5	52·28	10 21 9·9	15 52·2	1 38·62	0·702	10 18 44·91
27	10 0 8·8	52·68	10 0 10·0	15 52·4	1 21·57	0·719	10 22 41·46
28	9 38 59·7	53·07	9 39 0·7	15 52·7	1 4·12	0·734	10 26 38·01
29	9 17 41·5	53·44	9 17 42·2	15 52·9	0 46·31	0·749	10 30 34·57
30	8 56 14·4	53·81	8 56 14·9	15 53·1	0 28·15	0·764	10 34 31·12
Aug. 31	8 34 38·8	54·15	8 34 39·0	15 53·3	− 0 9·66	0·776	10 38 27·68
Sept. 1	N. 8 12 55·0	54·49	N. 8 12 54·8	15 53·5	+ 0 9·14	0·789	10 42 24·23
2	7 51 3·1	54·82	7 51 2·7	15 53·8	0 28·21	0·800	10 46 20·78
3	7 29 3·6	55·13	7 29 2·9	15 54·0	0 47·54	0·811	10 50 17·34
4	7 6 56·7	55·44	7 6 55·6	15 54·2	1 7·11	0·820	10 54 13·89
5	6 44 42·7	55·73	6 44 41·3	15 54·5	1 26·90	0·829	10 58 10·44
6	6 22 21·9	56·00	6 22 20·3	15 54·7	1 46·90	0·837	11 2 7·00
7	5 59 54·7	56·26	5 59 52·8	15 55·0	2 7·09	0·844	11 6 3·55
8	5 37 21·5	56·50	5 37 19·2	15 55·2	2 27·44	0·851	11 10 0·10
9	5 14 42·5	56·74	5 14 39·8	15 55·4	2 47·95	0·857	11 13 56·66
10	4 51 58·0	56·96	4 51 55·0	15 55·7	3 8·60	0·863	11 17 53·21
11	4 29 8·5	57·16	4 29 5·1	15 55·9	3 29·37	0·867	11 21 49·76
12	4 6 14·2	57·35	4 6 10·5	15 56·2	3 50·24	0·871	11 25 46·32
13	3 43 15·5	57·53	3 43 11·5	15 56·4	4 11·20	0·875	11 29 42·87
14	3 20 12·8	57·69	3 20 8·5	15 56·7	4 32·24	0·878	11 33 39·42
15	2 57 6·4	57·84	2 57 1·7	15 57·0	4 53·33	0·879	11 37 35·98
16	2 33 56·6	57·97	2 33 51·6	15 57·2	5 14·45	0·880	11 41 32·53
17	2 10 43·8	58·09	2 10 38·4	15 57·5	5 35·60	0·881	11 45 29·08
18	1 47 28·4	58·19	1 47 22·6	15 57·8	5 56·74	0·880	11 49 25·63
19	1 24 10·7	58·28	1 24 4·5	15 58·0	6 17·87	0·879	11 53 22·19
20	1 0 51·0	58·35	1 0 44·5	15 58·3	6 38·96	0·877	11 57 18·74
21	0 37 29·7	58·41	0 37 22·9	15 58·6	7 0·01	0·875	12 1 15·29
22	N. 0 14 7·2	58·45	N. 0 14 0·1	15 58·8	7 20·98	0·872	12 5 11·85
23	S. 0 9 16·1	58·48	S. 0 9 23·6	15 59·1	7 41·86	0·868	12 9 8·40
24	0 32 40·0	58·50	0 32 47·8	15 59·4	8 2·63	0·863	12 13 4·95
25	0 56 4·0	58·50	0 56 12·2	15 59·7	8 23·27	0·856	12 17 1·50
26	1 19 27·8	58·48	1 19 36·1	15 59·9	8 43·76	0·850	12 20 58·06
27	1 42 51·2	58·46	1 43 0·1	16 0·2	9 4·06	0·842	12 24 54·61
28	2 6 13·8	58·42	2 6 22·9	16 0·5	9 24·16	0·833	12 28 51·16
29	2 29 35·3	58·36	2 29 44·7	16 0·8	9 44·03	0·822	12 32 47·72
Sept. 30	2 52 55·3	58·30	2 53 5·1	16 1·0	+10 3·63	0·811	12 36 44·27

Apparent Noon. (N. A. p. I.)			Mean Noon. (N. A. p. II.)				
DATE.	DECLINATION.	VAR. IN 1 HOUR.	DECLINATION.	SUN'S SEMI-DIAMETER.	EQUATION OF TIME.	VAR. IN 1 HOUR.	SIDEREAL TIME.
	° ′ ″	″	° ′ ″	′ ″	m. s.	s.	h. m. s.
Oct. 1	S. 3 16 13·6	58·22	S. 3 16 23·7	16 1·3	+10 22·95	0·799	12 40 40·82
2	3 39 29·9	58·13	3 39 40·3	16 1·6	10 41·97	0·785	12 44 37·38
3	4 2 43·8	58·02	4 2 54·5	16 1·9	11 0·65	0·771	12 48 33·93
4	4 25 55·0	57·90	4 26 6·0	16 2·1	11 18·98	0·756	12 52 30·48
5	4 49 3·1	57·77	4 49 14·3	16 2·4	11 36·94	0·740	12 56 27·03
6	5 12 7·8	57·61	5 12 19·2	16 2·7	11 54·51	0·724	13 0 23·59
7	5 35 8·6	57·45	5 35 20·3	16 2·9	12 11·67	0·706	13 4 20·14
8	5 58 5·2	57·26	5 58 17·1	16 3·2	12 28·41	0·688	13 8 16·69
9	6 20 57·3	57·07	6 21 9·4	16 3·5	12 44·70	0·669	13 12 13·25
10	6 43 44·4	56·85	6 43 56·8	16 3·8	13 0·53	0·650	13 16 9·80
11	7 6 26·2	56·62	7 6 38·8	16 4·0	13 15·83	0·629	13 20 6·35
12	7 29 2·3	56·38	7 29 15·0	16 4·3	13 30·73	0·608	13 24 2·91
13	7 51 32·3	56·11	7 51 45·2	16 4·6	13 45·07	0·586	13 27 59·46
14	8 13 55·8	55·84	8 14 8·8	16 4·9	13 58·87	0·564	13 31 56·01
15	8 36 12·4	55·54	8 36 25·5	16 5·1	14 12·13	0·541	13 35 52·57
16	8 58 21·7	55·23	8 58 34·9	16 5·4	14 24·83	0·517	13 39 49·12
17	9 20 23·2	54·90	9 20 36·6	16 5·7	14 36·95	0·493	13 43 45·68
18	9 42 16·7	54·55	9 42 30·2	16 6·0	14 48·48	0·468	13 47 42·23
19	10 4 1·6	54·19	10 4 15·2	16 6·2	14 59·42	0·443	13 51 38·78
20	10 25 37·6	53·80	10 25 51·2	16 6·5	15 9·74	0·417	13 55 35·34
21	10 47 4·2	53·40	10 47 17·8	16 6·8	15 19·43	0·391	13 59 31·89
22	11 8 21·0	52·99	11 8 34·7	16 7·1	15 28·49	0·364	14 3 28·45
23	11 29 27·7	52·56	11 29 41·4	16 7·3	15 36·89	0·336	14 7 25·00
24	11 50 23·8	52·11	11 50 37·4	16 7·6	15 44·61	0·307	14 11 21·55
25	12 11 8·9	51·65	12 11 22·6	16 7·9	15 51·64	0·278	14 15 18·11
26	12 31 42·8	51·17	12 31 56·4	16 8·1	15 57·96	0·249	14 19 14·66
27	12 52 4·9	50·67	12 52 18·5	16 8·4	16 3·55	0·218	14 23 11·22
28	13 12 15·1	50·16	13 12 28·6	16 8·7	16 8·39	0·186	14 27 7·77
29	13 32 12·8	49·64	13 32 26·2	16 8·9	16 12·47	0·154	14 31 4·33
30	13 51 57·7	49·10	13 52 11·0	16 9·2	16 15·76	0·121	14 35 0·88
Oct. 31	14 11 29·5	48·54	14 11 42·7	16 9·4	+16 18·27	0·088	14 38 57·44
Nov. 1	S. 14 30 47·7	47·97	S. 14 31 0·8	16 9·6	+16 19·97	0·054	14 42 53·99
2	14 49 52·0	47·38	14 50 4·9	16 9·9	16 20·86	0·020	14 46 50·55
3	15 8 41·9	46·77	15 8 54·6	16 10·1	16 20·92	0·014	14 50 47·10
4	15 27 17·0	46·15	15 27 29·6	16 10·4	16 20·15	0·049	14 54 43·66
5	15 45 37·0	45·51	15 45 49·4	16 10·6	16 18·54	0·084	14 58 40·21
6	16 3 41·4	44·85	16 3 53·5	16 10·8	16 16·09	0·120	15 2 36·77
7	16 21 29·7	44·17	16 21 41·7	16 11·1	16 12·78	0·155	15 6 33·32
8	16 39 1·7	43·48	16 39 13·4	16 11·3	16 8·62	0·191	15 10 29·88
9	16 56 16·8	42·77	16 56 28·2	16 11·5	16 3·60	0·227	15 14 26·44
10	17 13 14·7	42·04	17 13 25·8	16 11·7	15 57·72	0·263	15 18 22·99
11	17 29 54·9	41·30	17 30 5·7	16 12·0	15 50·98	0·298	15 22 19·55
12	17 46 17·0	40·54	17 46 27·6	16 12·2	15 43·39	0·334	15 26 16·10
13	18 2 20·6	39·76	18 2 30·9	16 12·4	15 34·94	0·369	15 30 12·66
14	18 18 5·4	38·96	18 18 15·4	16 12·6	15 25·64	0·404	15 34 9·22
15	18 33 30·8	38·15	18 33 40·5	16 12·8	15 15·51	0·439	15 38 5·77
16	18 48 36·5	37·32	18 48 45·8	16 13·1	15 4·54	0·474	15 42 2·33
17	19 3 22·0	36·47	19 3 31·1	16 13·3	14 52·76	0·508	15 45 58·89
18	19 17 47·0	35·61	19 17 55·7	16 13·5	14 40·16	0·541	15 49 55·44
19	19 31 51·1	34·73	19 31 59·4	16 13·7	14 26·76	0·574	15 53 52·00
20	19 45 33·8	33·83	19 45 41·8	16 13·9	14 12·57	0·607	15 57 48·56
21	19 58 54·9	32·92	19 59 2·5	16 14·1	13 57·59	0·640	16 1 45·11
22	20 11 53·9	31·99	20 12 1·2	16 14·3	13 41·83	0·673	16 5 41·67
23	20 24 30·5	31·05	20 24 37·5	16 14·5	13 25·30	0·705	16 9 38·23
24	20 36 44·5	30·10	20 36 51·1	16 14·7	13 8·00	0·736	16 13 34·78
25	20 48 35·4	29·14	20 48 41·6	16 14·8	12 49·94	0·768	16 17 31·34
26	21 0 3·0	28·16	21 0 8·9	16 15·0	12 31·14	0·799	16 21 27·90
27	21 11 7·0	27·17	21 11 12·5	16 15·2	12 11·61	0·829	16 25 24·46
28	21 21 47·1	26·16	21 21 52·2	16 15·4	11 51·35	0·859	16 29 21·01
29	21 32 2·9	25·15	21 32 7·7	16 15·5	11 30·38	0·888	16 33 17·57
Nov. 30	21 41 54·2	24·12	21 41 58·7	16 15·7	+11 8·73	0·916	16 37 14·13

Apparent Noon. (N.A. p. I.)			Mean Noon. (N. A. p. II.)				
Date.	Declination.	Var. in 1 hour.	Declination.	Sun's semi-diameter.	Equation of time.	Var. in 1 hour.	Sidereal time.
	° ′ ″	″	° ′ ″	′ ″	m. s.	s.	h. m. s.
Dec. 1	S. 21 51 20·7	23·08	S. 21 51 24·8	16 15·8	+10 46·41	0·944	16 41 10·69
2	22 0 22·1	22·03	22 0 25·9	16 15·9	10 23·43	0.970	16 45 7·24
3	22 8 58·1	20·96	22 9 1·6	16 16·1	9 59·82	0·996	16 49 3·80
4	22 17 8·4	19·89	22 17 11·6	16 16·2	9 35·60	1·021	16 53 0.36
5	22 24 52·9	18·81	22 24 55·8	16 16·3	9 10·80	1·045	16 56 56·92
6	22 32 11·2	17·71	22 32 13·7	16 16·5	8 45·44	1·068	17 0 53·47
7	22 39 3·0	16·60	22 39 5·3	16 16·6	8 19·54	1·090	17 4 50·03
8	22 45 28·2	15·49	22 45 30·2	16 16·7	7 53·13	1·111	17 8 46·59
9	22 51 26·6	14·37	22 51 28·4	16 16·8	7 26·24	1·130	17 12 43·15
10	22 56 57·9	13·24	22 56 59·5	16 16·9	6 58·90	1·148	17 16 39·71
11	23 2 2·1	12·10	23 2 3·4	16 17·0	6 31·14	1·165	17 20 36·27
12	23 6 38·8	10·95	23 6 39·9	16 17·1	6 3·01	1·180	17 24 32·82
13	23 10 47·9	9·80	23 10 48·8	16 17·2	5 34·53	1·193	17 28 29·38
14	23 14 29·3	8·64	23 14 30·0	16 17·3	5 5·75	1·205	17 32 25·94
15	23 17 42·8	7·48	23 17 43·4	16 17·4	4 36·71	1·215	17 36 22·50
16	23 20 28·4	6·31	23 20 28·8	16 17·5	4 7·44	1·224	17 40 19·06
17	23 22 45·8	5·14	23 22 46·1	16 17·6	3 37·98	1·231	17 44 15·62
18	23 24 35·1	3·96	23 24 35·3	16 17·7	3 8·37	1·237	17 48 12·17
19	23 25 56·1	2·78	23 25 56·2	16 17·8	2 38·64	1·241	17 52 8·73
20	23 26 48·7	1·60	23 26 48·8	16 17·8	2 8·82	1·244	17 56 5·29
21	23 27 13·1	0·43	23 27 13·1	16 17·9	1 38·91	1·246	18 0 1·85
22	23 27 9·2	0·75	23 27 9·1	16 18·0	1 9·03	1·247	18 3 58·41
23	23 26 36·9	1·93	23 26 36·9	16 18·0	0 39·12	1·246	18 7 54·97
24	23 25 36·4	3·11	23 25 36·4	16 18·1	+ 0 9·25	1·244	18 11 51·52
25	23 24 7·6	4·29	23 24 7·6	16 18·1	− 0 20·56	1·240	18 15 48·08
26	23 22 10·6	5·46	23 22 10·6	16 18·1	0 50·27	1·236	18 19 44·64
27	23 19 45·4	6·63	23 19 45·5	16 18·2	1 19·85	1·230	18 23 41·20
28	23 16 52·2	7·80	23 16 52·4	16 18·2	1 49·27	1·223	18 27 37·76
29	23 13 30·9	8·97	23 13 31·3	16 18·2	2 18·50	1·214	18 31 34·31
30	23 9 41·8	10·13	23 9 42·3	16 18·2	2 47·51	1·204	18 35 30·97
Dec. 31	23 5 24·9	11·28	23 5 25·5	16 18·2	− 3 16·27	1·193	18 39 27·43

THE SUN'S RIGHT ASCENSION.

Apparent Noon.			Mean Noon.
Date.	Right Ascension.	Var. in 1 hour.	Right Ascension.
	h. m. s.	s.	h. m. s.
Jan. 11	19 32 2·75	10·838	19 32 1·27
Feb. 23	22 26 53·88	9·498	22 26 51·73
Mar. 14	23 37 33·44	9·151	23 37 32·02
15	23 41 12·94	9·141	23 41 11·57
Apr. 12	1 23 18·61	9·198	1 23 18·49
May 21	3 52 40·41	10·019	3 52 41·01
26	4 12 49·06	10·123	4 12 49·60
27	4 16 52·23	10·142	4 16 52·76
Sept. 6	11 0 19·83	9·017	11 0 20·10
7	11 3 56·15	9·010	11 3 56·46
Oct. 5	12 44 48·33	9·114	12 44 50·09
20	13 40 23·21	9·438	13 40 25·60
Dec. 3	16 39 2·17	10·856	16 39 3·98
11	17 14 3·93	11·024	17 14 5·12

THE MOON'S RIGHT ASCENSION AND DECLINATION.

DATE.			RIGHT ASCENSION.			VAR. IN 10 MIN.	DECLINATION.				VAR. IN 10 MIN.
	d.	h.	h.	m.	s.	s.	°	′	″		″
Jan.	27	8	2	27	14·68	19·929	10	4	29·4	N.	116·81
		9	2	29	14·26	19·930	10	16	8·5		116·22
Feb.	6	4	10	37	11·26	19·651	13	40	33·9	N.	102·31
		5	10	39	9·11	19·632	13	30	18·2		102·93
	28	3	6	10	11·00	21·515	23	35	38·4	N.	21·01
		4	6	12	20·10	21·519	23	37	41·1		19·89
Mar.	8	6	12	48	26·25	19·500	0	7	36·0	N.	130·28
		7	12	50	23·29	19·513	0	5	26·0	S.	130·37
	14	20	18	43	24·48	25·974	24	2	57·3	S.	4·02
		21	18	46	0·38	25·993	24	3	16·5		2·38
Apr.	5	18	13	46	5·96	20·449	6	19	21·0	S.	132·43
		19	13	48	8·75	20·483	6	32	35·0		132·23
	12	8	20	2	3·42	25·496	23	14	19·3	S.	45·35
		9	20	4	36·32	25·470	23	9	42·5		46·90
	30	0	11	19	5·03	19·394	9	56	39·6	N.	118·25
		1	11	21	1·38	19·390	9	44	48·5		118·78
May	5	9	15	45	48·34	23·503	17	32	13·3	S.	105·08
		10	15	48	9·54	23·563	17	42	40·9		104·11
	26	19	10	53	20·26	19·242	12	36	58·2	N.	109·35
		20	10	55	15·67	19·228	12	26	0·3		109·94
June	25	5	12	28	24·66	18·861	2	33	34·3	N.	128·40
		6	12	30	17·86	18·874	2	20	43·2		128·63
	28	15	15	14	25·14	22·467	14	59	39·8	S.	116·38
		16	15	16	40·16	22·540	15	11	16·1		115·71
Aug.	7	2	2	53	7·88	20·758	13	16	39·1	N.	115·88
		3	2	55	12·45	20·765	13	28	12·1		115·11
Sept.	22	5	18	43	9·58	25·814	24	56	52·5	S.	2·53
		6	18	45	44·54	25·840	24	57	2·7		0·88
	28	0	0	24	58·69	22·230	2	31	38·6	S.	154·46
		1	0	27	12·00	22·206	2	16	11·7		154·51
Oct.	20	5	19	30	18·15	25·593	24	47	42·4	S.	27·85
		6	19	32	51·69	25·586	24	44	50·4		29·50
Nov.	19	22	22	53	14·33	22·341	12	46	24·1	S.	132·35
		23	22	55	28·25	22·299	12	33	7·9		133·03
Dec.	16	13	22	20	3·92	23·363	15	53	3·5	S.	122·68
		14	22	22	23·92	23·303	15	40	44·8		123·54
	18	10	23	59	57·90	21·250	5	36	23·4	S.	146·24
		11	0	2	5·31	21·219	5	21	45·4		146·43
	19	7	0	44	1·13	20·763	0	27	2·6	S.	147·44
		8	0	46	5·66	20·748	0	12	18·2		147·36

THE MOON'S UPPER MERIDIAN PASSAGE.

DATE.		MER. PASS	
		h.	m.
Jan.	25	4	23·4
	26	5	9·8
	27	5	55·3
	28	6	40·7
	29	7	26·7
	30	8	13·9
Feb.	1	9	51·3
	2	10	40·6
	27	6	57·0
	28	7	46·2
Mar.	7	13	13·1
	8	13	57·1
Apr.	5	12	38·2
	6	13	25·4
	9	16	5·2
	10	17	4·4
June	28	8	32·5
	29	9	25·5
Sept.	27	11	30·2
	28	12	21·4
Oct.	4	17	26·7
	5	18	17·8
Dec.	19	6	50·4
	20	7	37·6

LUNAR DISTANCES.

Sun or Star's Name and Position		Date	d	h	Distance °	′	″	P.L. of Diff.
Spica	E.	Feb.	6	0	48	58	41	3013
				3	47	28	43	3009
				6	45	58	42	3006
Sun	E.	Mar.	14	15	77	33	10	2613
				18	75	54	33	2605
				21	74	15	45	2596
Sun	E.	Apr.	12	3	87	44	54	2615
				6	86	6	19	2613
				9	84	27	42	2612
Sun	W.	May	26	15	92	2	46	3347
				18	93	26	3	3337
				21	94	49	32	3327
Mars	E.	June	25	0	56	10	7	2861
				3	54	36	58	2850
				6	53	3	35	2838
Fomalhaut	W.	Aug.	6	21	69	56	24	2852
			7	0	71	29	44	2865
				3	73	2	48	2877
Venus	W.	Sept.	22	0	50	57	17	2698
				3	52	33	59	2683
				6	54	11	1	2669
Sun	W.	Oct.	20	0	80	28	23	2685
				3	82	5	23	2678
				6	83	42	32	2670
Altair	W.	Nov.	19	18	49	37	57	3819
				21	50	52	40	3746
				24	52	8	39	3678
Markab	E.	Dec.	16	9	33	31	50	4154
				12	32	22	40	4368
				15	31	16	50	4617
Aldebaran	E.	Dec.	18	6	72	29	15	2360
				9	70	44	43	2368
				12	69	0	23	2377

THE MOON'S SEMI-DIAMETER AND HORIZONTAL PARALLAX.

Date.		Semi-Diameter. Noon. ′ ″	Semi-Diameter. Midnight. ′ ″	Horizontal Parallax. Noon. ′ ″	Horizontal Parallax. Midnight. ′ ″
Jan.	25	15 53·7	15 45·7	58 14·0	57 44·9
	27	15 23·5	15 17·0	56 23·5	55 59·7
Feb.	6	14 55·1	14 57·9	54 39·5	54 49·7
	28	14 49·1	14 47·4	54 17·4	54 11·1
Mar.	8	15 17·7	15 21·7	56 2·3	56 16·7
	14	16 6·9	16 10·7	59 2·6	59 16·2
	15	16 14·0	16 17·1	59 28·6	59 39·7
Apr.	5	15 30·0	15 34·5	56 47·1	57 3·7
	6	15 38·9	15 43·1	57 19·7	57 35·1
	12	16 10·4	16 11·1	59 15·2	59 17·7
	30	15 7·5	15 12·4	55 24·7	55 42·9
May	26	14 52·4	14 55·5	54 29·5	54 40·7
	27	14 59·2	15 3·6	54 54·4	55 10·5
June	25	15 7·6	15 13·2	55 25·0	55 45·8
	28	15 48·5	15 56·3	57 55·2	58 23·7
	29	16 4·0	16 11·4	58 51·7	59 18·8
Aug.	7	15 31·2	15 24·4	56 51·6	56 26·9
	27	16 34·0	16 38·0	60 41·9	60 56·3
Sept.	22	16 4·6	16 9·9	58 54·1	59 13·3
	28	16 25·3	16 21·0	60 9·9	59 54·0
Oct.	20	16 3·2	16 6·3	58 49·0	59 0·1
Nov.	19	16 9·8	16 9·2	59 13·2	59 10·9
	20	16 8·3	16 7·1	59 7·6	59 3·1
Dec.	16	16 19·1	16 16·8	59 47·0	59 38·6
	17	16 14·0	16 10·8	59 28·4	59 16·9
	18	16 7·4	16 3·7	59 4·2	58 50·8
	19	15 59·9	15 55·9	58 36·7	58 22·3

THE PLANETS' RIGHT ASCENSION, DECLINATION AND HORIZONTAL PARALLAX.

PLANET.	DATE.	RIGHT ASCENSION AT NOON.	VAR. IN 1 H. OF LONG.	DECLINATION AT NOON.	VAR. IN 1 H. OF LONG.	H.P.
		h. m. s.	s.	° ′ ″	″	″
Venus	Sept. 13	14 14 25·59	+9·88	15 59 28·3 S.	+64·0	11·5
	22	14 49 50·57		19 35 12·6		12·9
	23	14 53 44·91		19 57 17·5		
	Nov. 5	17 7 36·62		28 1 48·5		24·5
	6	17 8 53·58		28 0 55·7		
	Dec. 7					33·1
Mars	June 25	15 42 28·53		22 40 42·6 S.		17·0
	26	15 41 56·99		22 40 30·6		
Jupiter	June 4	20 59 4·85		17 43 32·8 S.		2·0
	5	20 59 1·23		17 44 2·0		
	Nov. 5	20 28 37·23		19 52 9·4		1·7
	6	20 29 6·21		19 50 25·2		
	Dec. 1	20 44 16·47		18 53 42·9		1·6
	2	20 44 59·34		18 50 56·1		
	3	20 45 42·65		18 48 7·1		
	July 4		−1·01			

THE PLANETS' DECLINATION FOR LATITUDE BY MERIDIAN ALTITUDE.

PLANET.	DATE.	DECLINATION AT TRANSIT AT GREENWICH.	VAR. IN 1 H. OF LONG.	H.P.
		° ′ ″	″	″
Venus	Dec. 31	16 49 25·1 S.	− 2·3	24·3
Mars	Feb. 20	18 9 36·8 S.	+16·8	7·5
	July 6	22 45 47·2 S.	+ 2·9	16·0
Jupiter	Sept. 12	20 20 55·7 S.	+ 1·7	2·0
Saturn	Mar. 6	13 21 17·2 N.	+ 4·0	1·1

APPARENT PLACES OF STARS.

DATE.		STAR.	RIGHT ASCENSION.			DECLINATION.			
			h.	m.	s.	°	′	″	
Jan.	11	α Tauri (*Aldebaran*)...............	4	29	36·51	16	17	13·3	N.
Mar.	2				35·85			12·4	
Sept.	28				38·49			23·7	
Nov.	27				39·80			24·6	
Dec.	7				39·91			24·4	
	17				39·93			24·3	
Jan.	21	α Aurigæ (*Capella*)	5	8	33·99	45	53	11·5	N.
	31				33·86			12·6	
Aug.	9				34·03			1·2	
	19				34·43			1·3	
May	1	α Argus (*Canopus*)	6	21	29·13	52	37	84·0	S.
June	10				28·47			73·7	
Jan.	11	α Canis Majoris (*Sirius*)	6	40	18·55	16	33	61·9	S.
	21				18·56			63·9	
Dec.	7				20·81			55·4	
	27				21·12			60·2	
Jan.	1	α Canis Minoris (*Procyon*)	7	33	32·83	5	30	21·7	N.
	11				32·96			20·5	
Feb.	10				33·05			18·0	
Mar.	2				32·89			17·2	
Oct.	18				34·04			25·2	
Jan.	1	β Geminorum (*Pollux*)	7	38	35·30	28	17	26·4	N.
	11				35·46			26·6	
Jan.	1	α Leonis (*Regulus*)	10	2	30·61	12	29	76·7	N.
	11				30·88			75·4	
	21				31·11			74·3	
	31				31·29			73·5	
Mar.	2				31·54			72·6	
April	1				31·40			73·6	
	11				31·30			74·2	
Dec.	7				33·10			64·6	
Mar.	12	α Ursæ Majoris (*Dubhe*)	10	56	58·33	62	20	42·0	N.
	22				58·30			44·7	
July	10				55·36			53·3	
	20				55·17			51·5	
Sept.	28				55·42			30·8	
Dec.	7				58·73			12·5	
June	10	α¹ Crucis..............................	12	20	29·83	62	28	101·0	S.
Jan.	31	α Virginis (*Spica*)	13	19	23·55	10	35	10·7	S.
Feb.	10				23·83			12·6	
Dec.	17				25·24			21·8	
Aug.	9	η Ursæ Majoris (*Benetnasch*)...	13	43	12·49	49	51	56·5	N.
	19				12·27			55·2	
April	21	α Bootis (*Arcturus*)	14	10	39·80	19	44	73·8	N.
Dec.	7				39·22			64·2	
Mar.	2	α² Centauri	14	32	8·95	60	22	31·0	S.
June	30				10·24			60·5	
July	10				9·93			61·6	
Mar.	22	α Scorpii (*Antares*)	16	22	39·93	26	11	13·8	S.
April	1				40·22			14·6	
July	10				41·60			20·1	
Jan.	11	α Lyræ (*Vega*)	18	33	10·52	38	40	48·2	N.
Nov.	17	α Aquilæ (*Altair*)	19	45	25·81	8	34	51·7	N.
	27				25·71			50·5	
June	30	α Gruis	22	1	19·74	47	29	21·0	S.
July	10	α Piscis Australis (*Fomalhaut*)	22	51	35·85	30	12	4·4	S.
	20				36·13			3·8	
	30				36·39			3·6	
Aug.	9				36·61			3·7	
Dec.	17	α Pegasi (*Markab*)	22	59	18·43	14	36	66·0	N.

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